



**AIAA 98-5208**

**Transforming Aerodynamic Datasets into  
Parametric Equations for use in Multi-  
disciplinary Design Optimization**

J. M. Scott

J. R. Olds

Space Systems Design Lab

Georgia Institute of Technology

Atlanta, GA

**1998 Defense and Civil Space Programs  
Conference and Exhibit**

**October 28 -30, 1998 / Huntsville, AL**

# Transforming Aerodynamic Datasets into Parametric Equations for use in Multidisciplinary Design Optimization

Jeffery M. Scott<sup>†</sup>  
Dr. John R. Olds<sup>\*</sup>

Space Systems Design Laboratory  
School of Aerospace Engineering  
Georgia Institute of Technology, Atlanta, GA, 30332-0150

## ABSTRACT

This paper presents a method of transforming aerodynamic datasets generated in Aerodynamic Preliminary Analysis System (APAS) into parametric equations which may subsequently be used in a multidisciplinary design optimization (MDO) environment for analyzing aerospace vehicles.

APAS is an analysis code which allows the user to create a simple geometric model of a vehicle and then calculate the aerodynamic force coefficients of lift, drag, and pitching moment over a wide range of flight conditions. As such, APAS is a very useful tool for conceptual level vehicle designs since it allows the force coefficients for a given design to be calculated relatively quickly and easily.

However, APAS suffers from an outdated user interface and, because it is tedious to generate a new dataset during each design iteration, it is quite difficult to integrate into an MDO framework. Hence the desire for a method of transforming the APAS output into a more usable form.

The approach taken and described in this paper involves the use of regression analysis techniques and response surface methodology to accomplish the data transformation with two goals in mind. The first goal was to develop a parametric model for calculating the aerodynamic coefficients for a single unique geometry. The second goal was to extend this model to capture

the effects of changes in vehicle geometry. This paper presents the results and gives the model developed for analyzing a sample vehicle for both cases.

## NOMENCLATURE

APAS	Aerodynamic Preliminary Analysis System
AR	wing aspect ratio
$C_d$	coefficient of drag
$C_{d0}$	coefficient of drag at zero lift
$C_l$	coefficient of lift
$C_{l0}$	coefficient of lift at zero angle of attack
$C_m$	coefficient of pitching moment
HABP	Hypersonic Arbitrary Body Program
K	parameter relating $C_d$ to $C_l$
M	Mach number
MDO	multidisciplinary design optimization
POST	Program to Optimize Simulated Trajectories
S	lift slope parameter
UDP	Unified Distributed Panel
$\alpha$	angle of attack (degrees)
$\Lambda$	wing leading edge sweep angle (degrees)

## INTRODUCTION

Aerodynamic Preliminary Analysis System (APAS) is an industry-standard tool for calculating aerodynamic force coefficients for use in the conceptual design of aerospace vehicles. This tool, developed by NASA and Rockwell International for use in the design of the Space Shuttle, is useful for aerodynamic analysis, but due to its highly interactive nature, it does not lend itself well to an iterative or optimized overall vehicle design process.

Aerodynamic analysis of a vehicle using APAS first requires the user to define a geometric model of

<sup>†</sup> - Graduate Research Assistant, School of Aerospace Engineering, Student member AIAA.

<sup>\*</sup> - Assistant Professor, School of Aerospace Engineering, Senior member AIAA.

the vehicle. This is done by specifying various parameters for individual components such as wings, fuselages, vertical tails, etc. For example, wings are defined by specifying planform area, aspect ratio, thickness-to-chord ratio, taper ratio, sweep angle and dihedral; fuselages by specifying length, cross-sectional area at various points along the longitudinal axis of the fuselage, and width-to-height ratio of each defined cross-sections. Required geometric data is entered manually via the keyboard with limited graphical interface. The process is prone to error and is difficult to duplicate or repeat accurately. In addition, changes to the vehicle configuration require a new model to be created in APAS because component geometric parameters cannot be altered at the keyboard once created.

Analysis in APAS is accomplished by defining specific flight conditions and vehicle attitude at which to calculate the aerodynamic force coefficients (typically lift, drag, and pitching moment). The required inputs are Mach number, angle of attack, flap deflection angle, altitude, skin friction coefficient and sideslip angle. For advanced launch vehicles, the user typically defines a range of 10 - 15 Mach number and altitude pairs from liftoff to orbit that model the vehicle's expected flight path. Aerodynamic coefficients are then calculated at 8 - 12 angles of attack (or sideslip angles) for each flight condition. The result is a very large set of tabular aerodynamic data (over 100 data points for each coefficient table). The entire aerodynamic dataset must be regenerated each time the overall vehicle geometry changes other than photographically.

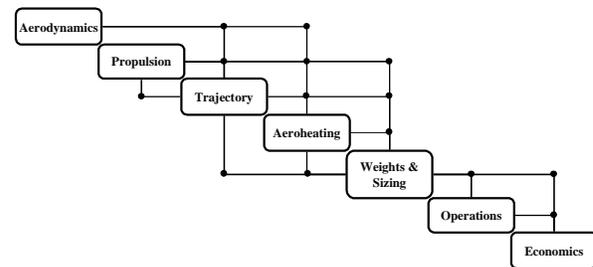
Two separate analysis codes are used to perform the actual flow calculations. The first is Unified Distributed Panel (UDP) which is a vortex paneling code capable of analyzing subsonic and low supersonic flow conditions. The second code is Hypersonic Arbitrary Body Program (HABP) which, as the name suggests, is used for hypersonic flow conditions and is based on local surface inclination methods in which pressure coefficients are calculated.

APAS is inherently a difficult program to use due to its outdated user interface and cumbersome interactive format for creating and modifying a vehicle model. For each geometry change, regenerating the aerodynamic database might take an experienced user

4 - 6 hours. These characteristics make integrating APAS into a design optimization process wherein the vehicle geometry is allowed to change very difficult. Because of this, the valuable analysis capability of APAS is often under-utilized at the conceptual design stage of an aerospace vehicle.

#### *Multidisciplinary Design Optimization Effort*

In conceptual design, the order of execution of the various disciplines is typically similar to what is shown in Figure 1, below. Here the design structure matrix serves to organize the flow of information from one discipline to another. The "inputs" from propulsion to trajectory optimization, for example, are shown as lines above and to the right of the boxes, and the "outputs," or feedback, are shown as lines below and to the left of the boxes.



**Figure 1 – Design Structure Matrix**

Overall vehicle layout and configuration is determined at the aerodynamic design and analysis stage, whereupon propulsion, trajectory, aeroheating and weights and sizing are carried out in a tightly coupled, iterative process. Aerodynamics is typically left out of the loop once initial analysis has been performed due to the time constraints involved in recreating geometric models in APAS. Once the aerodynamic analysis has been done, the vehicle is only permitted to scale up or down photographically which ensures that the non-dimensional aerodynamic force coefficients remain unchanged. Although this facilitates a rapid design process, it does not contribute to an overall optimized vehicle design since the aerodynamic coefficients are set at the beginning of the process and not allowed to vary as design knowledge increases.

In order to alleviate some of the problems associated with APAS, to make better use of its

capabilities, and to facilitate its integration into an MDO environment, this research project was undertaken with two goals in mind. The first goal was to develop a method of transforming APAS datasets into a parametric model for calculation of the aerodynamic coefficients for a single unique geometry, and the second goal was to extend this method to capture the effects of changes in vehicle geometry.

Using the parametric equations to approximate a vehicle's aerodynamics and the change in aerodynamics with change in geometry, a gradient-based optimizer would be able to determine the optimum geometric parameters given a desired objective function. For example, the methods outlined below were used to capture the effect of changing the wing aspect ratio. An optimizer could then vary the aspect ratio and determine the force coefficients according to the parametric equations rather than requiring actual APAS analysis of several manually-created models each with a unique aspect ratio. Extended to several geometric parameters, this would reduce the time and effort required to optimize a conceptual-level vehicle design.

One example of an optimization tool is NASA's Program to Optimize Simulated Trajectories (POST), a tool commonly used to perform the trajectory analysis and optimization for aerospace vehicles. POST requires as inputs the APAS tables of lift, drag and moment at specific Mach numbers, which it uses in calculating drag losses and lift forces along the vehicle's trajectory. In calculating the lift and drag forces throughout the trajectory, POST interpolates between explicit data points given in the aerodynamic tables as needed.

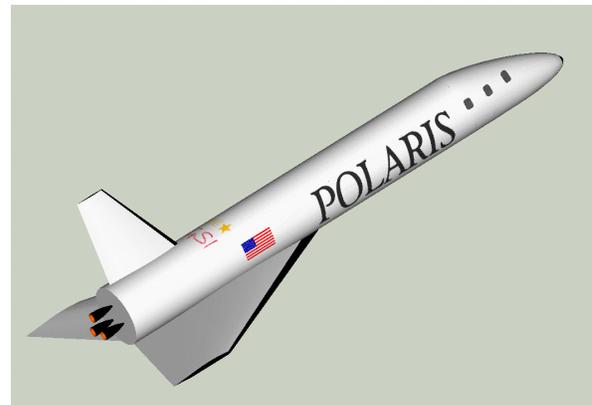
Given the parametric model developed here, the aerodynamic tables may be replaced with parametric equations. This will allow POST to internally calculate the force coefficients at run-time as opposed to repeatedly "looking up" values in a table, reducing CPU time. In addition, POST may use its internal optimization routines to vary the geometric parameters for which the parametric model was developed and calculate the optimum values of those parameters for a given objective function (such as minimum vehicle dry weight).

Use of the parametric model would thus provide a method of rapidly performing multiple design iterations in which vehicle geometry is changed, essentially bringing aerodynamics back "in the loop," i.e. including APAS analysis within a multi-disciplinary design optimization environment.

## DEVELOPMENT OF MODEL

### *Test Vehicle*

For this research effort, a representative aerospace vehicle designed by a team of graduate students at Georgia Tech was used as a test vehicle. This vehicle, named *Polaris*, is a rocket-powered reusable commercial launch vehicle designed for the space tourism market. It has a winged body configuration allowing horizontal take-off and landing, a fuselage fineness ratio of 8, swept wing (leading edge sweep angle =  $55^\circ$ ) with an aspect ratio of 1.86, and single vertical tail. A three-dimensional CAD model of the vehicle was produced using the I-DEAS solid modeling package. This model is shown in Figure 2.



**Figure 2 - Polaris launch vehicle**

### *APAS Analysis*

The aerodynamic analysis for the test vehicle was performed for a sub-orbital trajectory. The flight path was assumed to follow a trajectory from sea level to an altitude of about 250,000 feet, from Mach 0.3 at liftoff to Mach 18.0 at altitude. APAS was used to calculate the force coefficients for fourteen particular Mach numbers in the specified range, at nine angles of attack

for each Mach number, ranging from -15 degrees to +15 degrees. Thus a total of 126 data points for each force coefficient were obtained for use in the basic model. From this dataset, parametric equations were developed to calculate lift and drag as a function of Mach number.

Additional models of *Polaris* were generated in APAS in which the wing aspect ratio was varied. In this case, analysis was performed for models with wing aspect ratios of 1.5, 2.0 and 2.5 (in addition to the 1.86 AR wing). In total, 504 data points were generated for each force coefficient in order to carry out the regression analysis.

The appropriate form of the equations used to calculate lift and drag coefficients may be seen by plotting  $C_l$  vs. angle of attack and  $C_l$  vs.  $C_d$  (drag polar). These plots are shown for the test vehicle at three different Mach numbers: 0.3, 1.5 and 8.0.

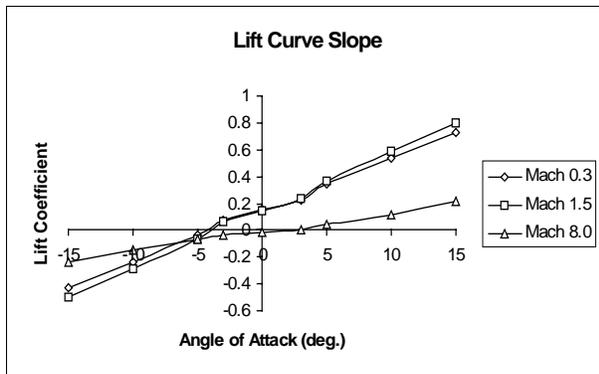


Figure 3 - Lift Curve Slope

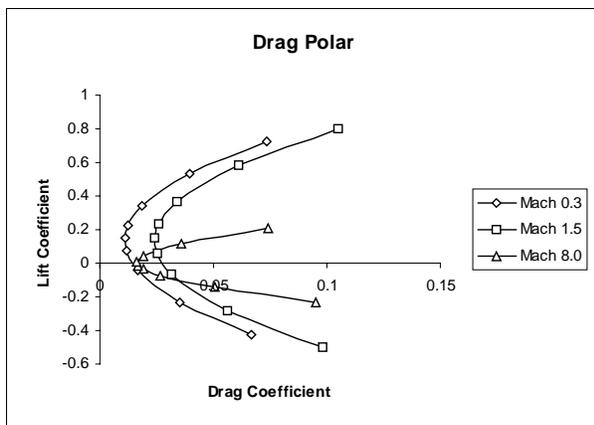


Figure 4 - Drag Polar

As indicated in the plot of lift curve slope (Figure 3), the relationship between lift and angle of attack is nearly linear for a given Mach number, hence the following equation applies:

$$C_l = C_{l_0} + S * \alpha \tag{1}$$

where  $C_{l_0}$  is the lift coefficient at an angle of attack of zero degrees,  $S$  is the lift slope curve parameter and  $\alpha$  is the angle of attack (either degrees or radians may be used in the regression; here angles in radians were used).

For a given Mach number, the relationship between lift and drag is non-linear, and the shape of the curves in the graph above suggests the use of an equation of the form:

$$C_d = C_{d_0} + K * C_l^2 \tag{2}$$

or, alternately:

$$C_d = C_{d_0} + K_1 * C_l + K_2 * C_l^2 \tag{3}$$

where  $C_{d_0}$  is the drag coefficient corresponding to zero lift condition and  $K$ ,  $K_1$  and  $K_2$  are parameters.

In these two equations, the constants  $C_{l_0}$ ,  $C_{d_0}$ ,  $K$ , and  $S$  expected in a conventional single Mach number analysis are replaced with quadratic or cubic polynomial equations that vary these coefficients with Mach number,  $M$ . For example, the equation relating  $K$  as a function of Mach number may take the form,

$$K = K(M) = \beta_0 + \beta_1 * M + \beta_2 * M^2 \tag{4}$$

Similar polynomial equations can be determined for  $C_{l_0}$ ,  $C_{d_0}$  and  $S$ . In this way, the simple relationships of equations (1) and (2) can be extended to model the entire flight regime.

The second goal of this research was to extend the multivariable regression analysis technique described above to include *geometric* variables in the parametric equations. For example, an equation was derived to calculate  $C_l$  and  $C_d$  over the entire flight regime as a function of the wing aspect ratio. APAS was used to generate tables of lift and drag coefficients for three different aspect ratios. Then, using the basic

relationships in equations (1) and (2), a new version of equation (4) was determined having the form,

$$K(M) = \beta_0 + \beta_1 * M + \beta_2 * AR + \beta_3 * M * AR + \beta_4 * AR^2 + \beta_5 * M^2 \quad (5)$$

where AR represents wing aspect ratio. Quadratic polynomial equations such as this are commonly used in regression analysis to capture the first and second order effect of each variable as well as the interaction effect between the two variables.

### Regression Analysis

To develop the desired parametric equations, methods of regression analysis were used to model the datasets generated in APAS. Specifically, the Least Squares method was employed to determine the relations between Mach number, lift and drag. The APAS data was imported into a Microsoft Excel spreadsheet and the regression analysis was carried out using Excel's built-in regression tool.

Equations of the form of (1) and (2) (and equation (3) when including both first and second order terms produced more accurate results) were derived for the set of  $C_l$  and  $C_d$  coefficients corresponding to each Mach number at which the test vehicle was analyzed in APAS. In other words, a regression was done at each Mach number with  $\alpha$  as the independent variable and  $C_l$  and  $C_d$  as response variables. From this, values of  $C_{l0}$ ,  $C_{d0}$ ,  $S$  and  $K$  were obtained at each Mach number.

The values of  $C_{l0}$ ,  $C_{d0}$ ,  $S$  and  $K$  were then used to perform a new regression for each of these parameters against Mach number, i.e. using Mach number as the independent variable and  $K$  and  $S$  as response variables. This regression permitted the determination of suitable equations describing the relationship between these four parameters and Mach number.

As discussed above, two separate codes, UDP and HABP, are used in conjunction with APAS to calculate the force coefficients for the subsonic and hypersonic regimes. Unfortunately, neither one is well suited for the transonic regime. UDP, a code based on a vortex panel method and slender body theory, is valid for linearized subsonic and low supersonic flow. HABP is valid for hypersonic flow, essentially for speeds above Mach 4, since it is based on various

impact methods such as Newton's sine squared law. Thus, the region between Mach 2 and Mach 4 is difficult to model accurately with APAS. Typically, UDP is used up to about Mach 1.5 - 2.0, and HABP is used above Mach 2.0 to essentially "split the difference."

Also, in order to model the mathematical discontinuity that occurs at Mach 1, two separate parametric models were developed. A complete set of parametric equations was derived for subsonic flow and a second set was derived for Mach numbers greater than 1.0. The resulting equations for each flow regime are given below.

## RESULTS

### Basic Geometry over Mach Number Range

The equations developed for *Polaris* in the subsonic flow regime using the regression analysis techniques are as follows:

#### Subsonic Flow

$$C_{l0} = 0.153 - 0.006 * M + 0.01 * M^2 \quad (6)$$

$$C_l = C_{l0} + S * \alpha \quad (7)$$

$$S = 2.293 - 0.666 * M + 1.071 * M^2 \quad (8)$$

$$C_{d0} = 0.014 - 0.0002 * M - 0.0016 * M^2 \quad (9)$$

$$K_1 = -0.047 - 0.003 * M + 0.005 * M^2 \quad (10)$$

$$K_2 = 0.177 + 0.005 * M - 0.011 * M^2 \quad (11)$$

$$C_d = C_{d0} + K_1 * C_l + K_2 * C_l^2 \quad (12)$$

The efficiency of each of these equations is shown graphically below in Figures 5 - 9. In each plot, the actual data points generated in APAS are shown in bold lines and the fitted curves are shown in dashed lines.

The coefficient of determination,  $R^2$ , is also shown for each curve fit. The coefficient of determination is an indicator of the measure of variability in the response variable that is accounted for by the predictor variable(s), and thus provides a measure of the validity of the regression model used. A coefficient of determination equal to one indicates a perfect fit of the equation to the data (all data points fall on the regression line) and a value equal to zero

indicates no relation whatsoever between the response variable and predictor variable(s).

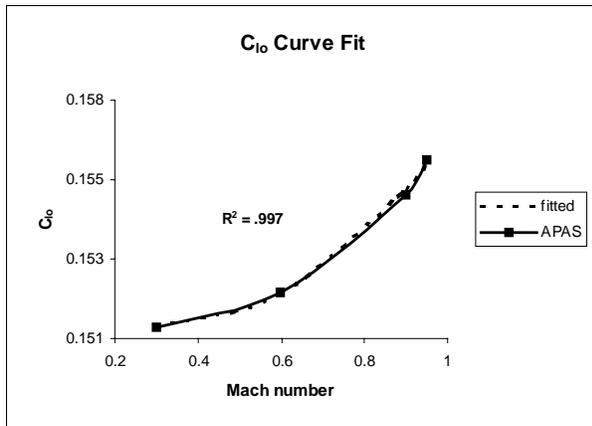


Figure 5 –  $C_{1o}$  Curve Fit

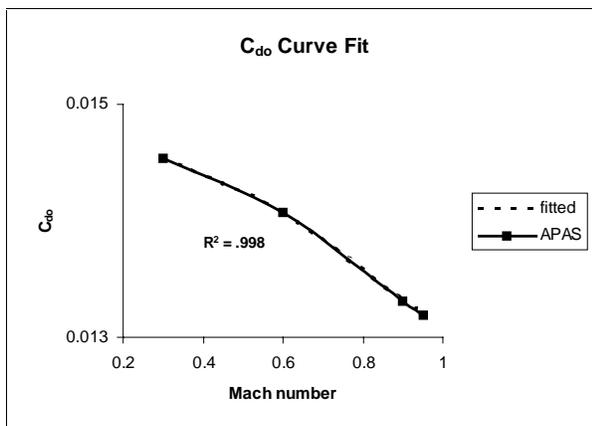


Figure 6 –  $C_{do}$  Curve Fit

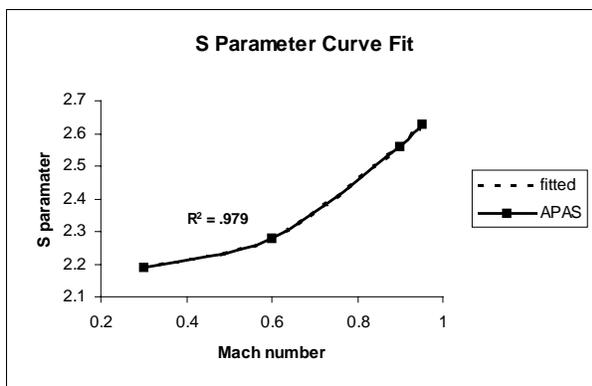


Figure 7 –  $S$  Parameter Curve Fit

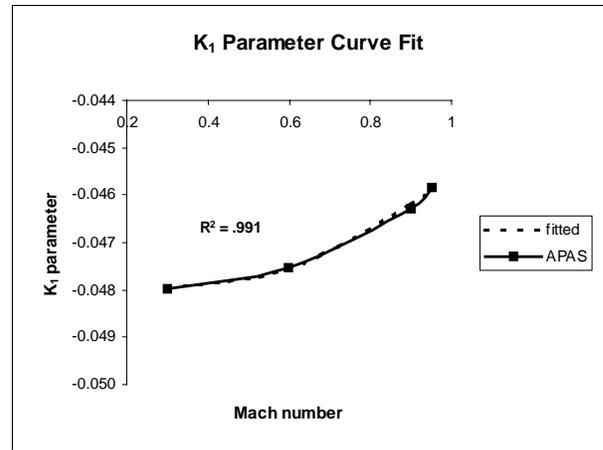


Figure 8 –  $K_1$  Parameter Curve Fit

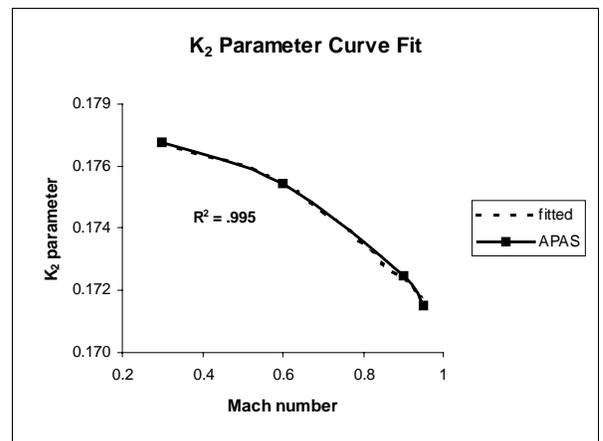


Figure 9 –  $K_2$  Parameter Curve Fit

As the plots show, excellent curve fits were obtained for all of the parameters as a function of Mach number. This indicates that the regression model used accurately represents the APAS tabular data and provides a means of accurately calculating the aerodynamic coefficients for the test vehicle.

The equations developed for the test vehicle in the supersonic flow regime are the following:

Supersonic Flow

$$C_{1o} = 0.214 - 0.082 * M + 0.008 * M^2 - 0.0002 * M^3 \quad (13)$$

$$C_l = C_{1o} + S * \alpha \quad (14)$$

$$S = 3.214 * M^{-0.639} \quad (15)$$

$$C_{do} = 0.04 - 0.005 * M + 0.0002 * M^2 \quad (16)$$

$$K = -0.291 + 0.396 * M - 0.029 * M^2 + 0.0007 * M^3 \quad (17)$$

$$C_d = C_{do} + K * C_l^2 \quad (18)$$

The curve fit for each of these equations is shown graphically below.

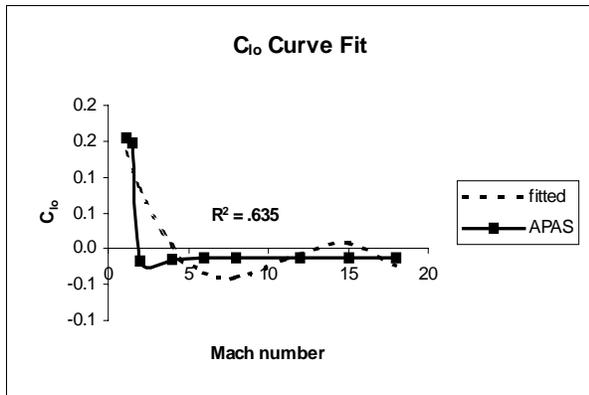


Figure 10 -  $C_{l_0}$  Curve Fit

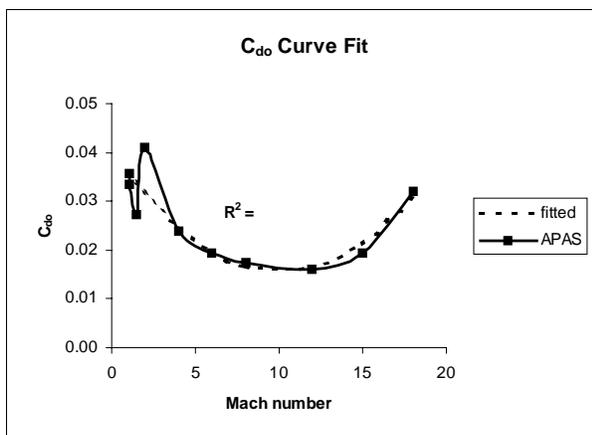


Figure 11 -  $C_{d_0}$  Curve Fit

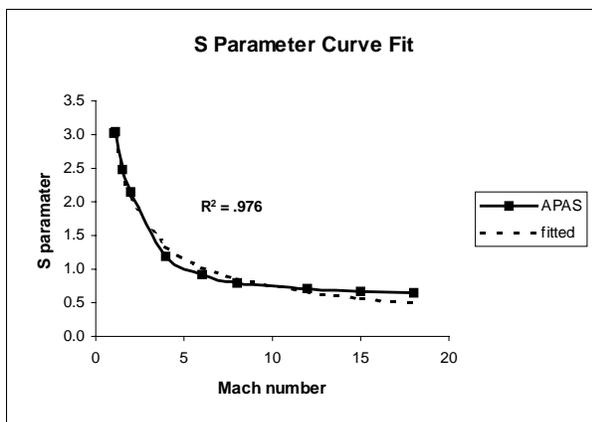


Figure 12 - S Parameter Curve Fit

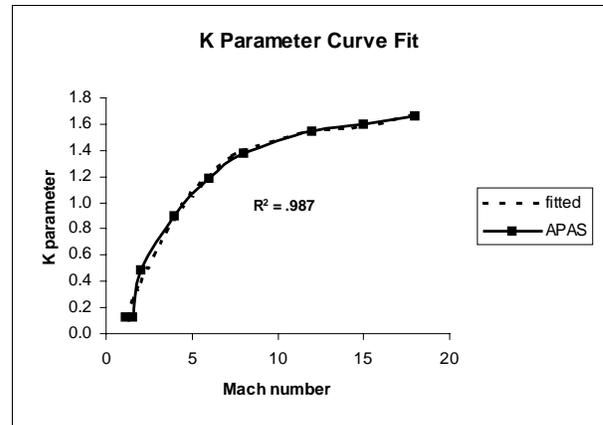


Figure 13 - K Parameter Curve Fit

As the above plots show, excellent curve fits were obtained for the S and K parameters in the supersonic flow model. However, it was somewhat difficult fitting a curve to the lift and drag coefficient data due to the different manner in which these values are calculated in UDP and HABP. The break in the data at Mach 2 where the analysis tool used changes from UDP to HABP is obvious.

For example, the jump in  $C_{d_0}$  at Mach 2 that is seen in Figure 11 is not an actual reflection of conditions encountered in flight, but simply a result of the different analysis method employed by the tool. In this case in particular, the smooth fitted curve may actually give a more accurate reflection of flight conditions.

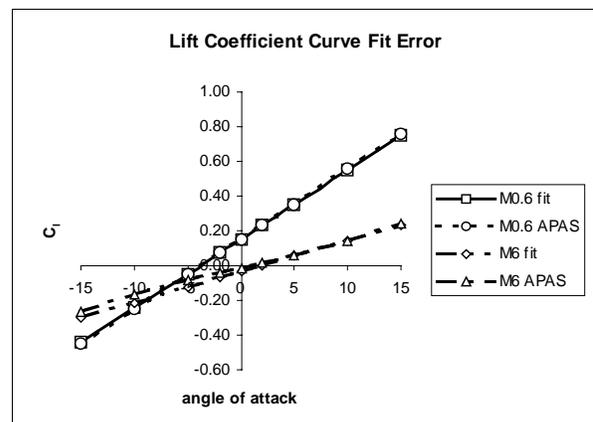


Figure 14 - Lift Coefficient Error

The error between the fitted curves and explicit APAS data was determined also in order to evaluate the accuracy of the parametric equations. The error in lift and drag coefficients at both subsonic and

supersonic flow conditions is graphically illustrated in Figures 14 and 15.

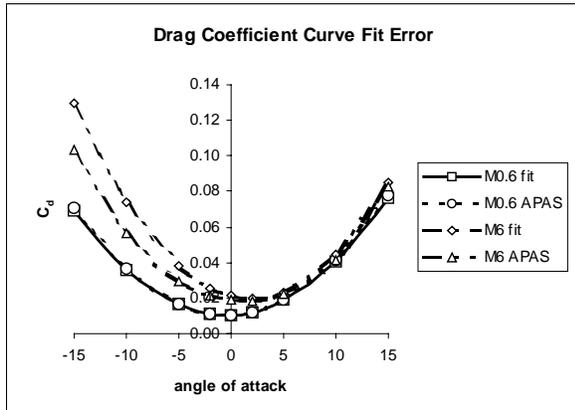


Figure 15 – Drag Coefficient Error

As may be seen in the above plots, the curve fits for subsonic flow velocities are extremely accurate.

The error between the fitted data and APAS data is on the order of 1% at a given data point. The results for supersonic flow conditions are also quite accurate for lift coefficient. The supersonic drag coefficient curve fit is less accurate, with most of the error occurring primarily at negative angles of attack.

### Wing Aspect Ratio

The techniques discussed above were applied to changing the aspect ratio of the test vehicle's wing. Three additional models were created in APAS, all identical except for aspect ratio. Aspect ratios of 1.5, 2.0 and 2.5 were used, and parametric equations were then developed to calculate  $C_{l_0}$ ,  $C_{d_0}$ ,  $C_l$ ,  $C_d$ ,  $S$ ,  $K_1$  and  $K_2$  as a function of wing aspect ratio and Mach number. The resulting equations are shown here:

#### Subsonic Flow

$$C_{l_0} = 0.11 - 0.013*M + 0.028*AR - 0.0003*M*AR + 0.0063*AR^2 + 0.0234*M^2 \quad (19)$$

$$C_{d_0} = 0.104 - 0.005*M + 0.002*AR - 0.0001*M*AR - 0.0004*AR^2 + 0.0027*M^2 \quad (20)$$

$$S = -0.664 - 1.606*M + 2.413*AR + 0.429*M*AR - 0.46*AR^2 + 1.046*M^2 \quad (21)$$

$$K_1 = -0.049 + 0.0023*M + 0.029*AR - 0.002*M*AR - 0.005*AR^2 + 0.004*M^2 \quad (22)$$

$$K_2 = 0.62 - 0.018*M - 0.34*AR + 0.014*M*AR + 0.058*AR^2 - 0.018*M^2 \quad (23)$$

#### Supersonic Flow

$$C_{l_0} = 0.099 - 0.022*M - 0.013*AR - 0.0002*M*AR + 0.004*AR^2 + 0.001*M^2 \quad (24)$$

$$C_{d_0} = 0.047 - 0.071*M + 0.016*AR + 0.0002*M*AR - 0.0006*AR^2 + 0.0004*M^2 \quad (25)$$

$$S = 0.747 - 0.339*M + 1.752*AR - 0.051*M*AR - 0.263*AR^2 + 0.018*M^2 \quad (26)$$

$$K_1 = -0.026 - 0.0005*M + 0.012*AR - 0.002*M*AR - 0.002*AR^2 + 0.00001*M^2 \quad (27)$$

$$K_2 = 0.268 + 0.262*M - 0.291*AR - 0.002*M*AR + 0.057*AR^2 - 0.01*M^2 \quad (28)$$

The accuracy of the above in the subsonic flow regime is again very good. However, the error of the supersonic equations is considerable for the lift coefficients. This is shown in Figures 16 and 17, where the force coefficients were calculated using an aspect ratio of 2.0. The error between fitted data points and APAS data points for lift coefficients was as high as 38%. In contrast, the accuracy of the equations are good for supersonic drag coefficients, with the error between a given fitted data point and the corresponding APAS data point being about 11% on average. This is an interesting result given that the drag coefficients are calculated from the lift coefficients.

### FUTURE WORK

Work is currently underway to develop a tool that would automatically carry out the regression analysis on APAS datasets. This tool will be written in Fortran and will allow the user to supply an APAS output file as an input, then receive a set of parametric equations as output. The Fortran code will use the regression model (i.e. equations of the form shown above) developed through this research effort. The Fortran code will rely on matrix operations and manipulation to determine the coefficients ( $S$ ,  $K_1$ ,  $K_2$ ,  $\beta_0$ ,  $\beta_1$ , etc.) in each of the parametric equations.

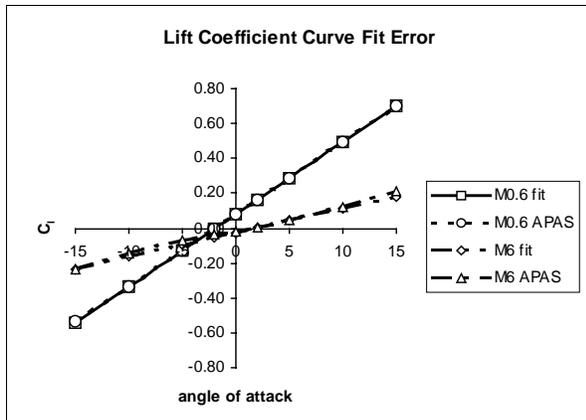


Figure 16 - Lift Coefficient Error

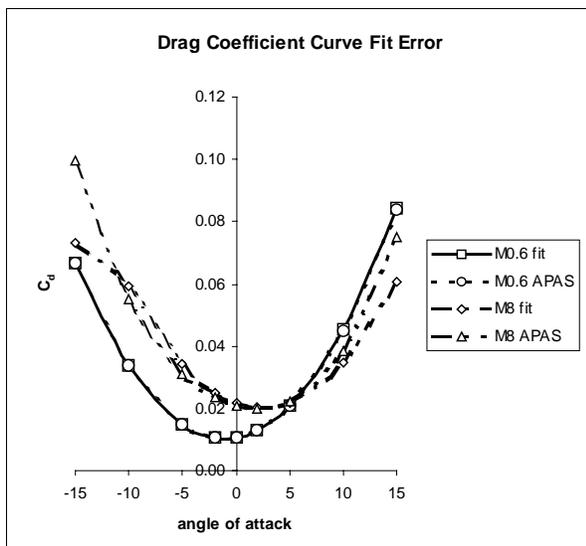


Figure 17 - Drag Coefficient Error

Once complete, this tool will be used to perform optimization studies of a reusable launch vehicle, Hyperion, designed at Georgia Tech. Hyperion is a horizontal take-off, horizontal landing, advanced aerospace vehicle using rocket-based combined cycle engines. The aerodynamic regression tool will specifically be used to determine the optimum nose cone angle of the vehicle.

## SUMMARY

This research has demonstrated that aerodynamic datasets generated in APAS for a given aerospace vehicle may be successfully reduced to a set of parametric equations through methods of linear regression. While the resulting accuracy of the

parametric equations is very good for dataset transformations involving force coefficients as a function of Mach number only, the accuracy of the equations generated by fitting force coefficients as a function of Mach number and a specific geometric parameter (wing aspect ratio in this case) is less accurate. It appears from this research that regression analysis is perhaps not applicable to the latter case, at least not using a regression model of the form of equation (5). Further research would be required to determine a more suitable model, perhaps using additional predictor variables, in order to obtain parametric equations whose accuracy is within an acceptable error range.

## REFERENCES

1. Schnackel, J; Dovenmuehle, R., "Statistical Experimental Design and its Role in Aerospace Vehicle Design Efforts," AIAA 90-1809, AIAA Aerospace Engineering Conference & Show, February 1990.
2. Neter, John; Kutner, Michael; Nachtsheim, Christopher, and Wasserman, William; Applied Linear Statistical Models, 4th Edition, Irwin, Chicago, IL, 1996.
3. Anderson, John D. Fundamentals of Aerodynamics, 2nd Edition. McGraw Hill, New York, NY, 1984.
4. Vanderplaats, Garret N., Numerical Optimization Techniques for Engineering Design, McGraw Hill, New York, NY, 1984.
5. Bonner, E., Clever, W., and Dunn, K., *Aerodynamic Preliminary Analysis System II, Part I - Theory*, NASA Contractor Report #182076, April 1991.
6. Sova, G. and Divan, P., *Aerodynamic Preliminary Analysis System II, Part II - User's Manual*, NASA Contractor Report #182077, April 1991.