A Work Potential Perspective of Engine Component Performance

Dr. Bryce Roth*
Aerospace Systems Design Laboratory
Georgia Institute of Technology, Atlanta, GA 30332-0150
www.asdl.gatech.edu

Abstract

There is a strong interest within the propulsion community in applying the concept of thermodynamic work potential as a universal figure of merit for gauging the performance of prime-movers. In particular, exergy, gas horsepower, and thrust work potential have shown considerable promise as work potential figures of merit for propulsion system design. However, the relationships between these measures of work potential and the classical measures of component performance (component efficiencies) are not widely known. The objective of this paper is to derive a series of relationships linking classical efficiency-based performance metrics to modern measures of work potential. Derivations for the most common component efficiencies encountered in aircraft engine design are given in terms of all three work potential measures previously mentioned. Finally, the classical efficiency-based models are compared and contrasted with modern work potential methods to highlight the strengths and weaknesses of each.

Nomenclature

C_{FG} = gross thrust coefficient
c_p = constant pressure specific heat
ex = mass-specific exergy
F_G = gross thrust
g = gravitational acceleration
ghp = mass-specific gas horsepower
h = mass-specific enthalpy
J = work equivalent of heat
P = pressure
PR = pressure ratio
q = mass-specific heat input
R = mass-specific gas constant
s = mass-specific entropy
T = temperature
V = propulsive stream velocity
w = flow-specific shaft work
wp = mass-specific thrust work potential
γ = ratio of specific heats
ε_T = turbine effectiveness
η_c = compressor adiabatic efficiency
η_R = inlet pressure recovery
η_T = turbine adiabatic efficiency
η_{comb} = combustion efficiency

Subscripts
actual = conditions corresponding to real flow process
amb = ambient (assumed to be reference)
ideal = conditions corresponding to ideal flow process
in = flux into component
loss = work potential destroyed (departure form ideal)
out = flux out of component
reject = heat rejected due to shaft parasite losses
0 = stagnation conditions

Introduction

There is a philosophical revolution of sorts taking place today in the field of thermodynamics, particularly in thermal systems analysis. The driving concept behind this revolution is the idea that every substance has a real and calculable potential to do work and this work potential is a very powerful tool in understanding the fundamental nature of thermal systems. The concept of work potential is philosophically different and distinct from the classical thermal sciences in that it gives a holistic view wherein *all* thermodynamic processes are gauged relative to a single, general figure of merit.

It follows that the concept of thermodynamic work potential holds promise as a universal figure of merit to gauge the performance of propulsion systems. Specifically, the application of these ideas to propulsion system performance analysis leads readily to generalized representations of engine component performance that are *directly comparable* to one another (unlike component efficiencies). These general representations are a *direct* measure of the fundamental quantity of interest to propulsion system designers: *transfer of work potential.*

Presently, expressions relating work potential to component performance are relatively unknown or have never been derived. The objective of this paper is to establish the links between work potential and the classical efficiency figures of merit (FoMs) for a variety of standard propulsion system components. Detailed derivations are presented and compared on a component-by-component basis, with emphasis on deriving expressions for transfer of exergy, gas horsepower, and thrust work potential.

Background

The fundamental principles underlying the concept of thermodynamic work potential have been under development for many decades, starting primarily with the work of J.W. Gibbs. It has been continuously updated and refined over the years and is today a well-

* Research Engineer, Aerospace Systems Design Laboratory (ASDL), Georgia Tech. Member, AIAA.

defined field of thermodynamics. The central concept of work potential methods is that every substance has a well-defined work potential stored in it. For instance, a rock at the top of a hill has potential to do work in being moved to the bottom of the hill. Likewise, fuel has a real and measurable quantity of work potential stored in the form of chemical energy of molecular bonds. This quantity is known as exergy.

Exergy is a thermodynamic property describing the maximum theoretical (Carnot) work that can be obtained from a substance in taking it from a given chemical composition, temperature, and pressure to a state of chemical, thermal, and mechanical equilibrium with the environment.[1] The general definition of exergy is given by

$$ex \equiv h - h_{amb} - T_{amb}\left(s - s_{amb}\right) \quad (1)$$

Note that while energy is a conserved quantity, exergy is *not*, and is always destroyed when entropy is produced. Exergy is the most comprehensive loss FoM of the three that are examined herein in that it captures the effect of *all* losses relevant to contemporary propulsive cycles, including non-equilibrium combustion, exhaust heat, and exhaust residual kinetic energy.

Gas horsepower is defined as the *ideal shaft work* that would be obtained by isentropic expansion of a gas from a specified temperature and pressure to a prescribed reference pressure (usually taken to be local atmospheric). The temperature at the imaginary expanded condition is a fall-out of the isentropic expansion process. Therefore, the definition of gas horsepower is independent of ambient temperature, unlike exergy. Gas horsepower is a special case of exergy wherein only mechanical equilibrium with the environment is enforced.[2] Gas horsepower can be thought of as a Brayton figure of merit because a perfect Brayton cycle will have no loss of gas horsepower, whilst any departure from the ideal Brayton cycle will appear as a loss.

Thrust work potential is defined as *ideal thrust work* that would be obtained in expanding a flow at a given temperature and pressure to ambient pressure using a thrust nozzle instead of an imaginary turbine. Therefore, thrust work potential is equal to ideal thrust multiplied by flight velocity.[3] Thrust work potential is a pure jet propulsion figure of merit because it is a direct index on the ability to *directly* produce thrust work. In effect, thrust work potential is a measure of ability to project thrust work into the Earth-fixed reference frame and is related to gas horsepower through propulsive efficiency. Thus, thrust work potential is a special case of gas horsepower, and by induction, a special case of exergy.

The above three measures of work potential are the basis for the following derivations for component loss as a function of classical efficiencies. Past derivation and discussion of work potential FoMs presented by Ackeret,[4] Clarke & Horlock,[5] Curran,[6] Riggins,[7] and Roth & Mavris[8,9] has touched upon the idea that component inefficiencies reduce the work potential available in a given cycle from the maximum theoretical to some lesser value. This effect can be thought of as a transfer function that takes maximum theoretical work provided by the cycle into the actual work output provided by the real machine. The difference between the work input and work output is loss, and is typically quantified in terms of a component efficiency. Component efficiencies are usually defined as a ratio of some actual to ideal quantity, and the exact definition of efficiency varies from component to component. Consequently, one component efficiency is not directly comparable to another.

Work potential can be used as a universal figure of merit for thermodynamic performance. One of the chief strengths of loss as a component figure of merit is that it provides an *absolute* measure of thermodynamic cost, which is something that efficiencies do not provide. Moreover, loss of work potential is a component FoM that is *directly comparable* between components. This paper will explore loss representations of component performance, and, where appropriate, discuss the work that has been done on this topic.

Derivations, Various and Sundry

A useful concept in studying work potential and losses thereof is the work transfer function, defined as the ratio of work potential output to work potential input. This function is bounded by 0 and 1 due to the laws of thermodynamics and can generally be expressed in terms of a few non-dimensional parameters. The non-dimensional presentation has the benefit that it lays bare the fundamental parameters impacting transfer of work potential. The following sections will present derivations of various work transfer functions for a few of the most prominent component efficiency figures of merit used in the propulsion industry today.

Loss Due to Nozzle Internal Aerodynamics

Loss due to nozzle internal aerodynamics is typically quantified in terms of a gross thrust coefficient (C_{FG}), defined as the ratio of actual thrust produced by the nozzle to ideal thrust. For air breathing propulsive applications, the prescribed boundary conditions are usually assumed to be known nozzle entrance conditions and exit pressure. Since thrust coefficient is defined as a ratio of actual to ideal thrust, it is directly a thrust work potential figure of merit. In other words, the thrust work potential transfer function is given by C_{FG} itself.

$$\frac{wp_{out}}{wp_{in}} = C_{FG} \quad (2)$$

An expression for gas horsepower transfer as a function of nozzle thrust coefficient is obtained by noting that ratio of actual to ideal thrust is directly proportional to exit velocity, and therefore, proportional to the square root of actual gas horsepower output to ideal output.

$$C_{FG} \equiv \frac{F_{G,actual}}{F_{G,ideal}} = \frac{\Delta V_{actual}}{\Delta V_{ideal}} = \frac{\sqrt{\Delta ghp_{out,actual}}}{\sqrt{\Delta ghp_{out,ideal}}} \quad (3)$$

Further, the ideal gas horsepower output is the same as the gas horsepower input, so the gas horsepower transfer function in terms of thrust coefficient is easily obtained.

$$\frac{ghp_{out}}{ghp_{in}} = C_{FG}^2 \quad (4)$$

Loss in gas horsepower due to nozzle thrust coefficient follows from the above expression.

$$ghp_{loss} = ghp_{in} - ghp_{out} = \left(1 - C_{FG}^2\right) ghp_{in} \quad (5)$$

To derive an expression for exergy transfer as a function of C_{FG}, it is convent to first note that the transfer function is related to total exergy loss in the nozzle.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{ex_{loss}}{ex_{in}} = 1 - \frac{ex_{in} - ex_{out}}{ex_{in}} \quad (6)$$

For a calorically perfect ideal gas, the exergy flowing into and out of the nozzle can be determined using the definition of exergy in conjunction with the second T-dS relation:

$$ex_{in} = c_p\left(T_{in} - T_{amb}\right) - c_p T_{amb} \ln\left(T_{in}/T_{amb}\right) + RT_{amb} \ln\left(P_{in}/P_{amb}\right) \quad (7)$$

and

$$ex_{out} = c_p\left(T_{out} - T_{amb}\right) - c_p T_{amb} \ln\left(T_{out}/T_{amb}\right) + \frac{1}{2gJ} V_{out}^2 \quad (8)$$

or, substituting the energy equation into the above:

$$ex_{out} = c_p\left(T_{in} - T_{amb}\right) - c_p T_{amb} \ln\left(T_{out}/T_{amb}\right) \quad (9)$$

Substituting these equations into Eq. (6), noting that:

$$c_p = \frac{\gamma R}{\gamma - 1} \quad \text{and} \quad \frac{P_{in}}{P_{amb}} = \left(\frac{T_{in}}{T_{amb}}\right)^{\gamma/\gamma - 1} \quad (10)$$

yields a compact expression for exergy transfer.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln\left(\frac{T_{out}}{T_{in}}\right) + \ln\left(\frac{T_{in}}{T_{amb}}\right)}{\frac{T_{in}}{T_{amb}} - 1} \quad (11)$$

T_{out} can be expressed as a function of C_{FG} and T_{in}/T_{amb} by first applying the energy equation and noting that $V_{out}=C_{FG}V_{out,ideal}$.

$$T_{in} = T_{out} + \frac{1}{2gJc_p} C_{FG}^2 V_{out,ideal}^2 \quad (12)$$

The later term is equivalent to the ideal gas horsepower input times the square of thrust coefficient. Substituting in the expression for gas horsepower of an ideal, calorically perfect gas

$$\begin{aligned} T_{out} &= T_{in} - \frac{C_{FG}^2}{c_p} c_p T_{in}\left[1 - \left(\frac{P_{amb}}{P_{in}}\right)^{\gamma/\gamma-1}\right] \\ &= T_{in}\left[1 - C_{FG}^2\left(1 - \frac{T_{amb}}{T_{in}}\right)\right] \end{aligned} \quad (13)$$

Finally, substituting this equation into Eq. (11) yields an expression for exergy transfer as a function of C_{FG} and T_{in}/T_{amb}.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln\left(1 - C_{FG}^2\left(1 - \frac{T_{amb}}{T_{in}}\right)\right) + \ln\left(\frac{T_{in}}{T_{amb}}\right)}{\frac{T_{in}}{T_{amb}} - 1} \quad (14)$$

The relative work potential as a function of nozzle thrust coefficient for all three work potential FoMs is plotted in Fig. 1. Transfer of thrust work potential has a one-to-one correspondence with thrust coefficient because C_{FG} is a thrust-based figure of merit. Gas horsepower transfer is exceptionally sensitive to nozzle thrust coefficient and is a limiting case for loss in work potential. Exergy is least sensitive to nozzle thrust coefficient because the residual heat in the exhaust efflux increases as C_{FG} decreases, thus providing a

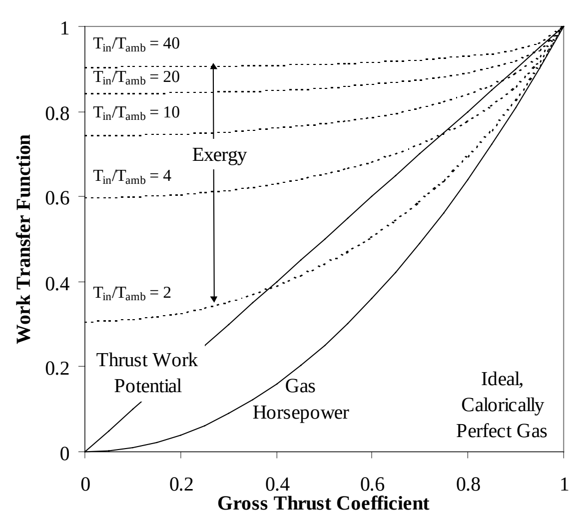

Fig. 1 Work Potential Transfer as a Function of Nozzle Thrust Coefficient.

recovery effect in which a portion of the lost work potential is reclaimed.† However, exergy becomes increasingly sensitive to C_{FG} as T_{in}/T_{amb} gets smaller, eventually approaching the gas horsepower curve as T_{in}/T_{amb} approaches 1.

Loss Due to Pressure Drop

Pressure drops in engine components are usually quantified in terms of percent drop in absolute pressure relative to the input pressure, denoted as $\Delta P/P_{in}$. Exergy loss due to a pressure drop is easily calculated by application of the Gouy-Stodola lost work theorem[1]

$$(\text{Lost Work}) = ex_{loss} = T_{amb}\Delta S = T_{amb} R \ln\left(P_{out} / P_{in} \right) \quad (15)$$

where ΔS is the change in entropy caused by the pressure drop, P_{in} & P_{out} are the pressure before and after the pressure drop, respectively, and R is the mass-specific gas constant. Next, expressing this equation in terms of $\Delta P/P_{in}$ yields:

$$ex_{loss} = T_{amb} R \ln\left(1 - \Delta P / P_{in} \right) \quad (16)$$

Recall that the expression for exergy transfer function in terms of exergy loss is:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{ex_{loss}}{ex_{in}} \quad (17)$$

so combining equations:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{RT_{amb} \ln\left(1 - \Delta P / P_{in} \right)}{ex_{in}} \quad (18)$$

Substituting the equation for exergy of a calorically perfect ideal gas and canceling RT_{amb} from the numerator and denominator yields an expression for exergy transfer as a function of pressure drop and non-dimensional inlet pressure & temperature.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln\left(1 - \Delta P / P_{in} \right)}{\frac{\gamma}{\gamma - 1}\left[T_{in}/T_{amb} - 1 - \ln\left(T_{in}/T_{amb} \right) \right] + \ln\left(P_{in}/P_{amb} \right)} \quad (19)$$

An analogous expression for ghp transfer across pressure drops can be derived using the expression for ghp of an ideal, calorically perfect gas.

$$ghp_{in} = c_p T_{in} \left[1 - \left(P_{in}/P_{amb} \right)^{\frac{1-\gamma}{\gamma}} \right] \quad (20)$$

Note that P_{out} can be expressed as:

$$P_{out} = P_{in} \left(1 - \Delta P / P_{in} \right) \quad (21)$$

Taking GHP_{in}/GHP_{out} and substituting for P_{out}:

$$\frac{ghp_{out}}{ghp_{in}} = \frac{1 - \left[\frac{P_{in}}{P_{amb}} \left(1 - \frac{\Delta P}{P_{in}} \right) \right]^{\frac{1-\gamma}{\gamma}}}{1 - \left(P_{in}/P_{amb} \right)^{\frac{1-\gamma}{\gamma}}} \quad (22)$$

Thus, gas horsepower transfer is a function of the ratio of specific heats, the non-dimensional inlet pressure, and the pressure drop.

Exergy and gas horsepower transfer are plotted as a function of component pressure loss in Fig. 2. The solid lines show gas horsepower transfer over a range of non-dimensional inlet pressures. Note that gas horsepower loss is highly sensitive to pressure loss at low inlet pressures, but becomes less sensitive as inlet pressure increases. The dashed lines show exergy transfer for a range of non-dimensional inlet temperatures at a single non-dimensional inlet pressure of 2. A family of such curves exists for each value of non-dimensional inlet temperature. Note that as inlet temperature increases, the exergy transfer function becomes insensitive to pressure drops. Also note that the pressure drop curve for P_{in}/P_{amb} ends at 50% because higher pressure drops imply outlet pressures below ambient. Exergy transfer is substantially the same as gas horsepower transfer when the non-dimensional inlet temperature is 1.0.

It is trivial to show that the work potential transfer function is related to the gas horsepower transfer function via the relation:

$$\frac{wp_{out}}{wp_{in}} = \sqrt{\frac{ghp_{out}}{ghp_{in}}} \quad (23)$$

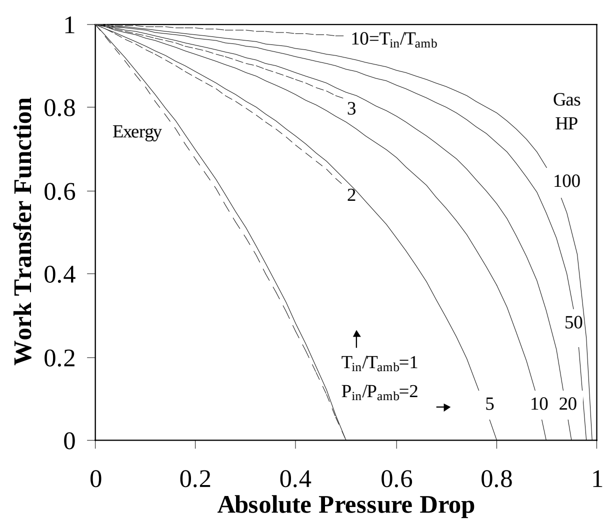

Fig. 2 Exergy and Gas Horsepower Work Potential Transfer as a Function of Component Pressure Drop.

† This comes from the fact that exergy presumes that waste heat in the exhaust efflux can be recovered to produce useful work.

This same relationship will be used in all following derivations as the basis to convert gas horsepower transfer functions into thrust work potential transfer functions.

Contours of thrust work potential transfer are plotted against contours of constant gas horsepower transfer in Fig. 3. Note that thrust work potential is generally less sensitive to pressure losses than is gas horsepower, especially at high inlet pressure. This result may at first seem counterintuitive given that a portion of the gas horsepower must inevitably be lost in the form of exhaust residual kinetic energy (propulsive efficiency effects). The answer lies in the realization that thrust work potential does not bookkeep exhaust residual kinetic energy as useful work potential.[2] Therefore, the exhaust residual kinetic energy loss is immaterial as far as ratios of losses are concerned. This sentiment is reflected in Fig. 3.

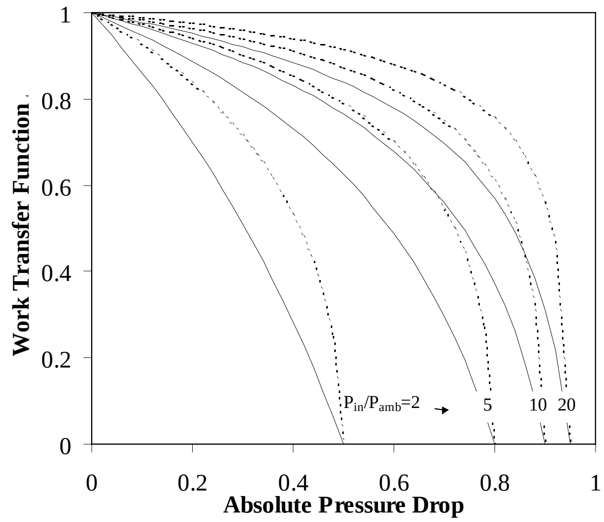

Fig. 3 Comparison of Thrust Work Potential (Dashed) and Gas Horsepower (Solid) Work Transfer as a Function of Pressure Drop.

Loss Due to Inlet Pressure Recovery

Expressions for work potential transfer as a function of inlet pressure recovery can be obtained directly from the equations developed in the previous section. From the definition of inlet pressure recovery:

$$P_{0,out} = \eta_R P_{0,in} \tag{24}$$

Substituting this expression in the previously developed equations readily yields formulas for work transfer as a function of inlet recovery.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\ln(\eta_R)}{\frac{\gamma}{\gamma-1}\left[T_{in}/T_{amb} - 1 - \ln\left(T_{in}/T_{amb} \right)\right] + \ln\left(P_{in}/P_{amb} \right)} \tag{25}$$

$$\frac{ghp_{out}}{ghp_{in}} = \frac{1 - \left[P_{in}/P_{amb}\, \eta_R \right]^{\frac{1-\gamma}{\gamma}}}{1 - \left(P_{in}/P_{amb} \right)^{\frac{1-\gamma}{\gamma}}} \tag{26}$$

Loss Due to Compressor Adiabatic Efficiency

To derive an expression for exergy transfer as a function of compressor adiabatic efficiency (η_c), recall that the exergy transfer function is related to total exergy loss via:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{ex_{loss}}{ex_{in}} = 1 - \frac{ex_{in} - ex_{out}}{ex_{in}} \tag{27}$$

For a calorically perfect ideal gas, the exergy loss can be determined using the Gouy-Stodola lost work theorem mentioned previously,

$$ex_{Loss} = T_{amb} \Delta S \tag{28}$$

where ΔS is the entropy change across the compressor. ΔS can be expressed in terms of temperature and pressure for an ideal, calorically perfect gas using the second TdS relation. Further, the definition of compressor adiabatic efficiency is the ratio of ideal work to actual work required to achieve a given pressure ratio.

$$\eta_c = \frac{h_{out,ideal} - h_{in}}{h_{out} - h_{in}} \approx \frac{T_{out,ideal} - T_{in}}{T_{out} - T_{in}} \tag{29}$$

Combining these four relations, it can be shown that:

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\eta_c T_{amb} \ln\left(T_{out}/T_{out,ideal} \right)}{T_{out,ideal} - T_{in}} \tag{30}$$

The quantity inside the parentheses can be expressed in terms of compressor efficiency by rearranging the terms in the definition of η_c..

$$\frac{T_{out}}{T_{out,ideal}} = \frac{1}{\eta_c} + \frac{T_{in}}{T_{out,ideal}}\left(1 - \frac{1}{\eta_c} \right) \tag{31}$$

Furthermore, the ratio of inlet to ideal outlet temperature can be expressed in terms of compressor pressure ratio (PR) using isentropic compressible flow relations.

$$\frac{T_{out,ideal}}{T_{in}} = PR^{\frac{\gamma-1}{\gamma}} \tag{32}$$

Substituting these two equations and rearranging yields an expression for compressor exergy transfer as a function of compressor adiabatic efficiency, pressure ratio, and non-dimensional inlet temperature.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\eta_c \ln\left(PR^{\frac{1-\gamma}{\gamma}}\left(1 - \frac{1}{\eta_c}\right) + \frac{1}{\eta_c} \right)}{\frac{T_{in}}{T_{amb}}\left(PR^{\frac{\gamma-1}{\gamma}} - 1 \right)} \quad (33)$$

This equation can be used to create a plot of exergy transfer as a function of compressor adiabatic efficiency, as shown in Fig. 4. This figure shows two families of curves. The solid lines show exergy transfer for a range of non-dimensional inlet temperatures with the pressure ratio fixed at 2. The dashed lines show exergy transfer for a range of pressure ratios with non-dimensional inlet temperature fixed at 1. Note that exergy transfer function approaches 1.0 as compressor adiabatic efficiency approaches 1.0. Further note that the exergy transfer is never less than the compressor efficiency itself, a well-known result in the thermodynamics community. It is clear from this plot that increasing non-dimensional inlet pressure decreases the relative proportion of exergy loss due to compression, as does increasing compressor pressure ratio. Conversely, it is clear from this figure that low pressure ratio devices (fans and propellers) will be heavily impacted by exergy losses if the compression system does not have a high adiabatic efficiency.

A surprising feature of Fig. 4 is the manner in which exergy transfer approaches 1.0 as compressor efficiency approaches zero. This trend is entirely counterintuitive, but can be explained as follows. The definition of compressor adiabatic efficiency is the ratio of ideal to actual work required to achieve *a given pressure ratio*. Therefore, as compressor efficiency approaches zero, the work required to achieve the pressure ratio becomes exorbitant, thus implying that compressor discharge temperature is also very high. Therefore, the exergy content of the compressor discharge at very low η_c is primarily made up of heat energy instead of compression work. In effect, at high compressor efficiency the primary mechanism of exergy transfer is via compression work. At very low efficiency the compressor is a flow heater, delivering exergy transfer via high discharge temperature.

An expression for gas horsepower transfer as a function of compressor adiabatic efficiency can be derived by substituting the expression for gas horsepower of an ideal gas (Eq. 20) into the definition of the gas horsepower transfer function.

$$\frac{ghp_{out}}{ghp_{in}} = \frac{c_P T_{out}\left[1 - \left(\frac{P_{out}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}}\right]}{c_P T_{in}\left[1 - \left(\frac{P_{in}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}}\right] + w_{in}} \quad (34)$$

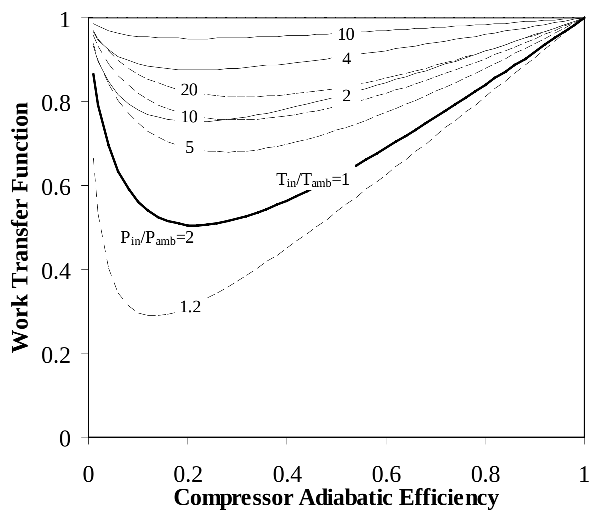

Fig. 4 Exergy Transfer as a Function of Compressor Efficiency for a Range of Pressure Ratios (Dashed) and Temperature Ratios (Solid).

w_{in} is the net shaft work into the compressor. This shaft work can be related to the ideal (isentropic) work through the definition of compressor efficiency.

$$w_{in} = \frac{w_{ideal}}{\eta_c} = \frac{c_P T_{out,ideal}}{\eta_c}\left[1 - \left(\frac{P_{out}}{P_{in}}\right)^{\frac{1-\gamma}{\gamma}}\right] \quad (35)$$

Substituting and re-arranging:

$$\frac{ghp_{out}}{ghp_{in}} = \frac{\frac{T_{out}}{T_{in}}\left[1 - \left(\frac{P_{out}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}}\right]}{1 - \left(\frac{P_{in}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}} + \frac{T_{out,ideal}}{T_{in}}\left[1 - \left(\frac{P_{out}}{P_{in}}\right)^{\frac{1-\gamma}{\gamma}}\right]} \quad (36)$$

The ratio of T_{out}/T_{in} can be expressed in terms of compressor efficiency and $T_{out,ideal}/T_{in}$ using the definition of compressor efficiency. Furthermore, $T_{out,ideal}/T_{in}$ can be expressed as a function of pressure ratio using Eq. (32). Substituting yields an expression for gas horsepower transfer as a function of non-dimensional inlet pressure, pressure ratio, gamma, and compressor efficiency.

$$\frac{ghp_{out}}{ghp_{in}} = \frac{\left[1 + \frac{1}{\eta_c}\left(PR^{\frac{\gamma-1}{\gamma}} - 1\right)\right]\left[1 - \left(PR\frac{P_{in}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}}\right]}{1 - \left(\frac{P_{in}}{P_{amb}}\right)^{\frac{1-\gamma}{\gamma}} + \frac{1}{\eta_c} PR^{\frac{\gamma-1}{\gamma}}\left[1 - PR^{\frac{1-\gamma}{\gamma}}\right]} \quad (37)$$

If the inlet pressure is assumed to be ambient pressure, this expression reduces to:

$$\frac{ghp_{out}}{ghp_{in}} = \frac{\eta_c + PR^{\frac{\gamma-1}{\gamma}} - 1}{PR^{\frac{\gamma-1}{\gamma}}} \qquad (38)$$

Note that this is a linear function of compressor adiabatic efficiency.

The above equation can be used to create a plot of gas horsepower transfer as a function of compressor adiabatic efficiency, as shown in Fig. 5. The dashed lines in this figure are the exergy transfer contours as a function of pressure ratio, repeated from Fig. 4. The solid lines are gas horsepower transfer as a function of compressor efficiency for the same range of pressure ratio. Note that the gas horsepower and exergy transfer converge at high efficiency, but diverge as efficiency decreases. While exergy transfer ultimately recovers to 1.0 as efficiency approaches zero, gas horsepower transfer can only recover a small portion of the heat addition in the form of usable work. Also, note that gas horsepower work transfer becomes increasingly sensitive to compressor efficiency as pressure ratio decreases. In the limit as pressure ratio approaches 1.0, the gas horsepower work transfer becomes equal to compressor adiabatic efficiency.

Finally, the impact of compressor efficiency on thrust work potential is easily estimated by taking the square root of the gas horsepower transfer function. Presuming that the inlet pressure is once again the same as ambient, the expression for thrust work transfer is:

$$\frac{wp_{out}}{wp_{in}} = \sqrt{\frac{\eta_c + PR^{\frac{\gamma-1}{\gamma}} - 1}{PR^{\frac{\gamma-1}{\gamma}}}} \qquad (39)$$

Fig. 6 shows a plot comparing gas horsepower contours shown previously against thrust work potential contours, shown dashed. Note that the thrust work transfer function is always higher than the gas horsepower transfer, though one should always bear in mind that total thrust work will actually always be less than total gas horsepower.

It is clear based on the equations developed in this subsection that compressor performance can be represented in terms of work potential and loss thereof. Given this idea, one is naturally led to inquire about the practicality of building a compressor "loss map" analogous to the standard compressor efficiency maps used today. Paulus et al.[10] have recently published investigations into this subject, though using entropy as an index of compressor loss instead of gas horsepower or exergy loss. Their results show that such component performance representations are indeed possible, and possibly even somewhat more in tune with the fundamental physics driving compressor performance than are existing methods.

Loss Due to Turbine Adiabatic Efficiency

Expressions for work transfer as a function of turbine adiabatic efficiency can be derived in parallel fashion to those for compressor work transfer. Starting with the well-known expression for "second law effectiveness" of a turbine:

$$\varepsilon_T = \frac{ex_{out}}{ex_{in}} = \frac{w_{out}}{ex_{in} - ex_{out}} = \frac{1}{1 + T_{amb}\dfrac{\ln\left(T_{out}/T_{out,ideal}\right)}{\eta_T\left(T_{in} - T_{out,ideal}\right)}} \qquad (40)$$

The term inside the natural logarithm can be expressed as a function of turbine pressure ratio (defined as P_{in}/P_{out} for a turbine) and turbine efficiency. With some

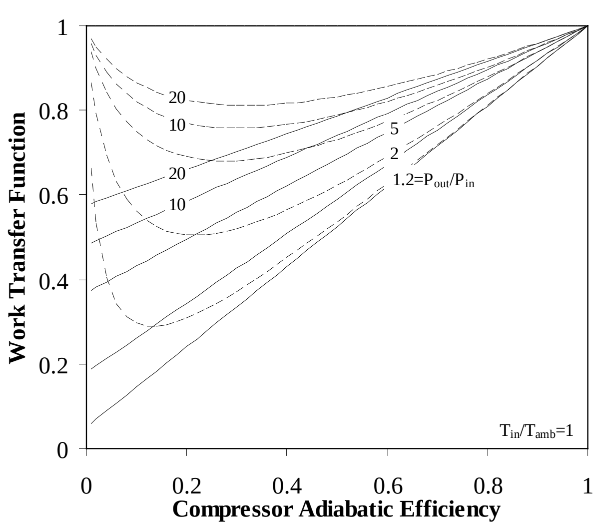

Fig. 5 Gas Horsepower (Solid) and Exergy Transfer (Dashed) as a Function of Compressor Efficiency for a Range of Pressure Ratios.

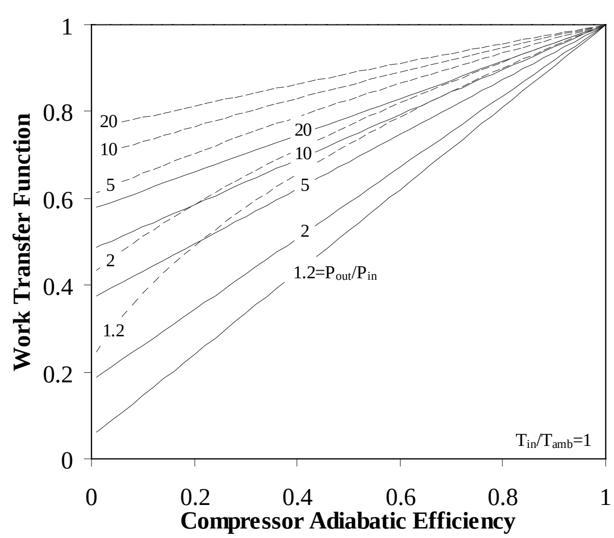

Fig. 6 Comparison of Gas Horsepower Work Transfer (Solid) Against Thrust Work Potential Transfer (Dashed) for a Range of Pressure Ratios.

rearrangement and substitution, this readily yields an expression for turbine exergy transfer as a function of turbine adiabatic efficiency, turbine pressure ratio, non-dimensional inlet temperature, and gamma.

$$\frac{ex_{out}}{ex_{in}} = \left[1 + \frac{\ln\left(PR^{\frac{\gamma-1}{\gamma}} (1-\eta_T) + \eta_T \right)}{T_{in}/T_{amb}\ \eta_T \left(1 - PR^{\frac{1-\gamma}{\gamma}} \right)} \right]^{-1} \qquad (41)$$

The impact of turbine exergy transfer as a function of adiabatic efficiency is shown in Fig. 7 for a range of inlet pressures and temperatures. Note that as inlet temperature increases, exergy transfer becomes less sensitive to turbine efficiency. Conversely, as turbine pressure ratio increases, exergy transfer becomes increasingly sensitive to turbine efficiency.

An expression for gas horsepower transfer can be derived using an approach directly paralleling that used previously for compressor gas horsepower transfer.

$$\frac{ghp_{out}}{ghp_{in}} = \frac{\left[1 - \eta_T \left(1 - PR^{\frac{1-\gamma}{\gamma}} \right) \right] \left[1 - \left(\frac{P_{in}}{P_{amb}} \frac{1}{PR} \right)^{\frac{1-\gamma}{\gamma}} \right]}{1 - \left(\frac{P_{in}}{P_{amb}} \right)^{\frac{1-\gamma}{\gamma}}} + \eta_T PR^{\frac{1-\gamma}{\gamma}} \qquad (42)$$

If P_{out} is assumed to be P_{amb}, the above expression reduces to:

$$\frac{ghp_{out}}{ghp_{in}} = \eta_T PR^{\frac{1-\gamma}{\gamma}} \qquad (43)$$

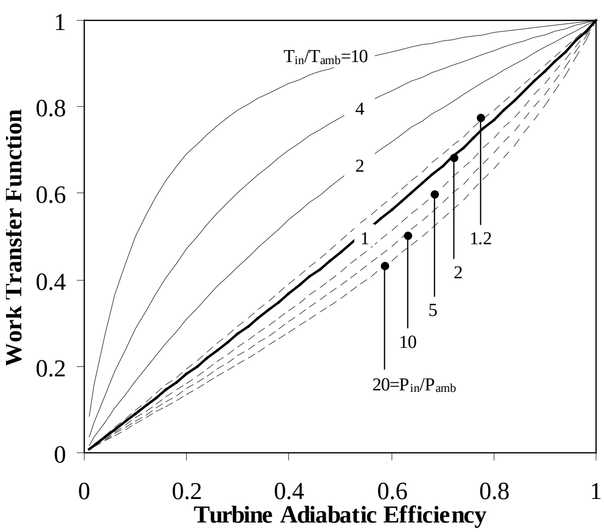

Fig. 7 Exergy Transfer as a Function of Turbine Adiabatic Efficiency for a Range of Inlet Pressures and Temperatures.

This is again a linear function of turbine adiabatic efficiency as it was in the case of compressor gas horsepower transfer. Once again, thrust work potential transfer is the square root of this quantity.

Loss Due to Incomplete Combustion

Loss due to incomplete combustion is typically quantified in terms of combustion efficiency, defined as actual heat released by the combustion process divided by the heat release of the ideal combustion process. Expressed mathematically,

$$\eta_{comb} = \frac{q_{actual}}{q_{ideal}} \qquad (44)$$

where q_{ideal} is the lower heating value of the fuel times the fuel/air ratio. An expression for gas horsepower transfer as a function of combustion efficiency is easily derived by simply applying Eq. 20 and recognizing that:

$$\frac{ghp_{out}}{ghp_{in}} = 1 - \frac{ghp_{loss}}{ghp_{in}} \qquad (45)$$

If ghp_{in} is taken to be the gas horsepower flowing into the combustor plus the ideal gas horsepower content of the fuel injected, the above expression becomes:

$$\frac{ghp_{out}}{ghp_{in}} = 1 - \frac{c_P \left(T_{out,ideal} - T_{out} \right) \left[1 - \left(P_{out}/P_{amb} \right)^{\frac{1-\gamma}{\gamma}} \right]}{c_P T_{out,ideal} \left[1 - \left(P_{in}/P_{amb} \right)^{\frac{1-\gamma}{\gamma}} \right]} \qquad (46)$$

If the combustion process is assumed to occur at constant pressure, it can be shown that gas horsepower transfer becomes:

$$\frac{ghp_{out}}{ghp_{in}} = 1 - \frac{T_{out,ideal} - T_{out}}{T_{out,ideal}} = \frac{f/a\,(LHV)\eta_{comb} + T_{in} c_P}{f/a\,(LHV) + T_{in} c_P} \qquad (47)$$

Therefore, gas horsepower is a linear function of combustion efficiency. Thrust work transfer is the square root of the above quantity. It should be noted that this derivation implicitly assumed constant specific heats, a very limiting assumption for most practical applications.

An expression for loss in gas horsepower can be derived by noting that the rise in available energy across the combustor, assuming no pressure drop, can be approximated as:

$$\Delta ghp_{ideal} = c_p \Delta T_{ideal} \left[1 - \left(\frac{P_{in}}{P_{amb}} \right)^{\frac{1-\gamma}{\gamma}} \right] \qquad (48)$$

where ΔT_{ideal} is the ideal combustor temperature rise. Now, making use of the definition of combustion efficiency once again:

$$\eta_{comb} = \frac{Q}{Q_{ideal}} \approx \frac{\Delta T_{actual}}{\Delta T_{ideal}} \qquad (49)$$

and substituting this back into the expression for ideal available energy rise:

$$\Delta ghp_{ideal} = \frac{c_p \Delta T_{actual}}{\eta_{comb}} \left[1 - \left(\frac{P_{in}}{P_{amb}} \right)^{\frac{1-\gamma}{\gamma}} \right] \qquad (50)$$

So loss in available energy due to incomplete combustion is approximated as:

$$ghp_{loss} = \Delta ghp_{ideal} - \Delta ghp_{actual} = \Delta ghp_{actual} \left(1 - \frac{1}{\eta_{comb}} \right) \qquad (51)$$

Nichols[11] has investigated the application of gas horsepower for experimental evaluation of combustor effectiveness when both pressure drop and combustion efficiency simultaneously play a role in determining combustor performance. The results of that study indicate that gas horsepower is a very desirable means for evaluating overall effectiveness of combustors.

If the exergy content of the fuel is approximated as the lower heating value, then exergy loss due to incomplete combustion is simply the difference between the ideal enthalpy rise across the combustor and the actual exergy rise:

$$ex_{loss} \approx \frac{f}{a} (LHV)(1 - \eta_{comb}) \qquad (52)$$

This can be used to create an expression for exergy transfer due to combustion efficiency by substituting the above as well as Eq. (7) into Eq. (6).

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{\dfrac{\frac{f}{a}(LHV)}{c_P T_{amb}} (1 - \eta_{comb})}{\dfrac{T_{in}}{T_{amb}} - 1 - \ln\left(\dfrac{T_{in}}{T_{amb}}\right) + \ln\left(\dfrac{P_{in}}{P_{amb}}\right) + \dfrac{\frac{f}{a}(LHV)}{c_P T_{amb}}} \qquad (53)$$

c_P in the above expression is the constant pressure specific heat of the flow entering the combustor. Note that if the quantity f/a(LHV) is much greater than the flow exergy entering the combustor, then Eq. (53) reduces to:

$$\frac{ex_{out}}{ex_{in}} \approx \eta_{comb} \qquad (54)$$

Note that this expression is an estimate on exergy loss due to unburned fuel only. It does not account for the destruction of exergy via non-equilibrium combustion in the combustor.

Loss Due to Shaft Power Extraction

In addition to the aerothermodynamic loss mechanisms previously described, mechanical elements in a propulsion system are also sources of loss. Typical loss mechanisms are windage, bearing friction, gear train losses, and shaft power extracted to drive engine accessories. Shaft power losses are usually measured in terms of absolute horsepower required, or in terms of the ratio of loss to total shaft power input. The latter quantity will be used herein by virtue of its non-dimensional nature. It is assumed that the shaft power lost due to parasitics is ultimately converted into heat, which may itself contain some work potential, but may or may not be usable. If the heat produced is not usable, then it is trivial to derive expressions for exergy, gas horsepower, and thrust work transfer as a function of percent power loss:

$$\frac{ex_{out}}{ex_{in}} = \frac{ghp_{out}}{ghp_{in}} = 1 - \frac{w_{loss}}{w_{in}} \qquad (55)$$

and

$$\frac{wp_{out}}{wp_{in}} = \sqrt{1 - \frac{w_{loss}}{w_{in}}} \qquad (56)$$

If the heat rejected can be recovered in some useful form, then the transfer function must also include a term to account for this. The first law of thermodynamics implies that the steady-state rate of heat rejection must be equal to the parasite power required. If the waste heat is rejected at a temperature T_{reject}, then the total exergy of the waste heat stream is:

$$ex_{reject} = w_{loss} \left(1 - \frac{T_{amb}}{T_{reject}} \right) \qquad (57)$$

Appending this term to Eq. 55 yields an expression for exergy transfer as a function of percent shaft power lost.

$$\frac{ex_{out}}{ex_{in}} = 1 - \frac{w_{loss}}{w_{in}} \frac{T_{amb}}{T_{reject}} \qquad (58)$$

It is clear from the above equation that the higher the rejection temperature, the more exergy can be recovered. This also applies for recovery of gas horsepower and thrust work potential, though the recovery capability will depend greatly on the specifics of any given scenario.

Comparison of Efficiency to Work Potential

Table 1 gives a summary of the various work potential transfer functions for a calorically perfect, ideal gas. This collection of work transfer functions is relatively comprehensive, though there are other expressions that could be derived and added to this table (work transfer as a function of heat exchanger effectiveness, for instance). However, the intent of this paper is to derive and investigate the most prominent component figures of merit with an eye towards exposing the relationships between classical component efficiency and work potential.

Table 1 Summary of Expressions for Loss as a Function of Component Efficiency.

Loss Source	***Definition***	***Exergy Transfer Function***	***Gas HP Transfer Function***	***Thrust Work Potential Transfer Fctn***
Inlet	$\frac{P_{0,2}}{P_{0,amb}}$	$1-\frac{\ln(\eta_R)}{\frac{\gamma}{\gamma-1}\left[T_{in}/T_{amb}-1-\ln\left(T_{in}/T_{amb}\right)\right]+\ln\left(P_{in}/P_{amb}\right)}$	$\frac{1-\left[P_{in}/P_{amb}\,\eta_R\right]^{\frac{1-\gamma}{\gamma}}}{1-\left(P_{in}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}}$	$\sqrt{\frac{1-\left[P_{in}/P_{amb}\,\eta_R\right]^{\frac{1-\gamma}{\gamma}}}{1-\left(P_{in}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}}}$
Compressor	$\frac{W_{in,ideal}}{W_{in,actual}}$	$1-\frac{\eta_c\ln\left(PR^{\frac{1-\gamma}{\gamma}}\left(1-1/\eta_c\right)+1/\eta_c\right)}{T_{in}/T_{amb}\left(PR^{\frac{\gamma-1}{\gamma}}-1\right)}$	$\frac{\eta_c+PR^{\frac{\gamma-1}{\gamma}}-1}{PR^{\frac{\gamma-1}{\gamma}}}$	$\sqrt{\frac{\eta_c+PR^{\frac{\gamma-1}{\gamma}}-1}{PR^{\frac{\gamma-1}{\gamma}}}}$
Turbine	$\frac{W_{out,actual}}{W_{out,ideal}}$	$\left[1+\frac{\ln\left(PR^{\frac{\gamma-1}{\gamma}}\left(1-\eta_T\right)+\eta_T\right)}{T_{in}/T_{amb}\,\eta_T\left(1-PR^{\frac{1-\gamma}{\gamma}}\right)}\right]^{-1}$	$\eta_T PR^{\frac{1-\gamma}{\gamma}}$	$\sqrt{\eta_T PR^{\frac{1-\gamma}{\gamma}}}$
Pressure Drop	$\frac{P_{0,in}-P_{0,out}}{P_{0,in}}$	$1-\frac{\ln\left(1-\Delta P/P_{in}\right)}{\frac{\gamma}{\gamma-1}\left[T_{in}/T_{amb}-1-\ln\left(T_{in}/T_{amb}\right)\right]+\ln\left(P_{in}/P_{amb}\right)}$	$\frac{1-\left[P_{in}/P_{amb}\left(1-\frac{\Delta P}{P_{in}}\right)\right]^{\frac{1-\gamma}{\gamma}}}{1-\left(P_{in}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}}$	$\sqrt{\frac{1-\left[P_{in}/P_{amb}\left(1-\frac{\Delta P}{P_{in}}\right)\right]^{\frac{1-\gamma}{\gamma}}}{1-\left(P_{in}/P_{amb}\right)^{\frac{1-\gamma}{\gamma}}}}$
Incomplete Combustion	$\frac{Q_{out,actual}}{Q_{out,ideal}}$	$1-\frac{\frac{f/a\,(LHV)}{c_P T_{amb}}\left(1-\eta_{comb}\right)}{\frac{T_{in}}{T_{amb}}-1-\ln\left(\frac{T_{in}}{T_{amb}}\right)+\ln\left(\frac{P_{in}}{P_{amb}}\right)+\frac{f/a\,(LHV)}{c_P T_{amb}}}$	$\frac{f/a\,(LHV)\eta_{comb}+T_{in}c_P}{f/a\,(LHV)+T_{in}c_P}$	$\sqrt{\frac{f/a\,(LHV)\eta_{comb}+T_{in}c_P}{f/a\,(LHV)+T_{in}c_P}}$
Nozzle Internal Losses	$\frac{F_{G,actual}}{F_{G,ideal}}$	$1-\frac{\ln\left(1-C_{FG}^2\left(1-\frac{T_{amb}}{T_{in}}\right)\right)+\ln\left(\frac{T_{in}}{T_{amb}}\right)}{\frac{T_{in}}{T_{amb}}-1}$	C_{FG}^2	C_{FG}
Shaft Power Extraction	$\frac{HP_{loss}}{HP_{in}}$	$1-\frac{w_{loss}}{w_{in}}\frac{T_{amb}}{T_{reject}}$	$1-\frac{w_{loss}}{w_{in}}$	$\sqrt{1-\frac{w_{loss}}{w_{in}}}$

A general trend is evident in the expressions of Table 1: exergy transfer is typically a function of non-dimensional inlet pressure and temperature, while the expressions for gas horsepower and thrust work potential is usually only a function of non-dimensional inlet pressure. This should be no surprise given that the definition of exergy is a function of reference temperature and pressure while the other FoMs depend only on reference pressure. This trend is therefore a manifestation of the definitions themselves, specifically the boundary conditions imposed by each FoM.

These boundary conditions are summarized in Table 2. Note that exergy is defined as the work obtained by isentropically taking a flow into thermal, mechanical, and velocity equilibrium with a prescribed reference. Gas horsepower only prescribes a pressure and velocity equilibrium condition, and thrust work potential prescribes only mechanical equilibrium.

	Reference Pressure	*Reference Temperature*	*Reference Velocity*
Exergy	Prescribed	Prescribed	Prescribed
Gas HP	Prescribed	Float	Prescribed
Thrust Work	Prescribed	Float	Float

Table 2 Relationship Between Reference Conditions and Work Potential Figures of Merit.

Conclusions

The various comparisons of component efficiencies in terms of work potential transfer shown in this paper clearly illustrate why classical models for component efficiency are not entirely satisfactory. Specifically, classical component performance FoMs are "custom made" using disparate definitions, they are not directly comparable, and they give little insight as to transfer of work potential. The work potential perspective can supplement current FoMs and provide a deeper understanding of transfer and loss of work potential in prime movers.

For instance, the analyses presented herein have shown that nozzle thrust coefficient is a work potential figure of merit, with gas horsepower transfer being a limiting case for exergy loss due to nozzle internal aerodynamics. Loss due to pressure drops is a function of both inlet temperature and pressure, with the magnitude of loss decreasing precipitously as these parameters increase. Exergy transfer in a compressor actually goes to 1.0 as compressor efficiency goes to 0. Exergy transfer in a turbine goes to 0 as adiabatic efficiency approaches 0. Gas horsepower transfer in compressors and turbines is a linear function of adiabatic efficiency. Exergy loss due to shaft parasite losses is a function of average heat rejection temperature. All of these phenomena are not evident using only classical component performance representations. However, they are plainly obvious when viewed from a work potential perspective.

Acknowledgements

The author would like to thank the National Science Foundation for supporting portions of this research under grant DMI 9734234 and the US Army for contributing under grant NAG3-2586. Thanks also to my co-workers at Georgia Tech, particularly Prof. Dimitri Mavris for his ever-present vision and guidance, and Prof. Prasana Kadaba for his instruction and guidance.

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