A Probabilistic Design Methodology For Commercial Aircraft Engine Cycle Selection

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ABSTRACT
The objective of this paper is to examine ways in which to implement probabilistic design methods in the aircraft engine preliminary design process. Specifically, the focus is on analytically determining the impact of uncertainty in engine component performance on the overall performance of a notional large commercial transport, particularly the impact on design range, fuel burn, and engine weight. The emphasis is twofold: first is to find ways to reduce the impact of this uncertainty through appropriate engine cycle selections, and second is on finding ways to leverage existing design margin to squeeze more performance out of current technology.

One of the fundamental results shown herein is that uncertainty in component performance has a significant impact on the overall aircraft performance (it is on the same order of magnitude as the impact of the cycle itself). However, this paper shows that uncertainties in component efficiencies, pressure losses, and cooling flow losses do not have a significant influence on the variance of aircraft performance. This paper also shows that the probabilistic method is very useful for formulating direct trades of design margin against performance or other figures of merit such as engine weight, thus enabling the existing design margin to be capitalized upon in the interest of obtaining better system performance.

In terms of a comparison between techniques, one can conclude that the probabilistic approach is inherently more computationally intensive that the deterministic approach. It therefore behooves the designer to choose wisely when setting up the problem in order to avoid unnecessary work. However, a properly formulated probabilistic method provides a much clearer picture of how the various system trades “stack up” against one another and enables the ultimate cycle selection to be analytically determined based on the level of risk that is consistent with program objectives.

NOMENCLATURE

AMV Advanced Mean Value
CDF Cumulative Distribution Function
CDP Compressor Discharge Pressure
DoE Design of Experiments
EPNldB Equiv. Perceived Noise Level (decibels)
FoM Figure of Merit
FPI Fast Probability Integration
FPR Fan Pressure Ratio
HPT High Pressure Turbine
KCP Key Control Parameters
KNP Key Noise Parameters
LP Low Pressure
LPT Low Pressure Turbine
OEW Operating Weight Empty, lb
OPR Overall Pressure Ratio
PQEXT Extraction Ratio (P16/P56, per SAE ARP755B)
\( P_{success} \) Probability of Success
RSE Response Surface Equation
RSM Response Surface Method
SFC Specific Fuel Consumption, hr
TH41 Max Turbine Inlet Temp, °F (per SAE ARP755B)
TOGW Takeoff Gross Weight, lb
\( \Delta P/P \) Pressure Loss, %
\( \mu \) Mean Value
\( \sigma \) Standard Deviation

INTRODUCTION
The focus of this paper is to explore ways in which probabilistic design methods can be applied to the aircraft engine cycle design process in order to account for the uncertainty inherent in preliminary-level component performance estimates. The idea is that benefits can be garnered in two ways: first, probabilistic design techniques can be used to estimate uncertainty in performance of a particular design. Second, probabilistic methods can be used to leverage the design margin available in order to achieve better design performance with the same technology level. This paper will examine each of these aspects in detail as applied to a large commercial engine suitable to power a large (~800,000 lb) commercial transport. The focus of this text is on the development of probabilistic methods suitable for engine cycle selection, and these methods
are subsequently applied to a notional commercial engine/aircraft to illustrate the process and provide some useful results.

Motivation for Probabilistic Cycle Design

At first glance, the business of aircraft engine preliminary design may seem to be quite well-defined and therefore in little need of probabilistic methods. After all, accurate predictions for the performance and weight of engine components as well as the performance of the overall system (at least within a couple percentage points or so) are possible using existing analysis techniques that have been developed over the past several decades. In reality, the cumulative effect of the many uncertainties in engine component performance may stack up to represent a significant uncertainty in the performance of the overall system. This idea is readily apparent in Figure 1 which compares cumulative probability distribution functions (CDFs) for aircraft design range of two representative engine cycles. The chart on the right side of this figure depicts the probability of failing to meet a design range target versus design range for two bounding engine cycles in an arbitrarily selected design space. This cycle design space is shown at the top left in the form of normalized ranges for cycle parameters (which the cycle designer can directly control) and a set of distributions for noise parameters (which are uncertain from the cycle designer’s point of view).

Consider first the deterministic case where the impact of uncertainty is ignored. If one varies the engine cycle parameters between the ranges shown in the upper left corner of this figure, the resulting locus of solutions for aircraft design range (at the 50% probability level) spans 4% of the total aircraft design range, as shown at right (i.e.- there is a 4% difference in design range from the best to worst cycles). This is the family of solutions that can be achieved using the nominal “best guess” values for component performance. Next, if one takes the best and worst design range cycles and introduces component performance uncertainty via the distributions shown in the bottom left of Figure 1, the aircraft design range becomes a distribution instead of a point value. The resultant design range distributions can be viewed in the form of CDFs, as plotted to the right. The distance from tail to tail on a given CDF is on the order of 5% of the aircraft design range, meaning that the impact of the combined uncertainty is easily on the same order of magnitude as the impact of the cycle design parameters!

To be fair, it should be pointed out that the probability of achieving a design that is on the extremes of the CDF tails is small. Solutions at the tails represent cases where either “everything came together beautifully” or “nothing went right” and this does not typically occur in an engine program. Furthermore, the relative importance of the cycle and uncertainty effects will depend on the width of the ranges selected for the cycle parameters. Nevertheless, one could reasonably expect variation on the order of 100 nmi on either side of the mean, and in today’s highly competitive marketplace, this is significant enough to warrant further consideration.

Typically, uncertainties in engine performance estimates are accounted for by introducing design margins based on hard-won experience. However, times are changing and there is currently much interest within the aircraft engine industry to apply robust and probabilistic methods. This interest stems from several sources, the most noteworthy are:

- Increased competitive pressures
- Demand for greater safety and higher mean time between failures
- Environmental consciousness
- Maturation of the jet engine and associated technology

These first three items have the combined effect of making the job of engine design more difficult, meaning the design freedom available to the designer is increasingly limited as time goes on. The last bullet, maturation of technology, refers to the fact that the pace of major technology developments has slowed somewhat over the past decade. As progress slows and
constraints become increasingly restrictive (particularly cost, acoustic noise, and emissions), the engine designer must find ways to squeeze every bit of performance from current technologies while simultaneously satisfying all requirements.

Based on this technology maturation argument, it is clear that designers of future engines will likely be required to find ways of obtaining superior performance without having the benefit of major technological advances. The way to accomplish this is by refining current designs (perfecting the trades between weight and specific fuel consumption (SFC), tightening tolerances, eliminating inefficiencies in the engine design and manufacturing processes, etc.) and trimming the design margins while staying within the safety requirements. The major contribution of robust and probabilistic design is to provide an analytical framework which allows the designer to leverage available design margin to improve performance by answering questions such as:

- How much design margin is really necessary?
- How do design parameters impact the uncertainty in performance?
- What can be done to reduce the impact of uncertainty?

Finding answers to these questions is the motivation and justification for introducing probabilistic design methods into the preliminary design process, for it is in the early stages of design where most of the critical decisions are made and where the design freedom available can be leveraged to achieve better performance.

This paper demonstrates a probabilistic design method applied to the engine preliminary design process for a high bypass engine as installed on a 400 passenger notional commercial aircraft configuration. Engine figures of merit (FoMs) such as fan diameter, weight, and SFC are tracked as are mission performance FoMs of the installed engine-aircraft configuration such as design range and fuel burn. Both show the impact of changing the engine cycle parameters of a scaleable, fixed-configuration engine on the performance of a fixed-size, four-engine aircraft.

**PROBABILISTIC DESIGN METHOD**

The approach employed in this paper is to use standard Response Surface Methodology (RSM) in conjunction with the Fast Probability Integration (FPI) method. FPI is an advanced probabilistic analysis method that was developed in the early 90’s at the Southwest Research Institute (SwRI) under contracts from NASA Lewis Research Center and is the latest tool to be added to the growing number of probabilistic analysis tools available to the designer. FPI works by using the actual analysis code and approximating the Monte Carlo analysis, as opposed to the RSE/Monte Carlo method which approximates the code and uses the actual Monte Carlo Analysis. The advantage of FPI is that it is fast and accurate. It typically takes 15 to 20 cases for FPI to compute a CDF for a 7 variable problem using the advanced mean value (AMV) method (used in this paper), which is far fewer than would be required for the pure Monte Carlo method (~10,000 cases) or for the RSE/Monte Carlo method (143 cases for a 7 factor central composite design). Additionally, the distribution is more accurate than the RSE/Monte Carlo method (particularly for problems with highly non-linear responses) because it uses the actual analysis code instead of a quadratic polynomial approximation. In short, the FPI method has both accuracy and speed, which is a very desirable combination of attributes.

The basic steps used in the probabilistic analysis method are shown in Figure 2. The table shown at the top of this figure is representative of the basic setup used for probabilistic analysis in this paper. This table consists of three sections: control factor settings, noise parameter settings, and response values. The leftmost section gives control factor settings and shows that the Key Control Parameters (KCPs) were varied according to a
factored experiment created using Design of Experiments (DoE) to create a set of cases (one case per row). Each of these cases represents a specific engine cycle, different from every other case (thus, each row is a unique engine design).

The Key Noise Parameters (KNPs) are shown in the middle section of the table and are assigned a distribution which does not change from case to case. For a given engine cycle (row), these distributions are used in FPI to compute a resultant response distribution. FPI does this by estimating the perturbations necessary in each uncertainty parameter to achieve a user-specified probability level, and then calling the analysis routines to calculate a response value for that combination of uncertainty parameter settings. This is done repeatedly (once for each p-level), and the response data is then used to construct the CDF. The result of the probabilistic analysis is a series of CDFs, one for each case (row). If one were to superimpose the CDFs generated from the control and noise factor settings given in the table, the result is a collection of CDFs as shown at the lower right of Figure 2. In addition, other responses and constraints are tracked by FPI for later use.

The next step is to use the results of the FPI analysis to construct response surface equations (RSEs) for specific probability levels of design range (for instance, one could construct an RSE for the 90% probability of exceeding the design range target as a function of the cycle parameters). These RSEs are then used to plot contours which depict the design space in a graphical and intuitive way, showing the constraints as well as where the best fuel burn, design range, and engine weight regions are located. In effect, these contours are a “slice” of CDF data at a given probability level (or p-level) with which RSEs representing design performance at that p-level are constructed.

This concept is illustrated in Figure 3 which shows an aggregate CDF plot in the lower right corner. The data at any given p-level (10%, for instance) can be collected and used as the data set for construction of an RSE which can then be plotted as a contour (as for the 10% probability of success contour shown in Figure 3). Thus, each p-level has its own corresponding RSE which can be plotted as shown in the figure. Taken together, these RSEs constitute a set of probability contours. Also, note that the contour plots show fan pressure ratio (FPR) versus extraction ratio at a constant maximum turbine inlet temperature (TH41), but one could easily produce contour plots with any cycle parameter on any combination of axis (FPR vs TH41, for example).

The probability contours are interpreted as shown in Figure 3 where each slice of the CDF corresponds to a contour for probability of failure (to meet design range target) or its compliment, probability of success (Psuccess). Note that the definition of Psuccess or failure is governed by whether the metric is maximized or minimized.

From this point, the problem becomes an exercise of trying to find the best balance between weight, SFC, and probability of meeting the design range target while simultaneously avoiding violation of any constraints (such as limits on fan diameter). The important difference between this approach and the deterministic approach is that the cycle designer can now analytically design the cycle such that it meets all constraints and has a probability of meeting program goals which is consistent with the level of risk tolerance that the program managers are willing to accept. Furthermore, this applies not only to design range, but also to any other response of interest. For instance, it is possible to design for an 80% probability of achieving a fuel burn target, engine weight target, acoustic noise target, shop cost target, etc.
CYCLE DESIGN FOR COMMERCIAL SYSTEMS

The main objective of engine cycle design is to find a good balance between the various requirements in terms of engine weight, fuel burn, and range. For the purposes of this paper, design range receives the preponderance of emphasis, mainly because there is a well-defined target for design range and because the best design range solution is inherently a compromise between engine weight and SFC. In addition, a probabilistic analysis of all FoMs necessary for "real world" cycle design would be too complicated to fit in one paper and would merely detract from the point of this paper, which is to illustrate a method for probabilistic cycle analysis.

The system and engine level figures of merit tracked are given in Table I. These responses include most of the basic metrics for evaluating conceptual aircraft at the system level, as well as basic engine FoMs and constraints. Noise and cost are not included in this paper since appropriate models were not available, but should be considered in the future. Note also that not all of the responses listed in Table I are system FoMs. These additional responses are included primarily as a means to check the soundness of the cases being run and are not included in the results section in the interest of brevity.

Engine Cycle/Mission Analysis Process Flow

The basic analysis flow is shown in Figure 4. First, FPI determines values for the noise parameters according to a set of user-defined input distributions while the control parameters are prescribed according to a design of experiments setup (in this case a 3-factor central composite face-centered design as shown in Figure 5). These values are passed to the case execution routine by the FPI/shell script routine (shown in dashed lines). The shell script then generates the cycle model, calculates aircraft operating empty weight (OEW) based on the calculated engine weight with appropriate pylon structural weight "ripple" effects. Next, aircraft nacelle drag is calculated as a function of fan diameter, and the engine is "flown" on the aircraft to estimate installed performance using an appropriate mission analysis model. Finally, the response data is parsed out of the output file and sent to FPI for probabilistic analysis. FPI repetitively calls this routine in order to get data for CDF generation. The end result is a CDF for the response of interest, be it fuel burn, engine weight, aircraft range, etc.

Baseline Aircraft Configuration

The baseline aircraft for this study is a notional four-engine large commercial transport capable carrying 420 passengers (@210 lb/pax) in a tri-class configuration and is in the 800-900,000 lb gross takeoff weight class. The baseline configuration is set up for a fixed OEW minus propulsion system weight with a variable design range (fuel volume limited aircraft). Baseline horsepower extraction and customer compressor bleed are based on typical customer requirements. As mentioned earlier, appropriate pylon weight "ripple" effects are applied to the baseline configuration to account for the effects of changing engine weight, and a delta on nacelle drag as a

Table I. Engine and Aircraft Figures of Merit

<table>
<thead>
<tr>
<th>Aircraft Performance Metrics:</th>
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</thead>
<tbody>
<tr>
<td>Design Range (nmi)</td>
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<tr>
<td>Fuel Burn for 3,000 and 6,000 nmi missions (lbs fuel consumption)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Engine Performance Metrics:</th>
</tr>
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<tbody>
<tr>
<td>Cruise SFC - 35,000 ft, Mach 0.85 @ 9000 lb thrust per engine</td>
</tr>
<tr>
<td>Fan Diameter (for aircraft ground clearance constraint, in)</td>
</tr>
<tr>
<td>Engine Weight (lbs)</td>
</tr>
<tr>
<td>Noise - EPNLdB (for future work)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Other Attributes:</th>
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</thead>
<tbody>
<tr>
<td>Exhaust Gas Temperature (deg C)</td>
</tr>
<tr>
<td>Core Flow (lbs/sec)</td>
</tr>
<tr>
<td>Bypass Ratio</td>
</tr>
<tr>
<td>OPR</td>
</tr>
<tr>
<td>Number of stages in each engine component</td>
</tr>
<tr>
<td>Aircraft Drag at Mach 0.85, 35000 ft</td>
</tr>
</tbody>
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Figure 4. Analysis Process Flowchart

Figure 5. DoE Analysis Case Setup
function of fan diameter is applied to the aircraft based on a set of regressed data. In addition to typical mission rules and weights at standard day conditions, typical mission profiles for the maximum range design mission and secondary missions (3K & 6K nmi) for a commercial aircraft were provided for use in this study.

**Baseline Engine Cycle**

The baseline engine cycle is based on a current-technology core which has a fixed configuration but is photographically scaleable, while the low pressure (LP) spool configuration is completely variable (but has an assumed fixed technology level, and a 2 stage booster). The model for the baseline engine is calibrated to match the performance capable using current technology, and compressor pressure ratio is fixed with overall pressure ratio (OPR) falling out (all engines ran to a compressor discharge temperature limit). The impact of changes in OPR due to changes in FPR are small due to the relatively narrow range of FPR selected. In addition, TH41 is specified and core size is allowed to fall out for all cycles studied in this paper.

The baseline engine cycle KCPs and KNPs used in this study are presented in Table II. The primary KCPs are FPR and extraction ratio (PQEXT) since they are strong drivers on specific thrust, SFC, and engine weight. Table II also shows that the range selected for PQEXT and TH41 is large, while that for FPR is narrow.

The seven Key Noise Parameters examined in this study were selected from a group of 15 parameters through the use of a screening test which showed these seven to have the greatest impact on variability in range and fuel burn. All noise parameter distributions used in this paper are specified as normal distributions, although, in reality, the shape of these distributions may be different. The reason for this is that the central limit theorem will tend to “smear” any irregularities that are present such that the impact of deviations from the normal will be “washed out”. Thus, this represents a first-pass approximation based on the best available knowledge of the distribution mean and standard deviation. Note that the upper and lower limits for the KNPs are based on a ±2σ variation around the mean value for each parameter, which captures 95.4% of expected uncertainty (meaning that there is a 95.4% chance that the performance of the hardware that goes to test will have a performance that is inside the upper and lower bounds defined here). The baseline values for the KCPs and KNPs are not given due to the proprietary nature of the data, though the range of deviation is shown in Table II. Note that the range selected for PQEXT and TH41 in this paper is fairly wide, while the range for FPR is fairly narrow.

**RESULTS**

The primary system figure of merit of interest for this problem is the vehicle design range for a fixed fuel weight. However, FPI is also applied to other system FoMs such as 3K and 6K fuel burn, fan diameter, and engine weight. Each of these FoMs has 15 cases associated with it, representing 15 distinct engine cycles specified by the 3 factor central composite design of experiments for FPR, TH41, and extraction ratio (PQEXT), as shown in Figure 5.

The resulting design range CDFs for the 15 engine cycles studied in this paper are given in Figure 6. It is clear from this figure that the impact of cycle uncertainty is roughly of the same order of magnitude as the impact of the cycle itself. Note that the changes in variance between cycles are not very significant, as shown by the fact that all of the CDFs in Figure 6 have roughly the same shape and slope. If the variance were different from case to case, the slope of the CDFs would be visibly different between cases. In fact, the relative change in variance over all cases is on the order of 10%. In effect, this indicates that the control (cycle) parameters have a weak impact on design range variance.

![Table II. Engine Cycle Parameter Range Specification](image-url)
The results for 3K and 6K fuel burn mirror those of the design range. Once again, the impact of component performance uncertainty is on the same order of magnitude as the cycle itself, and this is evident in Figure 7 which gives the results for both 3K and 6K missions. Also, there is almost no change in variance from case to case. Note that the 3K and 6K fuel burn results show essentially the same trends with almost no difference between them (except the absolute amount of fuel consumed). Also, the variance of fuel burn is nearly constant with respect to cycle uncertainties. It is interesting to note that there is one case which seems to stand out from the rest, that being the CDF on the far left of these plots. In this case, this effect is caused by the cycle analysis model change in the engine heat transfer design point midway through the CDF construction.

The cumulative distribution functions for fan diameter and engine weight differ markedly from those for range and fuel burn. The difference is evident in the CDFs for a representative set of cases for fan diameter and engine weight, shown in Figure 8. Note that the variance due to component performance uncertainty is far less than the impact of changes in cycle parameters. This result is not entirely unexpected given that component performance generally does not have a first-order effect on either engine weight or fan diameter. Clearly, engine weight and fan diameter are driven primarily by the cycle and not the noise parameters considered here.

An additional benefit of this small variance is the fact that weight and diameter need not be treated as probabilistic responses. It can be concluded that uncertainty in component performance has only a weak effect on weight and diameter, and is therefore unnecessary to expend additional analysis effort to treat these probabilistically for the current problem formulation.

**Cycle Design via Probability Contours**

The main objective of this analysis is to use probabilistic methods to leverage the design margin available to squeeze more performance out of existing technology. The way to do this is through a series of well-informed trades which take into account all considerations which impact the ultimate performance of the design. In point of fact, this is the way engines are designed today, but the lack of knowledge about uncertainty makes it very difficult to take advantage of available design margin because it is difficult to ascertain exactly how much margin is really available. The probabilistic approach described earlier allows these trades to take place through the use of probability contours.

A graphical representation of the probability contours for design range target along with contours of constant 3K fuel burn are shown in Figure 9. The best fuel burn solution is limited by the upper limit on fan diameter, imposed to assure that the engine nacelle has sufficient ground clearance. Thus, solutions lying in the darkened region of the design space have fan diameters larger than the maximum allowable for this aircraft. In the interest of clarity, TH41 is fixed at the value yielding best design range, the reason being that there is insufficient space and its impact on cycle specific thrust and SFC
tends to be weaker than that of FPR and extraction ratio. Note that the region for best design range has a higher FPR and extraction ratio than that for the best fuel burn (lowest SFC). This is exactly the same trend as one would expect to see, except that this time the design range is expressed as the probability of meeting a target rather than an absolute range. Note also that the best fuel burn design is estimated to have only a 30% chance of meeting or exceeding the design range target, whilst the best design range cycle has in excess of 60% probability of success.

It is possible to get a “back of the envelope” feel for the sensitivities by simply examining the spacing of the contours in Figure 9. In the vicinity of the best design range cycle, the probabilistic sensitivities are roughly 5% probability of success per 8.8 nmi range and 5% probability of success per 350 lb of 3K fuel burn. Likewise, Figure 10 can be used to estimate the impact of engine weight on probability of success by determining sensitivities based on the contours. In this case, a change of 5% probability of success is worth about 200 lb of engine weight in the vicinity of the best design range cycle. At first glance, it appears that it might be worthwhile to trade several percent probability of success in order to get a 200 lb reduction in engine weight. This is especially attractive in light of the fact that there is an attendant decrease in engine manufacturing cost when weight is reduced. However, in order to make an educated decision, one must include the acoustic noise because the lower engine weight also has a higher FPR which implies higher acoustic noise levels.

As mentioned earlier, acoustic noise and manufacturing cost were not included in this paper because the analysis tools were not available and the project would have been too complex to have been completed in a timely manner. However, it is easy to see how these aspects could have been included in this analysis had they been available, as illustrated in Figure 11. Since cost is typically highly correlated with weight, one can hypothesize that contours of constant manufacturing (shop) cost would look very much like contours of constant weight. Additionally, acoustic noise is driven primarily by FPR, but is also linked to extraction ratio. Therefore, a constraint on acoustic noise would probably look something like that shown in the example figure. The inclusion of acoustic noise now places an upper limit on FPR and prevents the designer from pushing the cost and weight down as far as is theoretically possible. Although Figure 11 is purely hypothetical, it should be a fairly accurate representation of reality, at least in terms of the contour shapes. Clearly, both acoustic noise and manufacturing cost are essential to making a well-informed decision as to the best compromise engine cycle.

The overall situation is nicely summarized in Figure 12, which depicts the best design as being a well-balanced solution that is a compromise between all of the opposing requirements. On one hand, if engine weight and cost receive too much emphasis, then SFC and acoustic noise margin will suffer. On the other hand, if too much attention is paid to reducing SFC, the result is a heavy and expensive design. The authors are not suggesting that this is a revelation due to the probabilistic methods offered here, (any experienced designer has seen these trends many times before). Rather, we are suggesting that these methods help to easily visualize the trades and also allow direct trades of design margins. Both of these capabilities are seriously lacking in today’s methods and tools.
CONCLUSIONS

The primary conclusion of this study is that component uncertainty has a significant impact on vehicle performance. One need only examine the design range and fuel burn CDFs to see this. In fact, the CDFs for this problem showed that the collective impact of component uncertainty is roughly the same order of magnitude as the cycle itself. As a result, it is imperative that the impact of uncertainty be taken into account if one desires to refine current designs by trading design margin for increased performance.

Second, the results presented in this paper show that there is little opportunity for reducing the impact of variance due to the seven noise parameters by manipulation of cycle parameters. Thus, the idea of a robust cycle design which has minimal variance is not a very useful concept for this specific problem. As a result, the probabilistic approach has received the preponderance of attention throughout this paper.

Third, probabilistic design methods certainly show promise in preliminary design applications, particularly in helping to quantify trades of design margin against performance. The probabilistic sensitivity methods explored in this study only scratch the surface of possibilities for this technique, and the authors are currently developing a more formal treatment which is mathematically precise and more exact in formulation. Additionally, it should be pointed out that this method will be quite useful for analysis of the acoustic noise and engine manufacturing cost aspects of this problem (and could be extended to include emissions as well).

Finally, the RSE formulation used herein is graphical and intuitive, thus enabling the easy presentation of all constraints and FoMs on a single chart. Furthermore, the designer can interactively change the cycle design point settings and get instantaneous estimates for the changes in all relevant constraints and FoMs throughout the design space of interest.

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