DETERMINISTIC MODELING OF A ROTARY LIP SEAL WITH MICROASPERITIES ON THE SHAFT SURFACE

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DETERMINISTIC MODELING OF A ROTARY LIP SEAL WITH
MICROASPHERITIES ON THE SHAFT SURFACE

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
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<tbody>
<tr>
<td>(A_1)</td>
<td>half amplitude of lip surface fluctuation, (A_1^<em>/h_r^</em>)</td>
</tr>
<tr>
<td>(A_2)</td>
<td>half amplitude of shaft surface fluctuation, (A_2^<em>/h_r^</em>)</td>
</tr>
<tr>
<td>(A_c)</td>
<td>contact area ratio, (A_c^*/(L^<em>B^</em>))</td>
</tr>
<tr>
<td>(a^*)</td>
<td>radius of contact region in Figure 3.3.2</td>
</tr>
<tr>
<td>(B^*)</td>
<td>width of solution space (sealing zone) in axial (y) direction</td>
</tr>
<tr>
<td>(C)</td>
<td>defined by Equation (3.2.4)</td>
</tr>
<tr>
<td>(D)</td>
<td>initial average film thickness when the lip is mounted to the shaft</td>
</tr>
<tr>
<td>(d^*)</td>
<td>depth of the geometric overlap for each individual asperity contact</td>
</tr>
<tr>
<td>(E^*)</td>
<td>elastic modulus</td>
</tr>
<tr>
<td>(E')</td>
<td>equivalent Young’s modulus, (E^<em>/(1-\nu^</em>^2))</td>
</tr>
<tr>
<td>(F)</td>
<td>fraction of sealing zone area that is cavitated</td>
</tr>
<tr>
<td>(f^*)</td>
<td>dry friction coefficient between shaft and lip</td>
</tr>
<tr>
<td>(h)</td>
<td>film thickness, (h^<em>/h_r^</em>)</td>
</tr>
<tr>
<td>(h_{avg})</td>
<td>dimensionless average film thickness, (h_{avg}^<em>/h_r^</em>)</td>
</tr>
<tr>
<td>(I_1)</td>
<td>influence coefficient for normal (radial) deformation</td>
</tr>
</tbody>
</table>
\( I_2 \) influence coefficient for shear (circumferential) deformation

\( K \) time ratio in transient lubrication model, \( t^*/T^* \)

\( L^* \) length of solution space in circumferential (x) direction

\( p \) dimensionless pressure, \( (p^*-p_c^*)/(p_s^*-p_c^*) \), where \( p^* = p^*_{\text{fluid}} \) in non-contact region and \( p^* = p^*_{\text{Hertzian}} \) in contact region

\( p^{**} \) dimensionless pressure for deformation analysis, \( [p(p_s^*-p_c^*)+p_c^*]/E^* \), where \( p = p_{\text{fluid}} \) in non-contact region and \( p = p_{\text{Hertzian}} \) in contact region

\( p^*_{\text{contact}} \) static contact pressure

\( p^{**}_{\text{contact}} \) dimensionless static contact pressure for deformation analysis, \( p^*_{\text{contact}}/E^* \)

\( p_{\text{fluid}} \) dimensionless fluid pressure, \( (p^*_{\text{fluid}}-p_c^*)/(p_s^*-p_c^*) \)

\( p_{\text{Hertzian}} \) dimensionless Hertzian contact pressure, \( (p^*_{\text{Hertzian}}-p_c^*)/(p_s^*-p_c^*) \)

\( Q \) reverse pumping rate, \( \mu^*B^*Q^*/(p_s^*-p_c^*)h_r^*3 \)

\( R' \) equivalent radius of curvature, \( 1/(1/R_{\text{tip}}^* + 1/R_{\text{shaft}}^*) \)

\( R_a \) average roughness height

\( r^* \) distance to the center point of each individual asperity contact circle

\( r_c \) contact load ratio, \( W_{\text{Hertzian}}/W \)

\( S \) cavitation index
t \quad \text{time, } U_r t^*/L^*

T^* \quad \text{acceleration time or deceleration time for transient mixed lubrication model}

U^* \quad \text{speed of shaft surface}

U \quad \text{dimensionless pressure, } U^*/U_r^*

W \quad \text{total load support, } W^*/(p_s^*-p_c^*)L^*B^*

W_{\text{Hertzian}} \quad \text{dimensionless asperity contact load support in the contact region,} \quad W_{\text{Hertzian}}^*/(p_s^*-p_c^*)L^*B^*

W_{\text{fluid}} \quad \text{dimensionless hydrodynamic load support in the non contact region,} \quad W_{\text{fluid}}^*/(p_s^*-p_c^*)L^*B^*

x \quad \text{circumferential coordinate, } x^*/L^*

y \quad \text{axial coordinate, } y^*/B^*

\Lambda \quad \text{aspect ratio, } L^*/B^*

\beta_1(t) \quad \text{dimensionless parameter in Equation (3.4.2), } 6\mu^*U^*(t)L^*/[h_r^*]^2(p_s^*-p_c^*)

\beta_2 \quad \text{dimensionless parameter in Equation (3.4.2), } 12\mu^*L^*U_r^*[h_r^*]^2(p_s^*-p_c^*)

\delta \quad \text{dimensionless circumferential displacement of lip, } \delta^*/L^*

\Phi \quad \text{variable representing pressure/average density, defined by Equation (3.1.5)}

\gamma \quad \text{dimensionless shaft speed, } 6\mu^*U_r^*L^*/[h_r^*]^2(p_s^*-p_c^*)
\( \lambda_{11} \) lip surface wavelength in x direction, \( \lambda_{11}^*/L^* \)

\( \lambda_{12} \) lip surface wavelength in y direction, \( \lambda_{12}^*/L^* \)

\( \lambda_{21} \) shaft surface wavelength in x direction, \( \lambda_{21}^*/L^* \)

\( \lambda_{22} \) shaft surface wavelength in y direction, \( \lambda_{22}^*/L^* \)

\( \mu^* \) viscosity of lubricant

\( \rho \) average density, \( \rho^*/\rho_r^* \)

\( \tau^* \) shear stress

\( \nu^* \) Poisson’s ratio

\( \xi \) defined by equation (3.4.4)

**Subscripts**

( )\(_0\) smooth shaft value

( )\(_a\) ambient

( )\(_\text{avg}\) average in both time t and x – direction

( )\(_c\) cavitation

( )\(_i\) evaluated at i node

( )\(_r\) reference
( )

sealed liquid side of seal

**Superscripts**

( )

dimensional quantity

( )

dimensionless parameter for deformation analysis

( )
equivalent value
SUMMARY

The rotary lip seal is the most widely used dynamic seal. It is used extensively in the automotive and appliance industries. Experimentally, it is well known that the microasperities on the shaft surface can significantly affect the performance of a lip seal, even though the shaft roughness, after run-in, is much smaller than the lip roughness. In the present study, several deterministic numerical models are developed to investigate the effect of shaft surface finish on rotary lip seal behavior, through an understanding of the basic physics of lip seal operation.

This project is performed in a step by step manner with gradually increasing complexity. Four models are included in this study: hydrodynamic analysis, elastohydrodynamic analysis for full film lubrication, mixed-EHL model for mixed lubrication with asperity contact, and transient dynamic mixed-EHL model for startup and shutdown processes. Those analyses allow the examination of some important seal characteristics, such as the load support sharing between hydrodynamic and contact pressure, contact and cavitation area ratio, reverse pumping rate, liftoff speed for tracing the liftoff process and average film thickness. The development of fluid, contact and cavitation areas as a result of the changing operation condition is also examined.

The results of the present deterministic modeling indicate that shaft surface roughness can produce significant desirable effects on lip seal behavior. An appropriate shaft surface profile could improve the sealing ability and prevent seal failure.
CHAPTER 1
INTRODUCTION

1.1 Problem Description

The rotary lip seal is the most widely used type of dynamic seal. It is used in the automotive industry as crankshaft seals, transmission seals, wheel seals, axle pinion seals and power steering seals. It is also used in other industries such as appliance and general machinery, e.g., as bearing seals, washing machine seals, dishwasher seals and gear box seals. Every year millions of these seals are manufactured; such extensive usage inevitably has led to a significant impact on the environment and economy. Leakage of oil from poorly designed seals is both an environmental and economic problem. The excessive frictional losses in failed seals and downtime due to seal failures are additional economic problems.

At the present time, rotary lip seals are designed exclusively by empirical methods, with expensive and time-consuming test programs because of the absence of analytical design tools. Although a considerable amount of research had been done on this type of seal since the 1950’s, it is only recently that a fundamental understanding of lip seal operation has been achieved. Still, there are a number of important aspects of lip seal behavior that remain unexplored. One of these is the role of the shaft surface finish.

A number of theoretical and numerical studies have recently been made to mathematically model the lip seal. However, all of those models treat the shaft surface as perfectly smooth due to the small roughness height on the shaft surface. Those models do not account for the role of the shaft surface microgeometry on the lip seal operation even though practical experience has shown that the roughness on the shaft surface plays an
important role. There is therefore a strong need for analytical design tools that account for the effect of the shaft surface finish on lip seal performance.

### 1.2 Objective of the Research

This thesis aims at developing numerical models in which the important characteristics for the lip seal in a given operation condition are determined to predict the lip seal performance. Interasperity cavitation and shear deformation are considered in this work. The topography of the roughness on both surfaces provides the regions of divergent film which cause cavitation. The effect of cavitation on flow must be considered in order to accurately predict lubricant performance. The shear deformation is also included in the numerical models to simulate the circumferential deformation on the lip surface.

The models numerically simulate the roughness on the tribo-pair surfaces (lip surface and shaft surface). The performance of the seal in the full film lubrication region or mixed-lubrication regime can then be characterized by the following quantities: the load support, the contact load support ratio (contact force/total force), reverse pumping rate, cavitation area ratio (cavitation area/ nominal area), and the contact area ratio (contact area/nominal area). In this work, the contact load ratio and the contact area ratio reflect the severity of the contact condition; the cavitation area ratio shows the severity of interasperity cavitation effect.
1.3 Brief Summary of Each Chapter

Chapter 1 contains a brief introduction to the project; the objective of the research is also contained in this chapter.

Chapter 2 focuses on the research background of each numerical model: the hydrodynamic model, the elastohydrodynamic model for full film lubrication; the mixed lubrication model with asperity contact and the transient mixed lubrication model for start up and shut down processes. This part gives a fundamental explanation for rotary lip seal operation in different regions.

Chapter 3 is the theoretical focus of the thesis. The mathematical and numerical model developments are presented in this chapter. In the hydrodynamic model, without considering the lip surface deformation, the fluid mechanics is the main issue. The numerical scheme for analyzing the fluid mechanics with an emphasis on cavitation is presented. The second model for full film lubrication is the elastohydrodynamic model in which the normal and shear deformation of the lip surface are considered. This model consists of two elements, fluid mechanics and deformation mechanics. These two parts are strongly coupled and have a significant effect on each other.

A more complex model for the lip seal is the mixed lubrication model. When the shaft rotates at a relatively low speed, the lip is not completely lifted off the shaft and the total load support is shared by the hydrodynamic load support and the contact load support. So a contact model is included in this mixed lubrication model to simulate the lip seal performance at low speed operation. The Hertzian contact model is utilized to calculate the contact pressure and the contact area in the contact region. This mixed lubrication model consists of three main parts: fluid mechanics, deformation mechanics
and the contact mechanics. All of those analyses are connected to each other and are included in an iterative computational scheme.

The last model deals with the case in which time dependent operating parameters are considered. In this transient model, the shaft speed is not a constant value anymore but varies with the time. This dynamic model is developed to simulate the start up and shut down processes.

Chapter 4 is a summary of the mathematical and numerical model development and results presented in the first three chapters of this thesis, and conclusions.
CHAPTER 2
RESEARCH BACKGROUND

2.1 Hydrodynamic and Elastohydrodynamic Models

Even though elastomeric rotary lip seals have been used since the 1930’s, a comprehensive physical understanding of the fundamental behavior of such seals has remained unclear until recently. A large variety of experimental observations has led to identification of the complex mechanisms of rotary lip seal operation, and the high computational power and advances in numerical modeling have contributed to a comprehensive understanding of lip seal operation.

Figure 2.1 contains a schematic of a typical lip seal. Since the pioneering work of Jagger (1957), it has been known that under typical steady state operating conditions, a micron scale lubricating film of liquid separates the lip from the shaft in such a seal. This film prevents damage to the lip from excessive heat generation and mechanical stresses at the lip - shaft interface. However, discovery of this film led to two important questions. What is the load support mechanism that produces elevated pressures large enough to lift the lip off of the shaft and allow formation of the film? What is the sealing mechanism that prevents liquid from flowing through the film and leaking through the seal?
As a result of empirical observations, controlled experiments and numerical analyses, it is now known that both the load support mechanism and sealing mechanism are intimately connected with the asperities on the lip surface (Salant, 1999). Although the lip surface of a new seal is very smooth, after the break-in process a successful seal has a multitude of asperities on its surface in the sealing zone, between the lip and the shaft (Horve, 1991).

These asperities produce load support by acting like mini slider bearings (Jagger, 1966; Johnston, 1978; Gabelli, 1988, 1992). When liquid is dragged over each asperity by the shaft rotation, the film thickness variation on the upstream side produces a pressure elevation. On the downstream side inter-asperity cavitation occurs, resulting in a net pressure elevation, which keeps the lip lifted off of the shaft and maintains the integrity of the film.
Jagger and Walker (1966) proposed three analytic models, none of which considers cavitation: an inclined bearing, a deformed cylinder and a point contact. All of them gave similar, reasonable results for friction coefficient and film thickness.

One of the shortcomings of the above analytic approaches is that they neglected the occurrence of the cavitation. Experimental research has shown significant degrees of cavitation near the valleys of the asperities (Nakamura and Kawahara, 1984). Due to the necessity of handling such cavitation in the complex environment under the normal operation condition, a numerical analysis is required to model the real lip seal.

Some studies numerically computed the pressure field under the lip (Gabelli, 1988; Gabelli and Poll, 1992). In these works, the asperities are modeled as a uniform distribution by using two-dimensional sinusoids. The fluid mechanics is performed by solving the Reynolds equation using finite difference methods and mass is conserved at the cavitation boundaries by using the JFO (Jakobsson-Floberg-Olsson) boundary conditions. The results numerically show that the asperities hydrodynamically generate pressures to maintain the film, and the film thickness results matched well with the experimental measurements. However, both the macrodeformation of the lip and the deformation of the asperities are not considered. Salant and Flaherty (1994, 1995) simulate the lip seal, considering the asperities and microundulations which deform circumferentially and axially with the bulk material of the lip. In this analysis, the Reynolds equation is solved and cavitation is taken into account with the JFO boundary conditions by using the Elrod algorithm; later study by Day and Salant (1999) also show the thermal effects on the lip seal operation. These analyses all yield reasonable values of the film thickness.
The asperities on the lip surface also are responsible for the sealing mechanism. They produce a reverse pumping action, which tends to pump liquid from the air-side of the seal toward the liquid-side of the seal, opposing the natural flow from the liquid-side toward the air-side, thereby preventing leakage (Stakenborg, 1988; Salant, 1996).

This reverse pumping is primarily generated by a viscous pump mechanism (Qian, 1984; Kammuller, 1986; Muller, 1987). This phenomenon is illustrated in Figure 2.2. Under dynamic conditions, the asperities on the lip surface are distorted into vane-like shapes due to the shear deformation of the elastomeric lip in the circumferential direction (Kawahara, et al., 1980). As liquid is dragged over each vane-like asperity by the shaft rotation, flow is induced in the axial direction. If the axial location of maximum shear deformation is closer to the liquid-side of the seal than to the air-side, as it is in all successful seals (van Leeuwen, et al., 1996), there is a net pumping of liquid from the air-side toward the liquid-side.

![Figure 2.2 Primary Reverse Pumping Mechanism](image)
To verify the previous concepts, a complete numerical analysis is needed since the sealing mechanism strongly depends on the shear deformation which is produced by the shear stresses in the lubrication film. Furthermore, the load support mechanism and sealing mechanism are coupled. Salant (1992) was the first investigator to consider the hydrodynamics of a seal with microundulations in a numerical model. This work assumed the circumferential deformation of the lip is proportional to the local shear stress and in that way the maximum circumferential displacement is closer to the liquid side of the seal than to the air side. The results generated high enough reverse pumping rates to prevent leakage.

The previous numerical models were actually preliminary since the important elastohydrodynamic interaction is not included. The first full elastohydrodynamic model of a seal with microundulations was presented by Salant and Flahety (1994); in their work, the hydrodynamic and the elastic analyses are coupled by the iterative computational procedure. The computed circumferential displacement has a peak closer to the liquid side of the seal than the air side and these results are consistent with the experimental work.

An extension of the above model was given by simulating the lip seal with asperities (Salant and Flaherty, 1995). It was found that the asperities generally produce lower pumping rates than undulations and the pumping rate depends on the number and shape of the asperities. The analyses of those two references both treat the air side of the seal as flooded in order to compute the reverse pumping rate. Salant (1996) presented a numerical model with a non-flooded air side. This model combined an elastohydrodynamic analysis with a simplified analysis of a meniscus separating the
liquid from the air on the air side of the seal. Stakenborg (1988) showed that a meniscus leads to a steady state non-leaking condition.

A main feature in all the above models is the asymmetric circumferential lip displacement distribution with the peak closer to the liquid side than to air side and that is necessary for reverse pumping occurring. Van Leeuwen and Wolfert (1996) measured the circumferential displacement distributions of new and run-lips and have found that the running-in process didn’t change the asymmetric character of the displacement distribution.

A secondary reverse pumping mechanism due to inter-asperity cavitation that supplements this viscous pump mechanism has recently been discovered (Shi and Salant, 2001). A slight axial variation in the average film thickness can produce a variation in the degree of inter-asperity cavitation which generates additional reverse pumping.

All of the theoretical and numerical studies of lip seal behavior, cited above, treat the shaft surface as perfectly smooth. This is because, ever since the work of Jagger, it has been well known that the shaft is much smoother than the lip (Jagger and Walker, 1966). Typically the Ra of the shaft surface is one-tenth that of the lip surface. However, practical experience indicates the shaft surface finish is very important to lip seal performance. It has long been known that if the shaft is too rough or too smooth, the seal will fail. This is reflected in the RMA (Rubber Manufacturers Association) standard on upper and lower limits on shaft surface Ra (Rubber Manufacturers Association, 1985). More recently, a number of studies have revealed that not only is the amplitude of the shaft surface fluctuations (as measured by Ra) important, but the shaft surface profile is
also important (Qu, 1995, 1996; Jackowski and Lavoie, 1999). This has led the RMA to revise their standards to include Rz and Rpm (Rubber Manufacturers Association, 1999).

One can conceive of several roles of the shaft, in which the shaft surface finish exerts a strong influence:

i. irregularities on the shaft surface provide reservoirs to store liquid under static conditions, which supply liquid to form the lubricating film at startup;

ii. the shaft surface conditions the lip during the break-in period, generating asperities by preferential wear;

iii. shaft surface fluctuations affect the load support and sealing mechanisms through the hydrodynamics (or elastohydrodynamics) of the lubricating film.

It is this last role of the shaft that is the principal subject of the present study.

In the hydrodynamic model and the elastohydrodynamic model, the separation between the lubricated, relatively moving surfaces is on the same order as the surface roughness. Once the fluid is traveling through a diverging gap which is on the order of the roughness, the fluid pressure will drop and may even decrease to a certain threshold pressure usually referred to as cavitation pressure, and then the resulting “inter-asperity” cavitation will significantly affect the fluid mechanics.

Gaseous cavitation is a familiar phenomenon in lubrication. It has been observed as a result of surface micro-geometry (Hamilton et al., 1965; Stakenborg, 1988). One of the earliest explanations for the load support generation which is observed in thrust bearings and seals with parallel surfaces (Jagger, 1957, 1966; Hamilton et al., 1965;
Lebeck, 1987a, 1987b) is the following: when the cavitation is absent, the hydrodynamic pressure profile is expected to be anti-symmetric and then there is no net load support. If there is cavitation, the cavitated pressure will be truncated and the load support will be generated, as shown in Figure 2.3.

![Figure 2.3 Cavitation Load Support about an Asperity](image)

The deterministic model can accurately predict the cavitation zone location. The pioneer work which is known as the JFO theory was developed by Jakobsson and Floberg (1957) and Olsson (1965) for dealing with the cavitation phenomenon. Their work takes account of the mass conservation and the physics of the fluid, and provides appropriate boundary conditions. Elrod and Adams (1974) provided a new numerical algorithm based on JFO boundary conditions. They use fluid compressibility and develop a universal differential equation governing the fluid behavior in the lubrication region and the fluid/gas mixture behavior in the cavitation region. Elrod (1981) extended this basic idea in his later work in which he used a switching function between the lubrication and
cavitation regions. A central differencing scheme is utilized for the flow in the full film region and a backward scheme is used in the cavitation region.

More work on solving the cavitation problem was developed by Payvar and Salant (1992) using their version of a universal differential equation. The basic idea is straightforward, easy for programming and can yield accurate solutions. Kumar and Booker (1991a,b) gave a finite element method used in the cavitation algorithm; later Yu and Keith (1994) introduced a boundary element method for the cavitation algorithm.

All the previously mentioned studies stem from Elrod’s idea of a universal differential equation in which the flow pressure and partial film content are connected by a switching function. The universal differential equation is actually an elliptic equation and a hyperbolic equation linked by the switching function.

2.2 Mixed Lubrication Model and Transient Mixed Lubrication Model

As described above, under typical steady state operating conditions, a liquid film separates the lip from the shaft. However, at very low speeds and during transient operation, asperity contact between the lip and the shaft occurs. To study such operation, a mixed lubrication model is required.

The mixed lubrication model, which involves asperity contacts between two relatively moving surfaces, has been considered one of the most challenging works in tribology in the last two decades. In general, a mixed elastohydrodynamic lubrication model contains several basic coupled submodels: 1. The fluid model, to analyze the effect of rough surfaces on the fluid mechanics of the lubricating film, 2. The deformation model, to study structure distortion and the surface deformation, 3. The roughness contact
model, to distinguish the contact region and fluid region from the whole domain, and to determine the contact area and contact load ratios.

All these models are coupled and are joined to each other through numerical iteration. Many researchers (Patir and Cheng, 1978; Elrod, 1979; Tripp, 1983; Greenwood and Williamson, 1996; Lee and Cheng, 1992; Ren and Lee, 1993; Ju and Farris, 1996; Stanley and Kato, 1997; Shi and Salant, 2000) have made important contributions to both the theoretical and numerical analysis of mixed lubrication modeling.

In some previous mixed lubrication models (Yamaguchi and Mastuokda, 1992; Wang and Cheng, 1995; Chang, 1995; Wang and Shi, 1997), the three submodels are not completely coupled. Those models simply assume the surface roughness, while the latter is actually an unknown function of average gap, due to the micro-EHL effect in the whole domain. The contact sub-model is separated from the fluid mechanics analysis in those models as well. Other mixed-EHL models (Lubrecht et al., 1988; Venne and Lubrecht, 1994; Zhu and Ai, 1997; Hua et al., 1997; Jiang et al., 1998) only consider a simple film thickness distribution appropriate to either point contact or line contact; neither interasperity cavitation nor surface shear deformation are taken into account.

A more complete deterministic modeling of the mixed lubrication problem has been developed by Shi and Salant (2000). Their model includes the hydrodynamic lubrication, cavitation, and rough surface contact. A simplified approach to determine the film rupture condition, when asperity contact occurs, is adopted to establish the link between micro-EHL and mixed-EHL. Based on a mathematical analogy between fluid film lubrication and contact problems, a generalized formulation has been derived to
solve these two seemingly different types of problems. But in that model, the shaft surface has been treated as perfectly smooth and therefore the effect of moving surface roughness on the lip seal operation performance has been neglected.

The present mixed lubrication model considers the roughness on both the lip and the shaft surface and consists of the fluid mechanics, deformation mechanics and contact mechanics analysis. Since the nonuniformities on the rotating shaft cause the flow field to be unsteady, the fluid and contact regions, as well as the fluid and contact pressures, change with time; this model is therefore more challenging than the previous work with smooth shafts.

Transient mixed lubrication models have been developed by several researchers. The first complete numerical solutions for transient models were reported by Chang et al (1989). They simulated the process of a surface irregularity which enters and passes through the otherwise perfectly smooth EHL conjunction in line contact. Three other research teams, Ai and Zhang (1989), Lee and Hamrock (1990), and Venner et al (1991), also reported results with 2D surface irregularities going in and through line-contact EHL conjunction. A further contribution to the transient model was made by Chang and Webster (1991). In their work, they simulated the interactions of two rough surfaces that form the line-contact EHL conjunction. The roughness was modeled by one-dimensional sinusoidal functions to generate 2D surface asperity-ridges.

A major assumption made in the previous models is that the surface roughness profiles do not vary in the transverse direction of the contact. However, the machining processes of tribo-components often produce surface finishes with significant three-dimensional features. The first solutions of a transient model with 3D surface asperities
were reported in Ai et al (1993). They developed a full 3D micro-EHL model and simulated the interactions of a pair of 3D asperities which form the line-contact conjunction. Subsequently, Ai and Cheng (1993) developed a point-contact micro-EHL model with 3D asperities and simulated the process of an asperity passing through the conjunction. Another methodology for the modeling of rough-surface EHL with 3D roughness topography was devised in Chang et al (1994). This model is formulated to capture the major local effects induced by the 3D surface features in a line-contact conjunction. However, none of those models considered the interasperity cavitation and surface shear deformation.

The present transient mixed lubrication model simulates the lip seal operation in the startup and shutdown period. During these transients, the lip is most vulnerable to wear and damage due to mechanical contact with the shaft; excessive leakage and the high contact force should be avoided.
CHAPTER 3
MATHEMATICAL AND NUMERICAL MODEL DEVELOPMENT

Since the performance of the lip seal may determine the operating condition of the machine, it is desired to improve the seal’s operating characteristic and increase its life. To achieve this objective, it is necessary to analyze the seal performance at different operating conditions in the sealing zone. However, the full film lubrication and the mixed lubrication phenomenon are so complicated that it is impossible to find an analytical solution; therefore, a numerical model is the choice to analyze this problem.

The present research is performed in a step-by-step manner, through the development of four models with gradually increasing complexity: hydrodynamic model, elastohydrodynamic model, mixed lubrication model and transient mixed lubrication model. First, the fundamental model is built to solve the hydrodynamic problem without considering the deformation and contact mechanism. Then more factors are added in, and the model is enhanced. Even though it is still a full film lubrication problem, the deformation of the lip surface in the normal and the shear direction is included. Next, the contact model is added to the system to simulate the lip seal performance in the mixed lubrication region. And the last model, which is called the transient mixed lubrication model, is built to analyze the lip seal when changes occur in the operating condition, especially during the startup and shutdown processes.
3.1 Hydrodynamic Model (Salant and Shen, 2002)

3.1.1 Objective and Assumptions

The objective of this hydrodynamic model is to determine the effects of the shaft surface microgeometry on lip seal operation through an understanding of the basic physics of full film lubrication without considering material deformation on the lip surface. This hydrodynamic model should predict the important characteristic parameters such as load support, reverse pumping rate, and cavitation area ratio in the full film lubrication region.

To achieve this objective with a reasonable computation time, it has been necessary to make a number of simplifying assumptions. These assumptions include:

i. The seal operates at steady speed.

ii. The viscosity of the lubricant is a constant since the temperature dependence of viscosity is neglected, thermal effects are not considered.

iii. The air side of the seal is flooded with lubricant, so the reverse pumping rate can always be calculated.

iv. The average film thickness is uniform in the axial direction, based on previous numerical and experimental results.

v. The asperity contact is not considered in this full film lubrication model.

vi. Circumferential shear deformation is assumed according to equation (3.1.9).
3.1.2 Solution Approach

3.1.2.1 Governing Equations

A model of the sealing zone between the lip and the shaft is shown schematically in Fig. 3.1.1. x is the circumferential direction and y is the axial direction. The upper stationary surface represents the lip surface, while the lower moving surface represents the shaft surface. If the fluctuations on the lower (shaft) surface are much smaller than those on the upper (lip) surface, one might ask how they can significantly affect the load support and/or sealing mechanisms. The answer to this question lies in the form of the governing equation of the sealing zone, and will be discussed following development of that equation.

The Reynolds equation for an incompressible lubricant in the full film zone is:

\[
\frac{\partial}{\partial x^*} \left[ h'^3 \frac{\partial p^*}{\partial x^*} \right] + \frac{\partial}{\partial y^*} \left[ h'^3 \frac{\partial p^*}{\partial y^*} \right] = 6\mu^* U^* \frac{\partial}{\partial x^*} \left[ h^* \right] + 12\mu^* \frac{\partial}{\partial t^*} \left[ h^* \right]
\]  \hspace{1cm} (3.1.1)
The fluid pressure in the cavitation zone is equal to a constant value (fixed cavitation pressure) due to the existence of the gas pockets. Therefore, the lubricant flow in the cavitation regions is solely Couette flow and must follow the continuity equation:

$$\frac{U^*}{2} \frac{\partial}{\partial x^*} (\rho^* h^*) + \frac{\partial}{\partial t^*} (\rho^* h^*) = 0$$  \hspace{1cm} (3.1.2)$$

These differential equations are nondimensionalized by introducing the following nondimensional quantities:

$$x = x^* / L^* \hspace{1cm} y = y^* / B^* \hspace{1cm} p = (p^* - p_c^*) / (p_s^* - p_c^*) \hspace{1cm} h = h^* / h_r^* \hspace{1cm} t = U^* t^* / L^*$$  \hspace{1cm} (3.1.3)$$

Where $x$, $y$, $p$, $h$, and $t$ are nondimensional circumferential coordinate, nondimensional axial coordinate, nondimensional pressure, nondimensional film thickness and nondimensional time respectively; $L^*$, $B^*$, $p_s^*$, $p_c^*$, $h_r^*$, $t^*$ are the length and width of the solution space, sealed pressure in the liquid side, cavitation pressure, reference film thickness and reference time respectively.

To derive a single dimensionless equation valid in both the full film and cavitation zones, a cavitation index, $S$ and a universal variable, $\Phi$, are defined as the following:

$$S = 1 \hspace{0.5cm} for \Phi \geq 0 \hspace{1cm} liquid\ region$$

$$S = 0 \hspace{0.5cm} for \Phi < 0 \hspace{1cm} cavitated\ region$$  \hspace{1cm} (3.1.4)$$

and

20
\[ p = S \Phi \]
\[ \rho = 1 + (1 - S) \Phi \]  \hspace{1cm} (3.1.5)

The nondimensional form of equation (3.1.1) and (3.1.2) can be re-written as:

\[
\frac{\partial}{\partial x} \left[ h^3 \frac{\partial S \Phi}{\partial x} \right] + A^2 \frac{\partial}{\partial y} \left[ h^3 \frac{\partial S \Phi}{\partial y} \right] = \gamma \frac{\partial \left\{ (1 + (1 - S) \Phi) h \right\}}{\partial x} + 2\gamma \frac{\partial \left\{ (1 + (1 - S) \Phi) h \right\}}{\partial t} 
\hspace{1cm} (3.1.6)
\]

where

\[ \Lambda = \frac{L^*}{B^*} \quad \gamma = 6\mu^* U^* L^* / h_r^{*2} (p_s^* - p_c^*) \]  \hspace{1cm} (3.1.7)

In non-cavitating liquid regions, \( \Phi \) represents the pressure, and the equation reduces to the conventional Reynolds equation. In cavitating regions, \( \Phi \) is linearly related to the average density, and the equation reduces to a continuity equation. The cavitation index, \( S \) switches the character of the universal equation from one appropriate in the full film regions to one appropriate in the cavitating regions.

Equation (3.1.6) is nonlinear in \( h \), both explicitly through the \( h^3 \) factor and implicitly through the occurrence of cavitation. Therefore there is the potential for small variations in \( h \), such as those produced by the shaft surface finish, to produce large effects on \( \Phi \), and to significantly affect the load support and the sealing mechanisms.

In a real lip seal, the fluid mechanics of the lubricating film is coupled with the elastic behavior of the lip, since the normal and shear deformations of the lip, which are produced by the pressure and shear stress distributions in the film, determine the film
thickness distribution. Thus, the Reynolds equation, Equation (3.1.6), is coupled to the elastic equations of the lip through $h$. However, to simplify the problem and isolate the hydrodynamic effects of the shaft surface finish, in the hydrodynamic model the Reynolds equation is decoupled by selecting an explicit expression for $h$, based on previous numerical and experimental results. Those results indicate that the film thickness, averaged over the fluctuations due to the asperities, is relatively uniform and on the order of a micron.

It is therefore reasonable to let the film thickness equal a constant plus a fluctuating term due to the asperities on the lip and a fluctuating term due to the shaft surface finish. The asperities on the lip are represented by a two-dimensional cosine function, with a phase shift in the $y$-direction to represent the effect of circumferential shear deformation that is necessary to produce reverse pumping.

In the present study, several different shaft surfaces are considered. The base shaft surface consists of a two dimensional cosine function (like the lip surface, but without the phase shift and moving in the $x$ direction). For this base surface, Surface #1, the film thickness distribution is given by:

$$
\begin{align*}
\theta & \cos \left( \frac{2\pi}{\lambda_{11}} \left( \frac{x}{\lambda_{22}} - \frac{y}{\lambda_{22}} \right) \right) \cos \left( \frac{2\pi y}{\lambda_{12}} \right) \\
\end{align*}
$$

where the function $g$, which represents the circumferential shear deformation of the lip, is given by:
\[ g = \frac{2y - y^2}{y_b^2} \quad \text{for } y \leq y_b \]
\[ = \frac{1 - 2y_b + 2y_by - y^2}{(1 - y_b)^2} \quad \text{for } y > y_b \]

and \( \theta \) is the shaft lead angle. \( y_b \) represents the axial location of the maximum circumferential shear deformation. The first term on the RHS of equation (3.1.8) represents the lip surface, while the second term represents the shaft surface.

Two other shaft surfaces are considered in the present study. Surface #2, without shaft lead, yields a film thickness distribution given by:

\[
\begin{align*}
    h &= A_1 \cos \left[ 2\pi \left( x - \frac{c}{\lambda_{11}} \right) \right] \cos \left[ \frac{2\pi y}{\lambda_{12}} \right] \\
    &\quad - A_2 \left( 1 - \cos \left[ 2\pi \left( x - \frac{c}{\lambda_{21}} \right) \right] \right) \cos \left[ \frac{2\pi y}{\lambda_{22}} \right] + 1.0 + 0.637A_2
\end{align*}
\]

Surface #3, also without shaft lead, yields,

\[
\begin{align*}
    h &= A_1 \cos \left[ 2\pi \left( x - \frac{c}{\lambda_{11}} \right) \right] \cos \left[ \frac{2\pi y}{\lambda_{12}} \right] \\
    &\quad - A_2 \left( 1 - \cos \left[ 2\pi \left( x - \frac{c}{\lambda_{21}} \right) \right] \right) \cos \left[ \frac{2\pi y}{\lambda_{22}} \right] + 1.0 + 0.361A_2
\end{align*}
\]

Surfaces #1, #2 and #3 are shown in Fig. 3.1.2
Figure 3.1.2 Shaft Surface Roughness Profile
3.1.2.2 Numerical Algorithms

The general Reynolds equation (3.1.6) is solved numerically using a micro control
volume finite difference scheme (Patankar, 1980) for a given film thickness distribution.
Equation (3.1.6) is discretized by considering an individual control volume and its
neighboring grid points (Figure 3.1.3). The center node is labeled with the capital letter P,
while the surrounding nodes are labeled N,S,E,W to show that they are in the north, south,
east, or west side of the center node P. The boundaries of the control volume are similarly
labeled with lower case letters, n, s, e, w represent the north, south, east and west walls of
the control volume. The pressure at center node P is related to the pressure of the
neighboring points.

Figure 3.1.3 Individual Control Volume and Neighboring Grid Points
Equation (3.1.6), together with equation (3.1.8), (3.1.10), (3.1.11) is solved with a finite difference scheme. The equation is integrated over the control space domain; since there is a squeeze term and time is involved, the time dependence is handled with the Crank-Nicolson method (Chapra and Canale, 1988). First, several fluidity coefficients are defined at the node locations as:

\[
K_p = h_p^3, \quad K_E = h_E^3, \\
K_N = h_N^3, \quad K_W = h_W^3, \quad K_S = h_S^3
\]

(3.1.12)

Since the film thicknesses at the boundaries of each control volume are not given directly, the fluidity coefficients at the boundaries \(K_e, K_n, K_w, K_s\) are then calculated by taking the harmonic mean of the fluidity coefficients on both sides of the boundary:

\[
K_e = \frac{2 K_p K_E}{K_p + K_E} \\
K_n = \frac{2 K_p K_N}{K_p + K_N} \\
K_w = \frac{2 K_p K_W}{K_p + K_W} \\
K_s = \frac{2 K_p K_S}{K_p + K_S}
\]

(3.1.13)

Also, \(h_e\) and \(h_w\) are given by arithmetic mean values between neighboring nodes:

\[
h_e = \frac{h_p + h_E}{2}, \quad h_w = \frac{h_p + h_W}{2}
\]

(3.1.14)
Since the Crank-Nicolson method involves the “old” time information (at the previous time step), the corresponding fluidity coefficients are defined below. The superscript \((\text{ })^0\) stands for old time quantity.

\[
K_0^0 = 2 \frac{K_0^P K_0^E}{K_0^P + K_0^E}
\]

\[
K_n^0 = 2 \frac{K_0^P K_0^N}{K_0^P + K_0^N}
\]

\[
K_w^0 = 2 \frac{K_0^P K_0^W}{K_0^P + K_0^W}
\]

\[
K_s^0 = 2 \frac{K_0^P K_0^S}{K_0^P + K_0^S}
\]

\[
h_0^e = \frac{h_0^P + h_0^E}{2}
\]

\[
h_0^w = \frac{h_0^P + h_0^W}{2}
\]

In addition, the following definitions are introduced:

\[
A_E = \frac{\Delta y \Delta t K}{2 \Delta x}
\]

\[
A^0_E = \frac{\Delta y \Delta t K^0_e}{2 \Delta x}
\]

\[
A_W = \frac{\Delta y \Delta t K^0_w}{2 \Delta x}
\]

\[
A_N = \Lambda^2 \frac{\Delta x \Delta t K^0_n}{2 \Delta y}
\]

\[
A_S = \Lambda^2 \frac{\Delta x \Delta t K^0_s}{2 \Delta y}
\]

\[
A_P = (A_E + A_W + A_N + A_S) F_p + \frac{\Delta y \Delta t \gamma_1}{2} (1 - F_p) h_e + \Delta x \Delta y \gamma_2 (1 - F_p) h_p
\]
\[ A_p^0 = (A^0_E + A^0_W + A^0_N + A^0_S) F_p + \frac{\Delta y \Delta t \gamma_1}{2} (1 - F^0_p) h^0_e - \Delta x \Delta y \gamma_2 (1 - F^0_L) h^0_p \]

\[ S_b = \frac{\Delta y \Delta t \gamma_1}{2} \left( (h_e + h^0_e) - [1 + (1 - F^0_w) \Phi_w] h_w - \left[ 1 + (1 - F^0_w) \Phi^0_w \right] h^0_w \right) \]

\[ + \Delta y \Delta t \gamma_2 (h_p - h^0_p) \]

(3.1.18)

By utilizing the previous definitions, the algebraic discretized version of equation (3.1.6) can be obtained as the following:

\[ A_p \Phi_p = (A^0_E \Phi_E^0 F_E^0) + (A^0_W \Phi_w^0 F_w^0) + (A^0_N \Phi_N^0 F_N^0) + (A^0_S \Phi_S^0 F_S^0) \]

\[ + (A^0_E \Phi_E^0 F_E^0) + (A^0_W \Phi_w^0 F_w^0) + (A^0_N \Phi_N^0 F_N^0) + (A^0_S \Phi_S^0 F_S^0) - A^0_p \Phi^0_p - S_b \]

(3.1.19)

For each node \( P \) in the domain, the above equation can be written with appropriate values of coefficients \( A \) and cavitation indices \( F \).

The solution domain extends ten lip surface wavelengths in both the axial direction (\( y \)) and circumferential direction (\( x \)). The boundary conditions imposed are: \( \Phi = p = 1 \) at \( y = 0 \) (air-side) and \( y = 1 \) (liquid side); cyclic boundary condition on \( \Phi \) at \( x = 0, 1 \).

The initial condition imposed is \( \Phi = p = 1 \) at \( t = 0 \), but the solution is considered only after the influence of the initial condition has died out. The spatial grid size is \( 81 \times 81 \), chosen after a grid refinement study. The temporal grid spacing is the same as the spatial grid spacing, i.e., \( \Delta t = \Delta x = \Delta y = 1.25 \times 10^{-2} \).

At each instant of time, the discretized equation is solved iteratively using the ADI (alternating direction, implicit) method. The ADI method solves the system of
equations for $\Phi$ at each grid point by first traversing each row of grid points one row at a time. This row by row solution is improved by solving the equations again column by column.

The ADI method (Patankar, 1977) alternates between the TDMA (tri-diagonal matrix algorithm) in the direction of the defined pressure bounds and the CTDMA (cyclic tri-diagonal matrix algorithm) in the direction of the cyclic conditions.

Once the Reynolds equation is solved for the pressure distribution, some important quantities, such as the hydrodynamic load support and reverse pumping rate can be evaluated by integration.

The hydrodynamic load support at each node can be obtained by multiplying the fluid pressure and the area of the control volume surrounding the node. The total load support is the sum of the hydrodynamic load support at all nodes. The integration for the hydrodynamic load support is the following:

$$W = \iint (p - p_a) dx dy$$  \hspace{1cm} (3.1.20)

After the fluid mechanics analysis is completed, the reverse pumping rate can be calculated from:

$$Q = \int - \frac{h^3}{12} \frac{\partial p}{\partial y} dx$$  \hspace{1cm} (3.1.21)

In the program, the trapezoidal rule is used to integrate the above equation.
A positive value of reverse pumping rate indicates a successful seal operation without leakage; while a negative value of Q shows that the reverse pumping action doesn’t occur. Due to the unsteady flow in the model, the final load support and the reverse pumping rate are averaged over time t.

3.1.2.3 Computational Scheme for Hydrodynamic Model

In the hydrodynamic model, initially Φ and F are estimated at all grid points. All F and Φ are initially assigned a value of 1, based on the applied boundary condition pressure.

The fluid mechanics analysis focuses on solving the general Reynolds equation. After the first ADI sweep of the rows and the columns, the values of Φ are generated and compared to their previous value. If the latest Φ has been changed, the values of F will be updated; if the latest Φ is negative at a grid point, the value of F will be assigned to 0 at that grid point, indicating that there is cavitation occurring in the surrounding control volume.

After checking the convergence of Φ and F, F and Φ are relaxed by using the under relaxation method and an additional ADI sweep is performed. The convergence criterion requires the fractional deviation between successive iterations in Φ to be less than 10^{-6}. After the initial condition has died out, Φ becomes periodic in time, and the final “steady state” results, such as the values of W, Q and F, are obtained by averaging over one period. The criterion for determining when the initial condition has died out is the requirement that the fractional change in Φ at two corresponding times in successive periods be less than 10^{-3}. 

30
The computational scheme for the hydrodynamic model outlining this procedure is in Figure 3.1.4.

Figure 3.1.4 Computation Scheme for the Hydrodynamic Model
3.1.3 Results and Discussions for Hydrodynamic Model

In the hydrodynamic study, results are obtained for a base case, and then for a number of variations. The base configuration consists of shaft surface #1 with no lead ($\theta = 0$), equation (3.1.8). The values of the dimensional parameters for the base case are: $L^* = B^* = 5 \times 10^{-4}$ m, $U_r^* = 4$ m/s, $\mu^* = 2.5 \times 10^{-2}$ Pa-s, $p_s^* = p_a^* = 1.02 \times 10^5$ Pa, $p_c^* = 0$ Pa, $h_r^* = 1$ $\mu$m, $\lambda_{11}^* = \lambda_{12}^* = 5 \times 10^{-5}$ m, $\lambda_{21}^* = \lambda_{22}^* = 10^{-4}$ m, $A_1^* = 0.5$ $\mu$m, $y_b^* = 3.5 \times 10^{-4}$ m.

The corresponding dimensionless parameters are: $\gamma = 2941.2$, $\Lambda = 1$, $\lambda_{11} = \lambda_{12} = 0.1$, $\lambda_{21} = \lambda_{22} = 0.2$, $c = 1.0$, $A_1 = 0.5$, $y_b = 0.7$.

Figure 3.1.5 illustrates the effect of the shaft surface fluctuations on the spatial pressure/density distribution in the lubricating film, for the base case. The figure shows the $\Phi$ variation in the circumferential direction at a point midway between the air- and oil-sides of the seal ($y = 0.5$), at a given instant of time. As pointed out above, the dimensionless wavelength of the lip surface is 0.1, while that of the shaft surface is 0.2. From the figure it is seen that with the smooth shaft, $\Phi$ fluctuates with the same wavelength as the lip surface. However, with a shaft Ra as small as 5% of the lip Ra, the $\Phi$ distribution is completely changed, so that the predominant wavelength is that of the shaft surface. This is a striking illustration of the nonlinearity of the Reynolds equation. As the shaft Ra is increased to 15% of the lip Ra, there is little additional change in the shape of the $\Phi$ distribution.
Figure 3.1.5  Effect of Shaft Surface #1 on $\Phi$ Distribution
The effect of the amplitude of the shaft surface fluctuations on the time-averaged load support, for the base case (surface #1), is shown in Figure 3.1.6. When \((Ra)_{\text{shaft}}\) is 5% of \((Ra)_{\text{lip}}\), the load support is increased by 37% (over \(W_0\) of 0.75), a very significant increase. When \((Ra)_{\text{shaft}}\) is 10% of \((Ra)_{\text{lip}}\) (a realistic value), the load support is increased by 45%, an even more significant increase. Further increases in \((Ra)_{\text{shaft}}\) produce relatively small incremental increases in load support.

![Figure 3.1.6 Effect of Shaft Surface on Load Support versus Shaft Roughness Height](image-url)
Figure 3.1.6 also shows corresponding results obtained for shaft surfaces #2 and #3, with all other parameters equal to the base case values. It is seen that for the same value of \((Ra)_{shaft}\), surface #2 produces a smaller increase in load support than surface #1, and surface #3 produces an even smaller increase. For example, for \((Ra)_{shaft} = 0.05\) \((Ra)_{lip}\), surface #2 produces an increase of 25\% (compared to 37\% for surface #1), while surface #3 produces an increase of 17\%. This is consistent with empirical observations that it is not just the amplitude of the shaft surface fluctuations that is important, but also the details of the surface profile. For the same value of \((Ra)_{shaft}\), the three different surfaces have very different values of kurtosis, skewness and other surface characterization parameters as shown in Table 3.1.

<table>
<thead>
<tr>
<th>Surface #1</th>
<th>A2</th>
<th>Ra</th>
<th>Rsk</th>
<th>Rku</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface #1</td>
<td>0.05</td>
<td>0.032</td>
<td>0</td>
<td>1.3340</td>
</tr>
<tr>
<td>Surface #2</td>
<td>0.119</td>
<td>0.032</td>
<td>-0.5</td>
<td>1.9299</td>
</tr>
<tr>
<td>Surface #3</td>
<td>0.119</td>
<td>0.032</td>
<td>0.4977</td>
<td>1.9299</td>
</tr>
</tbody>
</table>
From Figure 3.1.7 it is seen that the shaft surface fluctuations also produce a significant increase in the time-averaged reverse pumping rate for the base case, although not quite as large as the increase in load support. When \((Ra)_{shaft}\) is 5\% of \((Ra)_{lip}\), the reverse pumping rate is increased by 15\% above the smooth shaft value \((Q_0\) of 109), and when \((Ra)_{shaft}\) is 10\% of \((Ra)_{lip}\), the reverse pumping rate is increased by 16\%. As with the load support, further increases in \((Ra)_{shaft}\) produce negligibly small incremental increases in reverse pumping rate. Figure 3.1.7 also shows the corresponding results for surfaces #2 and #3. As with the load support, at the same value of \((Ra)_{shaft}\), surface #2 produces a smaller increase in reverse pumping rate than surface #1, and surface #3 produces an even smaller increase.

![Figure 3.1.7 Effect of Shaft Surface on Reverse Pumping Rate versus Shaft Roughness](image)

Figure 3.1.7 Effect of Shaft Surface on Reverse Pumping Rate versus Shaft Roughness
The effect of the amplitude of the shaft surface fluctuations on the time-averaged cavitation area, for surface #1 (base case) and for surfaces #2 and #3, is shown in Figure 3.1.8. In contrast to the effects on load support and reverse pumping rate, for a given \((Ra)_{shaft}\), surface #2 produces the largest increase in cavitation area above the smooth shaft value (F of 34%) , surface #3 produces the next largest, and surface #1 produces the smallest increase. For example, for \((Ra)_{shaft} = 0.05 (Ra)_{lip} \), surface #2 produces an increase of 32%, surface #3 produces an increase of 27%, and surface #1 produces an increase of 6%.

![Figure 3.1.8 Effect of Shaft Surface on Cavitation Area versus Shaft Roughness Height](image)

Figure 3.1.8 Effect of Shaft Surface on Cavitation Area versus Shaft Roughness Height
All of the results presented thus far are for a single shaft speed, $\gamma = 2941.2$. Figure 3.1.9 shows how the smooth shaft values of the load support, $W_0$, reverse pumping rate, $Q_0$, and cavitation area, $F_0$, vary with speed. As would be expected, all three increase with speed. Figure 3.1.10 to Figure 3.1.12 show the effects of surface #1 on load support, reverse pumping rate and cavitation area (compared to the smooth shaft values) as functions of speed for three different values of $(Ra)_{\text{shaft}}$. The effects on all three variables decrease with speed, but within the range investigated, those decreases are relatively small. For example, for $(Ra)_{\text{shaft}} = 0.05 (Ra)_{\text{lip}}$, doubling the speed from the base case value of 2941.2 to 5882.4 causes the increase in load support (above the smooth shaft value) to decrease from 37% to 33%. The decreases in the effects on reverse pumping rate and cavitation area are even smaller.

![Graph showing load support, reverse pumping rate, and cavitation area versus speed for a smooth shaft.]

Figure 3.1.9 Load Support; Reverse Pumping Rate and Cavitation Area versus Speed, for a Smooth Shaft
Figure 3.1.10 Effect of Shaft Surface #1 on Load Support versus Speed
Figure 3.1.11 Effect of Shaft Surface #1 on Reverse Pumping Rate versus Speed
Figure 3.1.12 Effect of Shaft Surface #1 on Cavitation Area versus Speed
The shaft surface wavelength for the results discussed above has the value of twice the lip surface wavelength. Figure 3.1.13 shows how the effects of shaft surface #1 vary over the range of $\frac{\lambda_{\text{shaft}}}{\lambda_{\text{lip}}}$ from 0.25 to 2.0. The largest variation occurs in the load support: for $(Ra)_{\text{shaft}} = 0.05 (Ra)_{\text{lip}}$, a decrease in $\frac{\lambda_{\text{shaft}}}{\lambda_{\text{lip}}}$ from 2.0 to 0.25 results in an increase in the effect of the surface from 37% (increase over the smooth shaft value) to 45%. Over the same range there is virtually no change in the effect on pumping rate, while the effect on cavitation area increases from 6% to 9%.

Figure 3.1.13 Effects of Shaft Surface #1 on Load Support, Reverse Pumping Rate and Cavitation Area versus Shaft Surface Wavelength
The axial location of maximum circumferential shear deformation of the lip, \( y_b \), is determined by the geometry of the lip and the location of the garter spring. It must lie between 0.5 and 1.0 for the seal to be effective, as discussed earlier. Based on empirical observations, \( y_b = 0.7 \) has been chosen for the base case as a reasonable representative value. The numerical simulations of the present study show that \( y_b \) has an insignificant effect on the influence of the shaft surface on load support and cavitation area. Figure 3.1.14 shows the effect of surface #1 on the reverse pumping rate, for several values of \( y_b \). It is seen that the shaft surface will have the largest effect when \( y_b \) is between 0.7 and 0.8. Smaller and larger values of \( y_b \) can significantly decrease the influence of the shaft surface.

![Figure 3.1.14 Effect of Shaft Surface #1 on Reverse Pumping Rate for Several Values of Location of Maximum Circumferential Shear Deformation of Lip](image-url)
All of the preceding results apply to a shaft with no lead. As discussed earlier, shaft lead can cause the shaft to act like a screw pump, and pump liquid from the liquid-side of the seal toward the air-side. If this effect is large enough, it can overcome the reverse pumping produced by the lip, and result in leakage. Figure 3.1.15 shows the reverse pumping rate versus lead angle for surface #1, for several values of \((Ra)_{\text{shaft}}\). As expected, the reverse pumping rate decreases with increasing lead angle in the range examined. It becomes negative when the lead angle exceeds 0.1 degree, indicating the occurrence of leakage. This is consistent with the RMA specification (Rubber Manufacturers Association, 1999) calling for a lead angle less than 0.05 degrees, to assure the absence of leakage. From Figure 3.1.15 it is seen that the value of \((Ra)_{\text{shaft}}\) has an insignificant effect on the critical lead angle. This is because of two competing effects. First, an increased \((Ra)_{\text{shaft}}\) means larger shaft surface fluctuations (asperities), which produce an increased reverse pumping rate, as discussed in connection with Figure 3.1.7. Second, an increased \((Ra)_{\text{shaft}}\) results in a deeper spiral pattern associated with lead, which produces an increased pumping rate in the opposite direction (from the liquid-side toward the air-side). These two effects apparently cancel each other out. The numerical computations also indicate that the presence of lead has no significant effect on the load support or the cavitation area.
In generating the above results, it was assumed that the maximum circumferential shear deformation of the lip surface is equal to one lip surface wavelength (c = 1.0), based on empirical observations. One might expect that maximum deformation to affect the critical value of the lead angle, since the reverse pumping of the lip is dependent on that deformation. However, Figure 3.1.16, which shows the reverse pumping rate versus lead angle for several values of the maximum circumferential shear deformation of the lip surface, indicates that the maximum deformation has little effect.
Figure 3.1.16 Effect of Lead Angle on Reverse Pumping Rate for Several values of Maximum Circumferential Shear Deformation of Lip, Shaft Surface #1

The results presented above demonstrate how the shaft surface finish can significantly influence lip seal operation through hydrodynamic effects. Due to the nonlinearity of the Reynolds equation, very small fluctuations on the shaft surface can create large changes in the pressure distribution within the lubricating film, which produce significant changes in the load support, reverse pumping rate and cavitation area. These changes are primarily dependent on the details of the shaft surface profile, the Ra
of the shaft and shaft speed. In addition, the reverse pumping is dependent on the shaft lead angle.
3.2 Elastohydrodynamic Model (Shen and Salant, 2003)

3.2.1 Objective and Assumptions

The objective of the elastohydrodynamic model is similar to the hydrodynamic model: to predict important operating characteristics such as load support, reverse pumping rate and cavitation area under full film lubrication conditions. The major difference from the hydrodynamic model is that this model includes a deformation analysis of the lip. Therefore the average film thickness and the liftoff speed can be computed, and the previous assumptions regarding the film thickness distribution and the circumferential shear deformation need not be made.

In the hydrodynamic analysis (Section 3.1), it was pointed out that since the Reynolds equation is nonlinear in the film thickness, small variations in the film thickness can produce large effects on the pressure distribution in the lubricating film. This was, indeed, supported by means of a numerical hydrodynamic analysis, which showed that the surface finish can have a substantial influence on the load support and reverse pumping rate. These results, however, are not conclusive because the analysis was strictly hydrodynamic, i.e., an explicit expression for the film thickness distribution, based on previous numerical and experimental results, was assumed. In a real lip seal, the fluid mechanics of the lubricating film is coupled with the elastic behavior of the lip, since the normal and shear deformations of the lip, which are produced by the pressure and shear stress distributions in the film, determine the film thickness distribution. It is therefore necessary to perform an elastohydrodynamic analysis.

The third and the fifth assumption in the hydrodynamic model are relaxed, so the average film thickness is no longer a constant; it will be involved in the iterative
calculation. The shear deformation in the circumferential direction of the lip surface can be calculated from the deformation analysis, which is not included in the hydrodynamic model. These give a more realistic approximation of the lip seal behavior. In the deformation mechanics analysis, it is assumed that the asperities on the lip surface do not affect the bulk deformations of the lip; therefore those deformations are axisymmetric.

3.2.2 Solution Approach

3.2.2.1 Governing Equations

In a real lip seal, the fluid mechanics of the lubricating film is coupled with the elastic behavior of the lip due to the normal and shear deformations, which are produced by the pressure and shear stress distributions in the film. In the present elastohydrodynamic model, the film thickness is again set equal to the sum of two different fluctuating terms (due to the asperities on both the lip and shaft surfaces) plus an average film thickness, $h_{avg}$, which is now a function of axial location and determined by the normal deformation of the lip. For the base case, the film thickness distribution is expressed by equation (3.2.1): The fluctuating term due to the asperities on the lip surface is a two-dimensional cosine function containing a phase shift proportional to $\delta$, which represents the circumferential displacement of the lip surface; this displacement is determined by the shear deformation of the lip. Thus it is assumed that the asperity pattern is carried along with the bulk deformation of the lip. The fluctuating term due to the asperities on the shaft is one of three cosine type functions representing three different shaft surface finishes. The film thickness distribution associated with shaft surface #1 is given by:
The film thickness distribution associated with surface #2 is given by:

\[
h = A_1 \cos \left(2\pi \left(\frac{x - \delta}{\lambda_{11}}\right)\right) \cos \left(\frac{2\pi y}{\lambda_{12}}\right) - A_2 \cos \left(\frac{2\pi}{\lambda_{21}}(x - t)\right) \cos \left(\frac{2\pi y}{\lambda_{22}}\right) + h_{avg}
\]

(3.2.1)

while that associated with surface #3 is given by:

\[
h = A_1 \cos \left(2\pi \left(\frac{x - \delta}{\lambda_{11}}\right)\right) \cos \left(\frac{2\pi y}{\lambda_{12}}\right) - A_2 \cos \left(\frac{2\pi}{\lambda_{21}}(x - t)\right) \cos \left(\frac{2\pi y}{\lambda_{22}}\right) + 0.637A_2 + h_{avg}
\]

(3.2.2)

while that associated with surface #3 is given by:

\[
h = A_1 \cos \left(2\pi \left(\frac{x - \delta}{\lambda_{11}}\right)\right) \cos \left(\frac{2\pi y}{\lambda_{12}}\right) - A_2 \left\{1 - \cos \left(\frac{2\pi}{\lambda_{21}}(x - t)\right)\right\} \left\{1 - \cos \left(\frac{2\pi y}{\lambda_{22}}\right)\right\} + 0.361A_2 + h_{avg}
\]

(3.2.3)

The three shaft surface roughness profiles are the same as those considered in the hydrodynamic model and pictorial representations of these three surface profiles are contained in Figure 3.1.2.

In the fluid mechanics analysis of the elastohydrodynamic model, the dimensionless general Reynolds equation (3.1.6) is solved in the same way as in the hydrodynamic model. The general Reynolds equation (3.1.6) is combined with the film
thickness equations (3.2.1), (3.2.2), or (3.2.3) and solved numerically by using a finite difference scheme. (Since the film thickness equations contain \( h_{\text{avg}} \) and \( \delta \), the latter two variables must be obtained from a deformation analysis, as described below.) The Crank-Nicolson method is used to deal with the squeeze film term. The discretized algebraic equations are solved for \( \Phi \) at each instant of time for each node by utilizing the iterative ADI method with relaxation of \( h \). After the initial condition has died out, \( \Phi \) becomes periodic in time.

Once the pressure distribution is obtained from the solution of the Reynolds equation, the dimensionless load support and reverse pumping rate can be computed from equations (3.1.20) and (3.1.21).

In the deformation mechanics analysis, the influence coefficient method (Salant and Flaherty, 1994), in which the deformations are linearly related to the appropriate surface forces, is used. The deformations are characterized by two influence coefficient matrices, \( I_1 \) and \( I_2 \), such that the average film thickness, \( h_{\text{avg}} \), can be expressed in discretized form (at the i axial node) as:

\[
(h_{\text{avg}})_i = A_1 + A_2 (1-C) + \sum_{k=1}^{a} (I_1)_{ik} (p_{\text{avg}}^{**} - p_{\text{contact}}^{**})_k
\]

(3.2.4)

where, \( A_1 \) and \( A_2 \) stand for the half amplitudes of the lip surface and shaft roughness fluctuations, respectively; \( C = 0 \) for surface #1, 0.637 for surface #2, 0.363 for surface #3. The parameter \( C \) is introduced because the three shaft surfaces have different mean values of roughness. For example, the mean value of the shaft surface #1 fluctuation is 0 and that for shaft surface #2 is 0.637, while for shaft #3 it is 0.363.
The circumferential displacement of the lip surface, $\delta$, can be expressed as:

$$
(\delta)_i = \sum_{k=1}^{n} (I_2)_{ik} (\tau_{avg})_k
$$

(3.2.5)

The influence coefficients $I_1$ (for the normal deformation) and $I_2$ (for the shear deformation) are obtained from an off-line finite element analysis of the seal. The detail of the axisymmetric FEA of the lip seal will be given in the next section in this chapter.

The dimensionless pressure $p^{**}$ (in the above deformation relations) is related to the dimensionless pressure $p$ (in the fluid mechanics relations) through:

$$
p^{**} = p \left( (p_s^* - p_c^*)/E^* + p_c^*/E^* \right)
$$

(3.2.6)

and the dimensionless shear stress is given by:

$$
\tau = (hh_r^* (p_s^* - p_c^*)/2E^* L^*)(dp/dx) + (\mu U^*/hh_r^* E^*)
$$

(3.2.7)

The subscript $(\ )_{avg}$ indicates an average both in time and in the x direction.

3.2.2.2 Computational Scheme for Elastohydrodynamic Model

Since the fluid mechanics of the film and the deformation of the lip are strongly coupled, an iterative computation procedure is required to solve equations (3.1.6), (3.2.4) and (3.2.5). The overall procedure is shown in the flowchart of Figure 3.2.1. First, with a guessed film thickness distribution, the Reynolds equation is solved with the initial
condition $\Phi = 1$, as mentioned earlier. The fluid mechanics analysis follows the same procedure as in the hydrodynamic model. The so called “steady-state” $\Phi$ and $F$ are determined after the initial condition has died out. The resulting pressure distribution is used as input to the deformation mechanics analysis. Next, the normal deformation in the radial direction and resulting average film thickness are computed. After a convergence check on the average film thickness, the shear deformation (in the circumferential direction) is calculated and the film thickness distribution is updated. The updated film thickness is then used as input to the fluid mechanics analysis (Reynolds equation). The $\Phi$ distribution computed in the previous iteration is used as the initial condition for the solution of the Reynolds equation. Iteration continues until the average film thickness distribution converges. It should be noted that there are two nested iteration loops in this procedure. The inner loop, not shown in the figure, is required for solving the Reynolds equation. In the outer loop, the average film thickness and circumferential lip surface displacement are adjusted. The solution yields the distributions of $\Phi$ (or $p$), $S$, $h$, $h_{\text{avg}}$, $\delta$, $Q$ and $W$. 
Input design and operating parameters

Input influence coefficients and contact pressure
(Calculated from a finite element analysis)

Initial guess of average film thickness and circumferential deformation

Fluid Mechanics Analysis
(Calculate pressure distribution by solving general Reynolds equation at given film thickness distribution)

Deformation Mechanics Analysis
(Calculate normal deformation of lip using influence coefficients and new average film thickness)

Check convergence

Relax average film thickness

No

Compute shear stress and circumferential deformation

No

Compute and average W, Q, F

Yes

Output results and post process

Figure 3.2.1 Computational Scheme for Elastohydrodynamic Model
3.2.2.3 Axisymmetric FEA of Lip Seal and Influence Coefficient Matrix

To calculate both the deformations of the bulk material and the deformations of asperities on the lip surface, the influence coefficient method is utilized in this model. Since the deformations are linearly related to the surface forces in this method, the corresponding characteristic influence coefficient matrices should be calculated first. A finite element analysis is used to calculate these matrices.

Even though the influence coefficient matrix only needs to be given in the sealing zone, the 3D structure of the rotary lip seal has to be modeled in the finite element analysis. FEA on such a 3D structure will take a large amount computer memory and a large computing time.

Due to the axisymmetric structure of the rotary lip seal, an axisymmetric 2D model is used to simulate the lip seal. The use of this model greatly increases the efficiency of the computation compared to that of an equivalent three-dimensional model. In the present study, the commercial program ANSYS is utilized for this model.

In the present study, a 2D radial cross section of rotary lip seal is generated first, and then the boundary condition is set at the two rigid, straight lines shown on Figure 3.2.2. The displacement of those two lines is zero. The ANSYS element PLANE42 is used to analyze the model. To simulate the spring which is installed on the lip seal, spring element COMBIN14 is used to generate the spring force, as shown on Figure 3.2.2.
Figure 3.2.2 Finite Element Analysis for Lip Seal
Two sets of influence coefficient matrices, $I_1$ and $I_2$, are obtained from the axisymmetric 2D model. Since the study of deformation requires the matrix be given to great accuracy, adaptive methods have been developed to aid in increasing the accuracy of computed solutions. In finite element theory, the adaptive method is referred to as an h method if mesh refinement is used. The ANSYS program provides approximated techniques for automatically estimating mesh discretization error for certain type of elements. Using this method, the program can then determine if a particularly mesh is fine enough. This process of automatically evaluating the mesh discretization error and refining the mesh is called adaptive meshing.

The adaptive meshing technique is able to provide the solution for the entire lip seal structure with high accuracy. However, the region we are most interested is the sealing zone, where the influence coefficient matrix is to be computed. The zoom-in process in the finite element analysis is called submodeling, which can produce more accurate results in the small region of interest.

Figure 3.2.2 shows the picture for deformation of the lip seal contacting with the rigid shaft (The straight line stands for the shaft). First, the lip surface is modeled before it is mounted on the shaft surface (without deformation); second, the shaft surface is displaced to the mounted position; then a non-zero net force is added to each node successively in the sealing zone to calculate the influence coefficient matrix. Under this condition, the nodes have both surface deformation and structure deformation, and the rotary lip seal structure deforms like a cantilever beam.

The influence coefficient matrix is not sensitive to the detailed geometry at the lip tip and the surface deformation always dies out over a small distance. Figure 3.2.3 shows
the plots for influence coefficient matrices, $I_1$ and $I_2$, which will be used in the equations (3.2.4) and (3.2.5).

Figure 3.2.3 Influence Coefficient Matrix for Lip Seal
### 3.2.3 Results and Discussion for Elastohydrodynamic Model

Computations have been performed for a representative lip seal under typical operating conditions. The base parameter values are: \( L^* = B^* = 5 \times 10^{-4} \text{ m}, U_r^* = 5 \text{ m/s}, \mu^* = 2.5 \times 10^{-2} \text{ Pa-s}, p_s^* = p_a^* = 1.02 \times 10^5 \text{ Pa}, p_c^* = 0 \text{ Pa}, h_r^* = 1 \mu\text{m}, \lambda_{11}^* = \lambda_{12}^* = 5 \times 10^{-5} \text{ m}, \lambda_{21}^* = \lambda_{22}^* = 10^{-4} \text{ m}, A_1^* = 0.5 \mu\text{m}, \text{shaft diameter} = 4.445 \times 10^{-2} \text{ m}, E^* = 6.2 \times 10^6 \text{ Pa}, \nu^* = 0.49, \) The spatial grid size is \( 81 \times 81. \) The corresponding dimensionless parameters are: \( \gamma = 3676.47, \Lambda = 1, \lambda_{11} = \lambda_{12} = 0.1, \lambda_{21} = \lambda_{22} = 0.2, A_1 = 0.5, p_a = p_s = 1. \)

Figure 3.2.4 shows the average film thickness \( h_{avg} \) as a function of axial location, for \( \text{(Ra)}_{\text{shaf}}/\text{(Ra)}_{\text{lip}} = 0.1 \) (other values of shaft roughness yield similar results). It is seen that the average film thickness is virtually constant in the axial direction, as was assumed in (Section 3.1). This result is consistent with experimental observations (Gabelli and Poll, 1992) as well as with previous numerical studies (Salant, 1997).

![Figure 3.2.4 Average Film Thickness Distribution, Surface #1, (Ra)$_{shaf}/(Ra)$_{lip} = 0.1](image)

Figure 3.2.4 Average Film Thickness Distribution, Surface #1, \( \text{(Ra)}_{\text{shaf}}/\text{(Ra)}_{\text{lip}} = 0.1 \)
The effect of average shaft roughness height on the time-averaged load support, for three different shaft surfaces, is shown in Figure 3.2.5. For surface #1, when \((Ra)_{\text{shaft}}\) is 5\% of \((Ra)_{\text{lip}}\), the load support is increased by 14\% over the smooth shaft value. When \((Ra)_{\text{shaft}}\) is 10\% of \((Ra)_{\text{lip}}\), the load support is increased by 16\%. These are substantial increases, although they are smaller than those computed in the previous hydrodynamic analysis (Section 3.1). (In the hydrodynamic analysis, the average film thickness is fixed at a given value for both the smooth and rough shaft computations, while in the present elastohydrodynamic analysis the average film thickness can vary.) Larger values of \((Ra)_{\text{shaft}}\) result in larger increases in load support, as would be expected.

![Figure 3.2.5 Effect of Shaft Surface Roughness Height on Load Support](image-url)

Figure 3.2.5 Effect of Shaft Surface Roughness Height on Load Support
From Figure 3.2.5 it is also seen that the load support curves for surfaces #2 and #3 exhibit the same trends as that for surface #1. However, the increases in load support for shaft surface #2 are smaller than those for shaft #1, while those for shaft #3 are even smaller. This is not surprising, since empirical observations indicate that the skewness (Rsk), kurtosis (Rku), and maximum valley depth (Rv), in addition to Ra, are important parameters governing seal operation, as described in the introduction chapter 1. The values of these parameters differ for the three surfaces investigated as shown in Table 3.1.

Figure 3.2.6 shows that the shaft surface fluctuations also have a significant impact on the time-averaged reverse pumping rate. For surface #1, when (Ra)_{shaft} is 5\% of (Ra)_{lip}, the reverse pumping rate is increased by 15.2\%, and when (Ra)_{shaft} is 10\% of (Ra)_{lip}, the reverse pumping rate is increased by 16.1\%, figures similar to those obtained from the hydrodynamic analysis (Section 3.1). This is important because the reverse pumping rate is a measure of the sealing ability of the seal. Larger values of (Ra)_{shaft} produce larger increases in reverse pumping rate. Figure 6 also shows the corresponding results for surfaces #2 and #3. For the same value of (Ra)_{shaft}, surface #2 creates a smaller increase in reverse pumping rate than surface #1, while surface #3 creates an even smaller increase. As with the increases in load support, it is not surprising that different shaft surface profiles with the same average roughness height produce different increases in reverse pumping rate.
Figure 3.2.6 Effect of Shaft Surface Roughness Height on Reverse Pumping Rate
The cavitation area in the sealing zone is also increased by the shaft surface roughness, as shown in Figure 3.2.7. For a given value of (Ra)_{shaft}, surface #2 produces the largest increase in cavitation area and surface #1 produces the smallest increase. This trend is quite different from that observed with the load support and reverse pumping rate, indicating the latter two characteristics do not correlate in a simple manner with the cavitation phenomenon. But the trend for the cavitation area ratio curve is similar to that in the hydrodynamic model.

![Figure 3.2.7 Effect of Shaft Surface Roughness Height on Cavitation Area](image)

Figure 3.2.7 Effect of Shaft Surface Roughness Height on Cavitation Area
As previously mentioned in connection with Figure 3.2.4, the average film thickness is substantially constant across the axial length of the seal. Figure 3.2.8 shows that this average film thickness is increased over the smooth shaft value as a result of shaft roughness; the larger the \((Ra)_{shaft}\), the larger the increase. As seen in Figure 3.2.8, surface #1 produces the largest effect and surface #3 produces the smallest. This trend is consistent with those observed with the load support and reverse pumping rate.

Figure 3.2.8 Effect of Shaft Surface Roughness Height on Average Film Thickness
All of the seal performance characteristics discussed above are related to the steady state operation of the seal. In addition, characteristics associated with the transient operation of the seal are also of interest. The most important of these is the liftoff speed. During the startup process when the shaft speed increases from zero, the seal initially operates in the mixed lubrication regime, viz., there is asperity contact in the sealing zone. As the pressure distribution develops, and the load support increases, the lip gradually lifts off of the shaft. Eventually a speed is reached at which the lip is completely lifted off of the shaft, there is no longer any asperity contact, and there is full film lubrication. The speed at which this transition to full film lubrication occurs is defined as the liftoff speed. To calculate the liftoff speed, first a shaft speed is guessed and the converged $h_{\text{avg}}$ is calculated. If this average film thickness $h_{\text{avg}}$ is larger than the critical average film thickness at which contact occurs, then the shaft speed is decreased. This process is repeated until the converged $h_{\text{avg}}$ is close to the critical average film thickness.

Since lip wear can occur during the time when mixed lubrication occurs, it is desirable for the liftoff speed to be as low as possible. The liftoff speed, as a function of $(Ra)_{\text{shaft}}$, is illustrated in Figure 3.2.9. As shown in the plot, the liftoff speed decreases with increasing $(Ra)_{\text{shaft}}$. This is consistent with the observation that increased shaft roughness leads to increased load support. For shaft surface #1, when $(Ra)_{\text{shaft}}$ is 5% of $(Ra)_{\text{lip}}$ the liftoff speed is reduced by a significant 17%, compared to the smooth shaft value, and when $(Ra)_{\text{shaft}}$ is 10% of $(Ra)_{\text{lip}}$, the liftoff speed is decreased by 21%. Shaft surfaces #2 and #3 also produce liftoff speed reductions, but they are not as effective as shaft surface #1.
Figure 3.2.9 Effect of Shaft Surface Roughness Height on Liftoff Speed
3.2.4 Random Sealing Lip Surface Generation and Results

The deterministic modeling for the present study requires the input of lip surface roughness profiles. The previous surface roughness profiles for the lip surface are regular profiles, while a realistic lip seal has a random roughness profile. To investigate the effect of the lip surface roughness profile on lip seal operation, a numerical simulation of a random roughness profile should be provided.

The random sealing surface is generated as follows: first, a random rough surface is generated with a given roughness, orientation and density function using the “linear transformation matrix” approach (Patir, 1978). Second, to generate a ‘neutral’ surface which has no preference to hydrodynamically drive flow perpendicular to the sliding direction, the generated random surface from the first step is reflected in both x and y directions.

After the random surface is generated, it is possible that high value of roughness height could occur at some points and these abruptly increasing values may cause numerical stability and efficiency problems for the fluid mechanics solution. In order to eliminate the higher values of the roughness, truncation is performed to smooth the roughness profile.

In the present study, the random surface is generated with the same Ra value as the previous regular (cosine function) lip surface, while the shaft surface profile is the same as the previous models (surfaces #1, 2 and 3). The Skewness of the random surface is 0 and Kurtosis value is 3.

Four random surfaces are created with the same value of Ra, as shown in Figure 3.2.10. After inputting them in the FORTRAN program, the corresponding results are
very close, so the average of the results is taken to compare with the results obtained for regular surfaces.
Figure 3.2.10 (a) Two Different Random Surfaces with Same Ra Value
Figure 3.2.10 (b) Two Different Random Surfaces with Same Ra Value
Figure 3.2.11 to Figure 3.2.14 show the effect of shaft surface roughness height on the average film thickness, load support, reverse pumping rate and cavitation area ratio. As shown in these plots, the random surface can generate a larger value of average film thickness, load support, reverse pumping rate and than the previous regular models. However, these results are on the same order as those for the regular lip surface and very close to the surface #1 result. (It is interesting to note that surface #1 has the same value of Skewness as the random surface). There is not much qualitative difference in the effect of the shaft surface on the lip seal operation, as shown in those figures, which indicates that using the regular roughness profile is reasonable to simulate the lip surface, even though the random surface is more realistic.
Figure 3.2.11  Effect of Shaft Surface Roughness Height on Average Film Thickness
Figure 3.2.12 Effect of Shaft Surface Roughness Height on Load Support
Figure 3.2.13  Effect of Shaft Surface Roughness Height on Reverse Pumping Rate
Figure 3.2.14 Effect of Shaft Surface Roughness Height on Cavitation Area Ratio
3.2.5 Comparison of the Hydrodynamic Model and Elastohydrodynamic Model

The hydrodynamic model and the elastohydrodynamic model are both valid for full film lubrication. As mentioned earlier, the hydrodynamic model deals with the fluid mechanics without considering the deformation of the lip surface, while the elastohydrodynamic model can handle the more realistic case which includes the deformation analysis. By comparing the results of the two models for the same operating conditions, the effect of deformation mechanics can be determined.

Figure 3.2.15 shows the effect of shaft surface roughness height on the average film thickness for the hydrodynamic model and the elastohydrodynamic model. The shaft speed is 5m/sec; a series of different shaft surface roughness amplitudes are inputted to the program. Since the average film thickness is assumed to be a constant value, the average film thickness ratio is 1 for the hydrodynamic model. Surface #4 stands for the random surface which is generated with the same Ra value as the regular surface.
Figure 3.2.15 Comparison of the Effect of Shaft Surface Roughness Height on Average Film Thickness for Hydrodynamic Model and Elastohydrodynamic Model
Figure 3.2.16 to figure 3.2.18 show the results for both the hydrodynamic model and the elastohydrodynamic model. The corresponding load support, reverse pumping rate, and cavitation area ratio are compared for both models. Even though the trends on those plots are very similar, the hydrodynamic model (without considering the effect of the deformation of the lip surface) produces larger values of these performance parameters than those for the elastohydrodynamic model. The plots show that the adjustment of the average film thickness between the lip and shaft surface will affect the shaft surface roughness contribution to lip seal operation.
Figure 3.2.16 Comparison of the Effect of Shaft Surface Roughness Height on Load Support for Hydrodynamic Model and Elastohydrodynamic Model
Figure 3.2.17 Comparison of the Effect of Shaft Surface Roughness Height on Reverse Pumping Rate for Hydrodynamic Model and Elastohydrodynamic Model
Figure 3.2.18 Comparison of the Effect of Shaft Surface Roughness Height on Cavitation Area Ratio for Hydrodynamic Model and Elastohydrodynamic Model
3.3 Mixed Lubrication Model (Shen and Salant, 2004)

3.3.1 Objective and Assumptions

As mentioned earlier, the rotary lip seal usually operates with full film lubrication. The previous hydrodynamic and elastohydrodynamic models are only valid for full film lubrication. However at low speeds, such as those encountered during startup and shutdown, mixed lubrication occurs and asperities on the lip contact the shaft. As shown in Figure 3.3.1, a rough, rigid shaft surface moves past a stationary, rough lip surface. When the shaft speed is very low, the hydrodynamic load support in the lubrication film is not high enough to completely lift off the lip from the shaft. During this period, asperity contact will occur in the dark areas where the film thickness vanishes, and the total load support will be shared by the contact load support and the hydrodynamic load support.

Figure 3.3.1 Schematic of Sealing Zone for the Mixed Lubrication Model
Modeling of mixed-film lubrication problems is a challenging task. The tribological conditions of the lip seal are primarily determined by the interactions of the lubrication, deformation and the contact mechanisms. An analysis which integrates the major effects of all three mechanisms is required.

A deterministic mixed lubrication model has been constructed to simulate this mixed lubrication condition at the interface between two relatively moving, rough surfaces. The mixed lubrication model aims at: 1. solving an elastohydrodynamic lubrication problem with cavitation, 2. including a contact model between two rough surfaces, 3. constructing a systematic solution scheme for tasks 1 and 2. The fluid mechanics of the lubricating film is described by a Reynolds equation that can handle interasperity cavitation. The bulk deformation of the lip is computed using influence coefficients, while the junctions between the asperities and the shaft are modeled as Hertzian contacts.

This simplified Hertzian contact analysis is used because the shaft is rough, the flow is unsteady and an unsteady analysis is required. A more exact, but time-consuming method involves matrix inversion. The surface deformation in the contact region and the surface pressure in the non-contact region are given, and then the contact pressure can be calculated by using the inverse influence coefficient matrix. Even though this method is easy to understand and simple to implement in a computer program, the contact computation has to be performed many times in the present unsteady analysis. The very high computational cost of this matrix inversion method prevents this method from being directly used. Very recently FFT (fast Fourier transformation)-based contact models (Ju and Farris, 1996 Stanley and Kato, 1997) have been developed. These models transform
the contact problem, with associated deformations, from the spatial domain to the
frequency domain. By using this approach, the computation time can be decreased
significantly. Nevertheless, this study involves an unsteady analysis and the
computational time is still impractical for the present problem. It will take extremely
large computation times if that method is adopted. So the simplified Hertzian contact
model is utilized to handle the contact problem in the present study.

The mixed lubrication model shows how the shaft roughness affects such seal
characteristics as load support, contact load ratio, contact area ratio, cavitation area ratio,
reverse pumping rate and average film thickness.

In the fluid mechanics analysis, the assumptions for the mixed lubrication model
are the same as those for the elastohydrodynamic model except that asperity contact
occurs.

In the contact mechanics analysis, the assumptions are listed as follows:

i. Each asperity contact acts independently of all other asperity contacts.

ii. Each asperity has a radius of curvature R.

iii. Each asperity, in contact elastically, deforms according to the Hertzian theory.

iv. The contact surfaces are frictionless in both x and y direction, as in Hertzian
theory, for the purpose of calculating the contact pressure and contact area.

3.3.2 Solution Approach

3.3.2.1 Fluid Mechanics Analysis and Contact Mechanics Analysis

The mixed lubrication zone contains three different regions: the contact region,
the liquid region and the cavitation region. For the non-contacting portion, the fluid
mechanics model yields the boundaries of the liquid and cavitation regions, and the fluid pressure obtained by solving the general Reynolds equation (3.1.6). For the contact region, the contact mechanics analysis yields the location of the contact region and the contact pressure. The bulk deformation of the elastic surface (lip) is calculated in the deformation mechanics analysis. The film thickness is set equal to the sum of two cosine terms to represent the asperities on both surfaces plus an average film thickness, \( h_{\text{avg}} \), which is determined by the normal deformation of the lip. \( \delta \) represents the circumferential displacement of the lip surface, which is determined by the shear deformation of the lip.

\[
h = A_1 \cos \left( 2\pi \left( \frac{x - \delta}{\lambda_{11}} \right) \right) \cos \left( \frac{2\pi y}{\lambda_{12}} \right) - A_2 \cos \left( \frac{2\pi}{\lambda_{21}} \left( x - r \right) \right) \cos \left( \frac{2\pi y}{\lambda_{22}} \right) + h_{\text{avg}} \tag{3.3.1}
\]

This equation corresponds to shaft surface #1 in the hydrodynamic and elastohydrodynamic models.

Asperity contacts occur when the computed film thickness, using equation (3.3.1), becomes zero or negative. In the fluid mechanics analysis, a very small value of the film thickness is set at the contact points when asperity contacts occur. At these points the computed values of \( p_{\text{fluid}} \) from equation (3.1.6) are replaced with zero due to the asperities contacting. The dimensionless fluid load support and reverse pumping rate in the non-contact region can be computed from:

\[
W_{\text{fluid}} = \iint_{\text{non-contact}} \left( p_{\text{fluid}} - p_{\text{avg}} \right) dxdy \tag{3.3.2}
\]
\[ Q = \int -\frac{h^3}{12} \frac{\partial p_{\text{fluid}}}{\partial y} \, dx \] (3.3.3)

In the contact region the asperities on the lip surface are squeezed against the shaft, and the junctions between the asperities of the lip and the shaft surface are modeled as Hertzian contacts, as shown in Figure 3.3.2. The Hertzian solution provides a closed-form expression for two surfaces in a purely elastic contact.
The contact pressure in the contact region is given by (Johnson, 1985):

\[
p_{\text{Hertzian}} = 2E'(d'/R')^{(1/2)} \left[ 1 - (r^*/a^*)^2 \right]^{(1/2)}/\pi \tag{3.3.4}
\]

and,

\[
W_{\text{Hertzian}} = \iint_{\text{contact}} (p_{\text{Hertzian}} - p_a) dxdy \tag{3.3.5}
\]

Thus the total load support, \( W \), is the summation of the load support due to fluid pressure and that due to asperity contact:

\[
W = W_{\text{Hertzian}} + W_{\text{fluid}} \tag{3.3.6}
\]

Since the geometric overlap region is not the real contact region in the Hertzian contact model, the film thickness in the overlap region, but not in the contact region, is determined from the following equation (Johnson, 1985):

\[
h = \{a^*(r^2 - a^2)^{1/2} - (2a^* - r^2) \tan^{-1}[((r^2/a^2) - 1)]^{1/2} \}/(\pi R'h^*_r) \tag{3.3.7}
\]

This expression is combined with the film thickness expression (3.3.1) in the non-overlap region for use in the fluid mechanics analysis.
3.3.2.2 Deformation Mechanics Analysis

To calculate the normal and shear deformations of the lip surface, the influence coefficient method is again utilized in the deformation mechanics analysis. The normal deformation of the lip surface determines the average film thickness, $h_{\text{avg}}$, and can be expressed (at the $i$ axial node) as:

$$
(h_{\text{avg}})_i = D + \sum_{k=1}^{n} (I_1)_{ik} (p_{\text{avg}}^{**} - p_{\text{contact}}^{**})_k
$$

(3.3.8)

where $D$, is the initial average film thickness when the lip is mounted to the shaft; it is equal to $0.57A_1$ (Shi and Salant, 2001). It should be noted that the dimensionless pressure $p$ is equal to $p_{\text{fluid}}$ in the non-contact region and is equal to $p_{\text{Hertzian}}$ in the contact region. In order to obtain the circumferential displacement of the lip surface, the dimensionless shear stress is set equal to the viscous shear stress in the fluid region:

$$
\tau = (hh_r^{*} (p_x^{*} - p_v^{*}) / 2E^*L^*)^* (dp_{\text{fluid}}^{*} / dx^*) + (\mu^{*}U^*/hh_r^{*}E^*)
$$

(3.3.9)

and to the dry contact friction in the contact region:

$$
\tau = f^*P_{\text{Hertzian}} / E^*
$$

(3.3.10)

The shear deformation is calculated from equation (3.2.5).
3.3.2.3 Computational Scheme for Mixed Lubrication Model

In the solution scheme for the mixed EHL model, there are three main parts: the fluid mechanics, deformation mechanics and the asperity contact analyses. All of them are coupled and an iterative computation procedure is required as shown in Figure 3.3.3. First, the design and operating parameters are input into the program, and the initial guesses for average film thickness and circumferential deformation are specified the same way as in the EHL model. Next, the fluid mechanics model is used to calculate the fluid pressure $p_{\text{fluid}}$ by solving the general Reynolds equation and the procedure is the same as in the hydrodynamic and elastohydrodynamic models. The resulting pressure distribution is outputted in the whole domain. Also, the submodel for asperity contact is implemented for solving the contact pressure $p_{\text{Hertzian}}$. The contact pressure is calculated in the contact domain. A variable $p$ is introduced in the program to combine the fluid pressure in the non-contact region and the contact pressure in the contact region. This pressure distribution $p$ is used as input to the deformation mechanics analysis. In the deformation analysis, the normal deformation, shear deformation and the average film thickness are computed. After a convergence check on the average film thickness, the program goes back to the mixed-EHD loop again until the film thickness converges.
Figure 3.3.3 Overall Solution Scheme for Mixed Lubrication Model
3.3.3 Results and Discussion for Mixed Lubrication Model

This mixed lubrication model is used to compute the performance of a representative lip seal at low speeds. The base parameter values are: \( L^* = B^* = 5 \times 10^{-4} \) m, \( \mu^* = 2.5 \times 10^{-2} \) Pa-s, \( p_s^* = p_a^* = 1.02 \times 10^5 \) Pa, \( p_c^* = 0 \) Pa, \( h_r^* = 1 \) \( \mu \)m, \( \lambda_{11}^* = \lambda_{12}^* = 5 \times 10^{-5} \) m, \( \lambda_{21}^* = \lambda_{22}^* = 10^{-4} \) m, \( A_1^* = 0.5 \) \( \mu \)m, \( A_2^* = 0.05 \mu \)m, shaft diameter = \( 4.445 \times 10^{-2} \) m, \( E^* = 6.2 \times 10^6 \) Pa, \( \nu^* = 0.49 \), \( f^* = 0.25 \). The corresponding dimensionless parameters are: \( \Lambda = 1 \), \( \lambda_{11} = \lambda_{12} = 0.1 \), \( \lambda_{21} = \lambda_{22} = 0.2 \), \( A_1 = 0.5 \), \( A_2 = 0.05 \), \( p_a = p_s = 1 \). Both sides of the seal are assumed to be flooded so that the reverse pumping rate can be calculated. A grid analysis has shown that a \( 101 \times 101 \) mesh is sufficient for the mixed lubrication analysis.

The time-averaged total load support (including the hydrodynamic load and contact load) for two cases, a smooth shaft and a rough shaft, are shown in Figure 3.3.4 as functions of dimensionless shaft speed \( \gamma \). At very low speed, there are substantial asperity contacts in the solution domain, and the contact load is dominant, as will be discussed below. As the speed is increased, the hydrodynamic load increases while the contact load support decreases, as will be discussed below, and the total load support increases. This figure shows that roughness on the shaft surface increases the total load support and eventually makes the liftoff process easier.
Figure 3.3.5 shows the time-average reverse pumping rate (including the hydrodynamic load and contact load) as a function of speed. Once the shaft starts rotating at very low speeds, the lip begins to liftoff due to the hydrodynamic pressure in the film, and a very small value of the reverse pumping rate is obtained. As speed is increased, the reverse pumping mechanism becomes more significant, and that leads to a larger Q. Figure 3.3.3 also shows that a rough shaft produces a larger reverse pumping rate than a smooth shaft. It is important to note that the shaft surface microgeometry actually improves the sealing ability of the seal.
The cavitation area ratio which is defined as the ratio of the cavitation area to the total domain area is shown as a function of speed in Figure 3.3.6. The curves follow similar trends as the load support curves; as speed increases, so does the cavitation area. For the same shaft speed, the rough shaft can produce higher value of cavitation area ratio than the smooth shaft case.
Figure 3.3.7 shows the variation of the average film thickness $h_{avg}$ as a function of speed. As would be expected, the average film thickness increases with speed over the range of interest. This trend is consistent with the total load support curves since a larger value of load support should produce a larger average film thickness. For the same speed, the rough surface on the shaft produces a higher value of the average film thickness, compared to the smooth surface, accelerating the liftoff procedure.
An important characteristic parameter for tracing the liftoff process during the startup period is the contact area ratio, $A_c$, defined as the ratio of the contact area to the total domain area. Figure 3.3.8 shows the contact area ratio as a function of speed over the entire speed range. It decreases with speed until it becomes zero at (and after) the liftoff speed. Once the lip completely lifts off, the interface is in the full film regime. Since lip wear occurs during mixed lubrication and is proportional to contact area, it is desirable for the liftoff speed and the contact area to be as low as possible. Figure 3.3.8 shows that shaft roughness can lessen the wear in the mixed lubrication region by reducing both the contact area and the liftoff speed.
Figure 3.3.8 Contact Area Ratio versus Speed

Figure 3.3.9 shows the contact load ratio, $r_c$, which is the ratio of the contact load support to the total load support, as a function of speed. The liftoff process is reflected in the contact load ratio, as well as in the contact area ratio. The contact load ratio decreases with the speed due to the liftoff of the lip from the shaft; this is consistent with the curves for the contact area ratio. The contact load ratio reaches zero once the lip completely lifts off from the shaft and asperity contact ceases. As shown in Figure 3.3.9, the rough shaft produces a lower contact load ratio than a smooth shaft except at a very low speed; that is also consistent with the curves of the contact area ratio.
The liftoff process can be traced by maps of the various regions at different speeds as shown in Figure 3.3.10. The black area represents the contact region; the gray area represents the cavitation region; while the white area represents the fluid region. At very low speed, the hydrodynamic load support and the average film thickness are very small, and the contact load support is dominant. As the shaft speed increases, the lip is gradually lifted off the shaft due to the higher hydrodynamic pressure generation. So there is less contact area but more cavitation area. As shaft speed increases further, the contact area continually decreases while the cavitation area increases until the lip surface is completely lifted off the shaft. After that point, there is no asperity contact in the domain.

Figure 3.3.9  Contact Load Ratio versus Speed
Figure 3.3.10 The Development of Fluid, Contact and Cavitation Area Changing with Shaft Speed (Black: contact area; Gray: cavitation area; White: fluid area)
3.3.4 Comparison of the Elastohydrodynamic Model and Mixed Lubrication Model

As mentioned earlier, the elastohydrodynamic model is only valid for full film lubrication, while the mixed lubrication model can handle the cases under very low shaft speed with contact. Figure 3.3.11 to Figure 3.3.14 show the comparison of the elastohydrodynamic model and mixed lubrication model for a series of shaft speeds. The shaft speed range is from 0.1 m/sec to 5 m/sec. During the normal operation speed, the load support, reverse pumping rate, cavitation area ratio and average film thickness from the EHD model are consistent with those from the mixed lubrication model.
Figure 3.3.11 Load Support vs. Shaft Speed for EHD and Mixed Lubrication Model
Figure 3.3.12 Reverse Pumping Rate vs. Shaft Speed for EHD and Mixed Lubrication Model
Figure 3.3.13 Cavitation Area Ratio vs. Shaft Speed for EHD and Mixed Lubrication Model
Figure 3.3.14 Average Film Thickness vs. Shaft Speed for EHD and Mixed Lubrication Model
3.4 Transient Mixed Lubrication Model

3.4.1 Objective and Assumptions

The objective of this part of the study is to develop a computational scheme to predict the performance history of a lip seal during transient operation. Such a model could be used in the design process to obtain an initial simulation of the lip seal’s transient performance, which may include a history of the average film thickness, load support, reverse pumping rate, cavitation area ratio and contact area ratio. This transient mixed lubrication model combines the fluid mechanics, asperity contact, and both the bulk material and asperity deformation analyses. The lip seal performs under a time-varying shaft speed condition, such as occurs during the start up and shut down processes.

The assumptions for the transient mixed lubrication model are the same as those for the mixed lubrication model except for the steady speed assumption.

3.4.2 Solution Approach

3.4.2.1 Fluid Mechanics Analysis

In the start up process, the shaft speed starts from zero and accelerates in a short time, finally reaching a steady state speed. During this period, the rotary lip seal will experience a transient process since the shaft speed changes with time. At very low speed, the lip surface is not completely lifted off the shaft, so contact occurs between the lip and the shaft surface. As the shaft speed increases, the effect of the fluid dynamics becomes significant even though the contact regions may still exist. The transient mixed lubrication model includes the fluid mechanics analysis, contact analysis and the
deformation analysis. Those three analyses are coupled and are linked through a numerical iteration process.

For the fluid mechanics analysis, the dimensional Reynolds equation, equation (3.1.1), and the continuity equation, equation (3.1.2), from the hydrodynamic model, are valid except that the shaft speed \( U^* \) is now function of time. As in the hydrodynamic model, the two differential equations can be combined into one universal equation.

Introducing nondimensional time defined as:

\[
t = t^* / (L^*/U_r^*)
\]

and using the other nondimensional quantities from the hydrodynamic model, yields the universal equation that applies to both non-cavitating and cavitating regions:

\[
\frac{\partial}{\partial x} \left[ h^3 \frac{\partial \Phi}{\partial x} \right] + \lambda^2 \frac{\partial}{\partial y} \left[ h^3 \frac{\partial \Phi}{\partial y} \right] = \beta_1(t) \frac{\partial}{\partial x} \left\{ \left[ \frac{1 + (1 - S) \Phi}{h} \right] k \right\} + \beta_2 \frac{\partial}{\partial t} \left\{ \left[ \frac{1 + (1 - S) \Phi}{h} \right] k \right\}
\]  

(3.4.2)

where, \( \beta_1(t) = 6\mu U^*(t)L^*/h_r^2(p_s^*-p_c^*) \) and \( \beta_2 = 12\mu L^*U^* [h_r^2(p_s^*-p_c^*)] \). The parameter \( \beta_1(t) \) is dependent on the time due to the shaft speed variation during the startup and shutdown periods. The value of \( \beta_2 \) is a constant.

To evaluate the time derivative in the previous equation, a fully implicit scheme is utilized. The transient Reynolds equation (3.4.2) is discretized by using micro control volumes. In this model, the coefficients \( K_c, K_n, K_w, K_s \) have the same definitions as shown in Equation (3.1.13) and the following definitions are introduced also:
\[ A_E = \frac{\Delta y \Delta t K_e}{\Delta x} \]
\[ A_W = \frac{\Delta y \Delta t K_w}{\Delta x} \]
\[ A_N = A^2 \frac{\Delta x \Delta t K_n}{\Delta y} \]
\[ A_S = A^2 \frac{\Delta x \Delta t K_s}{\Delta y} \] (3.4.3)

\[ \xi = \int_{new}^{old} U(t) dt \] (3.4.4)

\[ A_P = (A_E + A_W + A_N + A_S) F_P + \Delta y \gamma_1 (1 - F_P) h_e \xi + \Delta x \Delta y \gamma_2 (1 - F_P) h_P \]

\[ S_b = \Delta y \gamma_1 \xi \left[ h_e - \left( 1 + (1 - F_w) \Phi_w \right) h_w \right] \]

\[ + \Delta y \Delta x \gamma_2 (h_P - h^0_p) - \Delta x \Delta y \gamma_2 (1 - F^0_p) \Phi^0_p h^0_p \] (3.4.5)

It should be pointed out that the parameters \( A_E, A_W, A_N, A_S, A_P \) and \( S_b \) in the fully implicit method are set to different values than those used in equations (3.1.17) and (3.1.18) even though the same symbols are used.

By utilizing the proceeding definitions, the algebraic discretized form of equation (3.4.2) can be obtained as:

\[ A_P \Phi_P = A_E \Phi_E F_E + A_W \Phi_W F_W + A_E \Phi_E F_E + A_W \Phi_W F_W - S_b \] (3.4.6)
For each node P in the domain, the above equation can be written with appropriate values of coefficients A and cavitation indices F.

The system of linear equations is solved with the ADI method, as in the hydrodynamic model. The tridiagonal matrix algorithm (TDMA) is used in the axial direction and the cyclic tridiagonal matrix algorithm (CTDMA) in the circumferential direction.

Once the fluid mechanics is solved, the reverse pumping rate is evaluated using equation (3.3.3). The fluid load support at each node is obtained by multiplying the pressure and the area of the control volume surrounding the node, as in equation (3.3.2).

The contact and the deformation analyses are similar to those in the mixed lubrication models.

3.4.2.2 Computational Scheme for Transient Mixed Lubrication Model

As mentioned earlier, the fluid mechanics, the contact mechanics and the deformation analysis are strongly coupled; the changes of film thickness will cause changes of fluid and contact pressure and vice versa. The major difficulty in the transient mixed lubrication model is the coupling of the fluid mechanics/contact mechanics and the deformation analysis. Therefore, a good scheme to deal with the coupling is the key to successful transient analysis of the lip seal.

The computational procedure for the transient model is shown in Figure 3.4.1. At the beginning of the program, the design and operating parameters are inputted and the average film thickness and circumferential deformation are initialized. At this time step
and given film thickness distribution, the fluid mechanics and the contact mechanics are solved, and then the deformation mechanics is solved. This is done iteratively for each time step until the film thickness is converged. For the next time step, the program goes through the time marching loop and solves these three main parts again.
Figure 3.4.1 Flow Chart for Transient Mixed Lubrication Model
3.4.3 Results and Discussion for Transient Mixed Lubrication Model

This transient model is built to simulate the performance of a representative rotary lip seal. The base parameter values are: \( L^* = B^* = 5 \times 10^{-4} \text{ m} \), \( \mu^* = 2.5 \times 10^{-2} \text{ Pa-s} \), \( p_s^* = p_a^* = 1.02 \times 10^5 \text{ Pa} \), \( p_c^* = 0 \text{ Pa} \), \( h_r^* = 1 \mu\text{m} \), \( \lambda_{11}^* = \lambda_{12}^* = 5 \times 10^{-5} \text{ m} \), \( \lambda_{21}^* = \lambda_{22}^* = 10^{-4} \text{ m} \), \( A_1^* = 0.5 \mu\text{m} \), \( U_r^* = 5\text{m/s} \), shaft diameter = \( 4.445 \times 10^{-2} \text{ m} \), \( E^* = 6.2 \times 10^6 \text{ Pa} \), \( \nu^* = 0.49 \), \( f^* = 0.25 \), \( T = 0.01 \text{sec} \). The corresponding dimensionless parameters are: \( \Lambda = 1 \), \( \lambda_{11} = \lambda_{12} = 0.1 \), \( \lambda_{21} = \lambda_{22} = 0.2 \), \( A_1 = 0.5 \), \( p_a = p_s = 1 \). Both sides of the seal are assumed to be flooded so that the reverse pumping rate can be calculated. For the transient model, the time step for each case was selected such that it was small enough to yield an accurate solution but large enough to avoid and excessive computation time.

Figure 3.4.2 shows the total load support \( W \) as a function of time ratio \( K \) (time/acceleration time) for four different shaft surfaces which are used in the EHD model (Figure 3.1.2). As mentioned earlier, the three rough shaft surfaces have the same Ra value but different surface profiles. This plot traces the transient total load support result for a startup process in which the shaft speed accelerates from a very low speed (0.1m/s) to a normal operating shaft speed (5m/s). The total acceleration time for the base case is 0.01sec. The range of the corresponding dimensionless time ratio \( K \) is from 0 to 1. In order to detect the transient effect, the shaft rotates at the same speed after it reaches the normal operation speed (5m/s) in the range of time \( K \) from 1 to 1.5. It is seen that the total load support increases with time during the acceleration process. At the same shaft speed, the case with shaft surface #1 produces the largest value, while the cases with surfaces #2 and #3 produce lower values. As expected, the case with the smooth shaft
produces the lowest. In each case, at the end of the transient the load support overshoots the “steady state” value, after which it approaches the “steady state” value.

Figure 3.4.2 Total Load Support versus K for Transient Mixed Lubrication Model in Startup Process, Base Case
The reverse pumping rate for the four different cases is shown in Figure 3.4.3 as a function of dimensionless time ratio $K$. At the very beginning, the shaft rotates at a very low speed. The reverse pumping rate is small (close to zero) since the contact mechanism is dominant. As the shaft speed is increased, the reverse pumping rate increases for the four different cases. Figure 3.4.3 also shows that the roughness on the shaft surface can help increase the reverse pumping rate. The reverse pumping rate for shaft surface #1 is higher than the corresponding results for surfaces #2 and #3, while the smooth shaft produces the smallest value. The transient effect is also seen in this plot.

![Figure 3.4.3 Reverse Pumping Rate versus K for Transient Mixed Lubrication Model in Startup Process, Base Case](image-url)
The cavitation area in the sealing zone also increases with shaft speed, as shown in Figure 3.4.4. For a given value of shaft speed, surface #2 has the largest increase in cavitation area and surface #1 has the smallest increase compared with the smooth shaft surface. The transient effect is very similar to those for the load support and reverse pumping rate. The results of cavitation area ratio in the constant shaft speed period are consistent with those from the EHD model.

Figure 3.4.4 Cavitation Area Ratio versus K for Transient Mixed Lubrication Model in Startup Process, Base Case
An important parameter of rotary lip seal operation is average film thickness. Figure 3.4.5 shows that the average film thickness increases during the acceleration process. As seen in this plot, surface #1 produces the largest effect on the average film thickness and the smooth shaft case produces the smallest. This trend is consistent with those observed with the total load support and reverse pumping rate. The transient effect is also seen in this plot.

Figure 3.4.5 Average Film Thickness versus K for Transient Mixed Lubrication Model in Startup Process, Base Case
The contact area ratio is defined as the ratio of the contact area to the total domain area. Figure 3.4.6 shows the contact area ratio $A_c$ as a function of time ratio over the entire speed range. It decreases with speed until it becomes zero after the liftoff speed is reached. When the hydrodynamic pressure in the film is large enough to completely lift off the lip from the shaft, the interface is in the full film lubrication region. Figure 3.4.6 shows that shaft surface #1 reaches the zero value of the contact area ratio earliest, and that indicates that surface #1 has the smallest liftoff speed. Shaft surfaces #2 and #3 have larger values of the liftoff speed respectively, while the smooth shaft has the largest one. This result is consistent with the plots for load support and average film thickness. Also, the results of the liftoff speed in this figure are consistent with the computations of liftoff speed in the EHD model.
Figure 3.4.6 Contact Area Ratio versus $K$ for Transient Mixed Lubrication Model in Startup Process, Base Case
Figure 3.4.7-Figure 3.4.9 show another group of results on the effect of the shaft roughness amplitude on the lip seal performance. (Ra)\text{ratio} is defined as the ratio of the Ra of the shaft to the Ra of the lip. For surface #1, three cases with different (Ra)\text{ratio} values (0.2, 0.1 and 0.05) are considered. Figure 3.4.7-Figure 3.4.9 give the corresponding results for total load support, reverse pumping rate and cavitation area ratio for those cases. The case with largest roughness (Ra)\text{ratio} value (0.2) produces largest value of load support, reverse pumping rate and cavitation area ratio compared with the previous results, even though the trends are very similar. The case with the smallest roughness (Ra)\text{ratio} value (0.05) yields the smallest values of those variables.
Figure 3.4.7 Total Load Support versus K for Transient Mixed Lubrication Model in Startup Process with Various \((Ra)_{ratio}\)
Figure 3.4.8 Reverse Pumping Rate versus K for Transient Mixed Lubrication Model in Startup Process with Various (Ra)Ratio
Figure 3.4.9 Cavitation Area Ratio versus K for Transient Mixed Lubrication Model in Startup Process with Various (Ra)$_{ratio}$
Figures 3.4.10, 3.4.11 and 3.4.12 show results for transient cases with different acceleration times. The previous figures all have the smallest acceleration time (0.01 second). This group of figures includes two slower acceleration processes, with acceleration times of 1 second and 10 seconds. \((Ra)_{ratio}\) is kept constant at 0.1 and the initial and final speeds are kept the same as the base case. Surfaces #1, #2, #3 and the smooth shaft surface are considered and each surface group has three curves corresponding to three different acceleration times. Figure 3.4.10 to Figure 3.4.12 show the total load support, reverse pumping rate and cavitation area ratio as a function of the time ratio \(K\). It is seen that for each surface group, the results with larger acceleration times (1 second and 10 second) are similar to those with the smallest acceleration time (0.01 second) except the transient effect is not as pronounced. During the period with constant speed, the results for load support, reverse pumping rate and cavitation area ratio eventually converge to values which are consistent with those from the EHD model.
Figure 3.4.10 Total Load Support Rate versus K for Transient Mixed Lubrication Model in Startup Process with Various Acceleration Times
Figure 3.4.11 Reverse Pumping Rate versus K for Transient Mixed Lubrication Model in Startup Process with Various Acceleration Times
Figure 3.4.12 Cavitation Area Ratio versus K for Transient Mixed Lubrication Model in Startup Process with Various Acceleration Times
Figure 3.4.13 and the subsequent figures deal with the shut down process. The subsequent figures deal with the shut down process. The base case values are the same as those for the startup process, except that the shaft speed decelerates from 5m/sec to 0.1m/sec and the deceleration time is 0.01 second. As in the start up process, the period following the transient operation is shown to display the transient effect. Figure 3.4.13 shows that the total load support decreases with time, and the results are consistent with those from the start up process. The case with shaft surface #1 produces the largest effect on the load support compared to the other shaft surfaces, and the smooth shaft produces the smallest value of total load support. The transient effect can be seen in this figure.

Figure 3.4.13 Total Load Support versus K for Transient Mixed Lubrication Model in Shut down Process, Base Case
The reverse pumping rate as a function of time is shown in the Figure 3.4.14. The curves for the four different shaft surfaces follow similar trends; as the shaft speed decreases, so does the reverse pumping rate. Again, shaft surface #1 has the largest reverse pumping rate while the smooth surface has the smallest. The transient effect is again seen in this figure.

Figure 3.4.14 Reverse Pumping Rate versus K for Transient Mixed Lubrication Model in Shut down Process, Base Case
Figure 3.4.15 shows the variation of the cavitation area ratio as a function of time. As would be expected, the cavitation area ratio decreases with time over the range of interest. Shaft surface #2 produces the largest effect on the cavitation area ratio. Again, the transient effect on the cavitation area ratio can be seen in the constant speed range.

![Figure 3.4.15 Cavitation Area Ratio for versus K for Transient Mixed Lubrication Model in Shut down Process, Base Case](image)

Figure 3.4.15 Cavitation Area Ratio for versus K for Transient Mixed Lubrication Model in Shut down Process, Base Case
Figure 3.4.16, 3.4.17 and 3.4.18 show four groups of results for transient cases with different deceleration times. The previous figures for the shut down process have the smallest deceleration time (0.01sec); these figures include two other slow deceleration processes with deceleration times of 1 sec and 10 sec. The (Ra) ratio is kept the same at 0.1 and the initial and final speed is kept the same as well. Surfaces #1, #2, #3 and the smooth shaft surface are inputted into the program and each surface group has three curves corresponding to three different acceleration times. Figure 3.4.16 to Figure 3.4.18 show the total load support, reverse pumping rate and cavitation area ratio as a function of time ratio. It is seen that for each surface group, the results with the larger acceleration times (1sec and 10 sec) are similar to the previous case with deceleration time (0.01sec) except the transient effect is not as obvious. During the period with constant speed, the results for load support, reverse pumping rate and cavitation area ratio eventually converge to the same value which is consistent with those in mixed lubrication model.
Figure 3.4.16 Total Load Support versus K for Transient Mixed Lubrication Model in Shut down Process with Various Acceleration Times
Figure 3.4.17 Reverse Pumping Rate versus K for Transient Mixed Lubrication Model in Shut down Process with Various Acceleration Times
Figure 3.4.18 Cavitation Area Ratio versus K for Transient Mixed Lubrication Model in Shut down Process with Various Acceleration Times
CHAPTER 4
CONCLUSIONS

From previous test results, it is well known that shaft surface finish can significantly affect the performance of the rotary lip seal operation, even though the shaft roughness, after run-in, is much smaller than the lip roughness. In the present research, four numerical models are developed to predict the performance of the rotary lip seal with different shaft roughness profiles. The most important measures of seal performance are load support; reverse pumping rate, cavitation area ratio and average film thickness. The models are able to predict these characteristics of the rotary lip seal under different operating conditions and to determine the effect of the shaft roughness on the lip seal performance.

The hydrodynamic model is used in the full film lubrication condition without considering the deformation of the lip surface. The main issue for this model is to solve the fluid mechanics numerically. The results demonstrate how the shaft surface finish can significantly influence lip seal operation through hydrodynamic effects. Due to the nonlinearity of the Reynolds equation, very small fluctuations on the shaft surface can create large changes in the pressure distribution within the lubricating film, which produce significant changes in the load support, reverse pumping rate and cavitation area. These changes are primarily dependent on the details of the shaft surface profile, the Ra of the shaft and shaft speed. In addition, the reverse pumping is dependent on the shaft lead angle. The results suggest that microasperities on the shaft surface improve lip seal
performance, but this conclusion is not conclusive because EHD affects have not been taken into account in this deterministic hydrodynamic model.

The elastohydrodynamic model incorporates both the fluid mechanics of the lubricating film and the elastic behavior of the lip. The results of this model indicate that shaft surface roughness, viz. fluctuations on the shaft surface, can produce significant desirable effects on lip seal behavior, which depend on the shaft surface profile. This analysis has shown that fluctuations in the lubricating film thickness, produced by the fluctuations on the shaft surface, generate increases in the average film thickness, load support, reverse pumping rate and a decrease in liftoff speed, all very favorable effects.

At low speeds such as those encountered during startup and shutdown, mixed lubrication occurs and asperities on the lip contact the shaft. A mixed EHD model is constructed to simulate this condition. The fluid mechanics of the lubrication film, the deformation mechanics and the contact mechanics analysis are included in this mixed lubrication model. The results demonstrate that an unsteady mixed EHL model can generate the important performance characteristics of a lip seal, such as load support, contact load ratio, contact area ratio, cavitation area ratio, reverse pumping rate and average film thickness. The mixed lubrication model also shows that microasperities on a shaft surface can improve sealing ability and accelerate the liftoff process.

The transient process of the lip seal is simulated by the transient mixed lubrication model. This transient model combines the partial film lubrication, asperity contact and both the bulk material and asperity deformations. Start up and shut down processes are used to trace the lip seal performance. The results demonstrate that the shaft surface roughness profile affects the characteristic performance of the lip seal during the transient
process. The roughness on the shaft surface could help reduce the wear and contact between the lip and shaft surface during the low speed period.

In summary, the research described in this thesis has produced an effective tool for studying the elastohydrodynamic effects of shaft surface roughness on lip seal behavior. The results indicate that increased shaft surface roughness generally improves lip seal performance, within the range of roughness amplitudes studied. This is in agreement with experimental observations. However it is known that at larger roughness amplitudes, outside of the range studied, shaft surface roughness is detrimental to seal performance. This may be due to factors other than elastohydrodynamics, as discussed in Chapter 2, which are beyond the scope of this thesis.
REFERENCES


VITA

Dawei Shen, daughter of Jiewen Qi and Jinsheng Shen, was born in Changchun, China in 1975. She grew up in Jilin and received her primary and secondary education there. She attended Harbin Institute of Technology from September 1991 to July 1995, where she earned her Bachelor of Science (1995) in mechanical engineering. After graduation with the bachelor’s degree, she joined the precision instruments lab in Harbin Institute of Technology as an assistant research engineer. During that time, she designed an electrical control system for an aircraft simulator and experimentally tested the electrical performance of the prototypes with other group mates. In 2000, she came to the G. W. Woodruff School of Mechanical Engineering at Georgia Institute of Technology to pursue a Master and a Ph.D. degree. At Georgia Tech, She earned her Master of Science in mechanical engineering in 2002 and conducted research on fluid sealing technology, under the guidance of Prof. Richard Salant.