Closeout Notice Date 20-OCT-1998

Project Number E-16-M19
Center Number 10/24-6-R7228-0A0
Project Director MENON, SURESH
Project Unit AERO ENGR
Sponsor NAVY/OFC OF NAVAL RESEARCH
Division Id 3314
Contract Number N00014-91-J-1963 Contract Entity GTRC
Prime Contract Number

Title RESEARCH IN UNDERWATER EXPLOSIONS
Effective Completion Date 30-SEP-1998 (Performance) 30-SEP-1998 (Reports)

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Comments

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NOTE: Final Patent Questionnaire sent to PDPI
Title: RESEARCH IN UNDERWATER EXPLOSIONS

Sponsor: NAVY/OFC OF NAVAL RESEARCH

Division Id: 103 / 3314
Award period: 01-JUN-1991 to 30-SEP-1998 (performance) 30-SEP-1998 (reports)

Sponsor amount
Contract value: 0.00
New this change
Funded: 12,000.00
Total to date
568,001.00
568,001.00
Cost sharing amount: 0.00
Total to date
22,565.00

Does subcontracting plan apply?

Title vests with: G

Active comments -
on No. A00006 extends period of performance to 30 September 98.
October 7, 1992

Annual Letter Report
N00014-91-5-1963

TO: Dr. Spiro G. Lekoudis, Director
    Mechanics Division, Office of Naval Research
    Arlington, VA 2217-5000

FROM: Warren C. Strahle, Regents' Professor
      Principal Investigator

SUBJECT: End of Fiscal Year Letter Report
         "Underwater Explosion Research"

A) Scientific Research Goals:

1) To understand reactive underwater explosion bubble behavior with an
   end view of tailoring the bubble energy.

2) To understand several water-gas interface instabilities with a goal
   of determining bubble cycle energy loss.

B) Significant Results of the Past Year:

1) The underwater explosion tank (2m x 2m x 1m) was finished.

2) Glass globe explosion experiments using H₂ and O₂ and CO and O₂
   explosives were initiated. The experiments record visual bubble
   behavior with a 10,000 fps image intensified CCD TV camera. Bubble
   internal pressure and temperature are measured.

3) Peculiar behavior has been noted, but as yet unexplained, in
   comparisons of the H₂-O₂ and CO-O₂ explosions. The hydrogen system
   is less powerful than calculated, whereas the CO-O₂ system follows
   theory quite closely.

4) The bubbles fall apart (due to a combination of evaporation and
   Rayleigh-Taylor instability) after one cycle.

5) A theory for the interface behavior has been constructed and
   calculations are underway.

6) Parameter variations, using inert diluents, have began in the
   experiment.
C. Plan for Next Year's Research:

1) Introduce aluminum into the glass globes and incorporate two-phase flow into the calculations.

2) Complete the calculations on evaporative and Rayleigh-Taylor interface behavior.

3) Initiate optimization theory for optimal heat release rate to maximize bubble performance.

See attachments for other items requested.
R&T Number: N00014-91-J-1963

Contract/Grant Title: Underwater Explosions Research

Scientific Officer: Richard S. Miller

Principal Investigator: Warren C. Strahle

Mailing Address: School of Aerospace Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332-0150

Phone Number: (404) 894-3032

FAX Number: (404) 894-2760

E-Mail Address: wstrahle@gtri01.gatech.edu

a. Number of Papers Submitted to Referred Journal but not yet published: 0

b. Number of Papers Published in Referred Journals: 0 (List Attached):

c. Number of Books or Chapters Submitted but not yet Published: 0

d. Number of Books or Chapters Published (List Attached): 0

e. Number of Printed Technical Reports & Non-Referred Papers (List Attached): 0

f. Number of Patents Filed: 0

g. Number of Patents Granted (List Attached): 0

h. Number of Invited Presentations at Workshops or Professional Society Meeting (List Attached): 0

i. Number of Presentations at Workshops or Professional Society Meetings (List Attached): ONR "Underwater Explosions" 1 Workshop, Pullman, WA, September 1992.

j. Honors/Awards/Prizes for Contract/Grant Employees: (List Attached, may include Society Awards/Offices, Promotions, Faculty Awards/Offices, etc.) 0
LIST OF PUBLICATIONS/REPORTS/PATENTS/GRADUATES

1. Papers Published in Referred Journals:
   None

2. Books (and sections thereof) Published:
   None

3. Technical Report, Non-Refereed Papers:
   None


5. Patents Granted:
   None

6. Degrees Granted (name, date, degree):
   Mr. Steven Arnios  MS  6/92

Enclosure (2)
OTHER SPONSORED RESEARCH
(Include title, sponsors's name, dollar amount and start and end dates for the award)

None

Enclosure (4)
September 29, 1993

TO: Dr. Spiro G. Lekoudis, Director
    Mechanics Division, Office of Naval Research
    Arlington, VA 2217-5000

FROM: Warren C. Strahle, Regents' Professor
      Principal Investigator

         End of Fiscal Year Letter Report - "Underwater Explosion
         Research"

A) Scientific Research Goals:

1) To understand reactive underwater explosion bubble behavior and its interaction with nearby target bodies with an end view of tailoring the bubble energetics.

2) To understand several water-gas interface instabilities with a goal of reducing the cycle to cycle oscillation energy loss.

B) Significant Results of the Past Year:

1) Rayleigh-Taylor instability has been found to be insignificant as a bubble breakup mechanism through both experiment and analysis.

2) Birkhoff and, perhaps, evaporative instability have been identified as the dominant mechanisms of bubble break-up.

3) Surface tension has been ruled out as a contributor to the instability mechanism by experimental observations after modifying the water surface tension by up to a factor of two.
4) Explosive parameter variations (by diluent addition and mixture ratio variations) have, in accord with theory, shown little effect, because of the insensitivity of bubble behavior to the explosion pressure.

5) The amplitude of the bubble corrugations goes down as the wavenumber increases, which is contrary to theory in almost all known instability mechanisms.

6) Explosions adjacent to a steel plate have demonstrated an extremely large plate impact pressure due to the jetting effect; efforts at understanding of the scaling rules for this impact pressure are underway.

C) Plan for Next Year's Research

1) Continue the calculation for the evaporation behavior during a bubble cycle of oscillation. Some numerical difficulties have been uncovered in this calculation, which have delayed progress here.

2) Through more detailed theoretical investigations, determine methods of controlling Birkhoff instability and, perhaps, evaporative instability. Experimentally verify these control procedures.

3) Initiate optimization theory for optimal heat release rate to maximize bubble performance.

4) Experimentally investigate the ocean sand digging capability of the bubble impact upon sand.

See the attachments for the other items requested.
September 8, 1994

Dr. Spyridon Lekoudis  
Office of Naval Research  
Code 1132 F  
800 N. Quincy Street  
Arlington, VA 22217-5000

Dear Spiro:

Enclosed please find 2 copies of the year-end letter report. I have also e-mailed you and ASCII version of the text.

As you know, I took over this project in March 1994, after the untimely demise of Warren Strahle. Dr. Dick Miller has conveyed to me that the final increment of this project will be funded as of September 1994. I plan to continue this project and accomplish the goals that were set by Warren.

Thank you for your support. Please feel free to call me (404-853-9160) if you need any further information.

Sincerely,

Suresh Menon  
Associate Professor

SM/tp
UNDERWATER EXPLOSION STUDIES
GRANT No. N00014-91-J-1963
Principal Investigator: Warren Strahle, deceased
New Principal Investigator: Suresh Menon

A. DESCRIPTION OF THE SCIENTIFIC RESEARCH GOALS

When an explosion occurs underwater, the resulting energy release creates an expanding gas bubble behind the propagating shock wave. This gas bubble undergoes multiple expansion/collapse processes loosing energy continuously until, finally, it breaks down due to instability. The bubble oscillation process, the process of initiation, the source of the instability leading to bubble collapse, and the energy loss mechanism are not well understood, especially when the bubble size is relatively large. Extrapolation of the results from earlier studies of microbubbles does not necessarily provide the proper scaling information. Another practical issue of relevancy to the Navy is the behavior of the bubble in close proximity to a rigid surface. In this case, the attraction of the bubble to the surface results in the formation of a coherent water jet that impinges on the surface with pressure substantially higher than the explosion pressure. This feature has obvious practical applications, such as, mine targeting and destruction. To address these issues, an experimental and numerical investigation is underway at Georgia Tech aimed at understanding the fundamental physics of underwater bubble oscillations and the instability mechanism. Research effort is also being directed to investigate the jetting phenomena near rigid surface.

B. SIGNIFICANT RESULTS IN THE PAST YEAR

The progress in the experimental and analytical/numerical program is briefly discussed below. Note that, so far, the primary focus of this research has been in the experimental program and only limited analytical studies have been performed.

Experimental Results - Free Bubble Explosion Behavior

Underwater explosion experiments were carried out in a tank of dimensions 2 x 1.5 x 1.5 m. The explosion bubble is generated by igniting an explosive gas mixture in a glass globe using fuel-air mixture such as Hydrogen-Oxygen and Hydrogen-CO-Oxygen in various mixture ratios. For all the experiments, the pressure inside the globe prior to the explosion was around 1 bar and due to facility size constraints, the experiments were carried out at a depth of 1 m. This is inconsistent with real world situations of bubble explosions in deep water and must be addressed, perhaps, by repeating these tests in a larger facility (currently being discussed with the sponsor). The pressure inside the globe was measured by using a dynamic pressure transducer and hydrophone was used inside the water tank as an event marker to monitor the strength of the pressure waves reflected from the wall. Wall-reflected acoustic waves can modify the bubble oscillation process. However, results suggest that the amplitude of this reflected wave is relatively low and, thus, may not be affecting the bubble oscillation process.

The bubble looses energy in each cycle and the loss is much larger than that can be explained using only acoustic radiation losses. It would appear that some other mechanism (possibly interface instability) is responsible for this loss. Flow visualization of the bubble behavior was con-
ducted using laser sheet illumination and images were recorded using an intensified high speed video camera with a frame rate of 2000 frames/sec. Typical bubble motion history (radius vs. time) was obtained by examining the images. One of the fundamental issue regarding bubble oscillation and its subsequent breakdown is the source of the instability. It appears that Rayleigh-Taylor instability coupled with a condensation/evaporation (Landau-Darrieus) instability are the primary causes of the bubble breakdown. To determine this, bubble radius variation as a function of the azimuthal angle was determined from the images and FFT analysis was carried out to determine the spectral density of the surface shape of the bubble. The results were surprising in that the amplitude of the bubble corrugation goes down as the wavenumber increases. This observation is counter to what would have been achieved if the R-T instability was the primary mechanism. The radius variation was also used to estimate the growth of the instability (in terms of its amplitude). It appears that the prediction based on Birkhoff instability agrees reasonably well with the experimental observation. However, the Birkhoff instability is a subset of the R-T instability and predicts an algebraic growth, whereas, the R-T instability has an exponential growth rate. Theoretical studies of the bubble oscillation suggests that as the bubble collapses from the maximum radius, surface waves can first grow algebraically in amplitude (Birkhoff instability), then exponentially (R-T instability) if the second derivative of the bubble radius in time becomes sufficiently positive near the minimum radius. Since the number of images analyzed were limited, so far, this phenomenon has not yet been clearly resolved but will be addressed in the near future using higher speed imaging to resolve the instants close to bubble minimum.

Comparisons of the results for bubble oscillation using different gas mixtures showed that the molecular weight of the gas mixture plays a significant role in the bubble oscillation. The results suggest that with decrease in the molecular weight, the energy lost during the oscillation decreases. This is yet to be fully explained and is under investigation. Preliminary analysis suggests that since the molecular weight appears in the equations governing both R-T and L-D instability (through the species density), the molecular weight is affecting the onset and behavior of the instability.

To study the effect of surface tension on the bubble behavior, water surface tension was reduced by a factor of two using commercial surfactant. No substantial difference in the bubble behavior was observed. Surface tension was, therefore, ruled out as a major contributor to the instability mechanism.

**Experimental Results - Bubble Explosion near a Rigid/Porous Surface**

The behavior of the bubble explosion near a rigid/porous surface was also investigated. The jetting behavior induced by the presence of a steel plate near the bubble was studied. The distance between the bubble and the rigid surface, and the surface properties (porosity) was varied to study the jetting behavior. Results show that a peak impact pressure is 5-6 times the explosion pressure can be measured on the plate when the plate is located close to the bubble. Varying the molecular weight of the fuel-air mixture showed that decreasing the molecular weight of the mixture increases the peak impact pressure. This is more understandable since the impact force is proportional to the pressure which is inversely related to the molecular weight (by the equation of state). When the steel plate was replaced by sand (a highly porous media), no clear crater was created and the jet appears to be directed upward! This suggests that the surface characteristics can play a
major role in determining the behavior of the jet. Interestingly enough, when a layer of clay was placed above or below the sand surface, deep craters of the same scale as the glass globe were created.

**Analytical Results**

Thermodynamic properties of many explosive systems were calculated for a constant volume explosion by modifying the NEWPEP software (which actually computes the propellant performance under constant pressure conditions). In addition, the one-dimensional equation for the bubble dynamics in spherical symmetry was numerically solved including the effects of acoustic radiation losses. A perfect gas law was used to relate the product gas pressure with bubble radius, and a constant specific heat ratio was assumed (these assumptions, however, may not be appropriate). The onset of the bubble instability was also studied. The criteria for R-T and B instability was evaluated. The governing equations were simplified by neglecting surface tension effects (which were determined based on experimental results). The period during which each of the instabilities becomes dominant was determined numerically. Although reasonable agreement was obtained for the first bubble oscillation (except near the bubble minimum), many details were missing in the above analysis. The combined effect of R-T and L-D instabilities still needs to be addressed.

**C. PLANS FOR NEXT YEAR'S RESEARCH**

**Bubble Dynamics and Instability Mechanism**

Explosion pressure calculated by the NEWPEP program was always less than the value obtained from the actual tests. It remains to be determined if the computer program or some other phenomena associated with the actual test is the reason(s). The program NEWPEP is designed for constant pressure calculations, whereas the current tests are for nearly constant volume process. More detailed calculations using other codes, such as DYNA 2D, will be carried out to determine if the numerical method is the source of this discrepancy. In addition, more controlled experiments are needed to ensure that there are no special effects of the tests causing the measured pressure value. For example, the pressure reflection from the walls of the test facility, from the inside of the glass globe, and from the sting that holds the globe may be modifying the measured pressure value. To test this, pressure measurements outside the glass globe need to be carried out and the effects of wall reflection and sting (globe support) need to be addressed. Currently, a new method is being devised to hang the globe away from the supporting sting so that the interference effect is minimized. Pressure measurements outside the bubble will also be carried out.

There is also some discrepancy in the measured pressure corresponding to the first bubble minimum. The data shows much higher pressure (for some tests, the pressure even exceeded the explosion pressure) than the value predicted by analytical methods. Therefore, the numerical method based on the Trilling-Herring equation needs to be re-evaluated by including both acoustic and heat loss effects. Furthermore, the assumption of perfect gas law needs to be verified. In any event, it is not clear if the current problem can be reasonably approximated using this one-dimensional model. Therefore, more detailed calculations using a 2D code will be necessary. Results also suggest that the molecular weight of the explosive mixture plays a role in the bubble
dynamics. Changing the fuel mixture using CO may have resulted in changes in the total water vapor produced in the combustion process. At present, it is not clear how the changes in the amount of water vapor in the combustion products play a role or if the observed 'molecular weight' effect is due to the changes in the water vapor content. Tests are needed to separate the effects of molecular weight and water vapor in the product. Inert gases such as Argon and Helium will be used to change the molecular weight (without affecting the water vapor content).

The exact mechanism of bubble breakup and the various factors affecting it, such as, wall reflection, glass globe fragments and bubble depth, etc. have not yet been systematically investigated. For example, even though the pressure wave amplitude from the wall reflection is relatively much smaller than the gas pressure over most of the bubble period, the acoustic wave reflections from the walls may be playing a role in accelerating or initiating the bubble instability leading to its eventual collapse. This issue cannot be addressed in the current facility due to size limitations. However, tests in a larger facility (e.g., NSWC) may be able to clarify this issue. The effect of glass globe fragments is not considered a problem with the current tests, since, in real detonation, the metal case will also result in fragments. However, different materials will cause different degrees of influence on the heat/mass transfer problems. Eventually, these issues may have to be taken into account.

The exact process of bubble breakdown and the source of the instability is of great fundamental interest in this research. Results obtained so far do not provide a clear indication of the source of the instability. Although Birkhoff instability appears to trigger the instability process, due to the exponential growth of the R-T instability, it is conceivable that R-T instability governs the final stages of the breakdown. These issues will be addressed in the coming year.

**Bubble Dynamics near a Surface**

The molecular weight effects observed in the bubble explosion also appears to have some observable effects on the jet behavior. Again, the role of water vapor in the jet behavior needs to be quantified. The impact pressure distribution needs to be carefully determined to understand the mechanism of jet impact. So far, only a one point pressure measurement has been carried out. We are now exploring new methods to obtain the pressure distribution on the impact surface. This would allow proper estimate of the behavior of the impact process.

The effect of changing the surface property such as from sand to sand/clay to non porous surfaces has resulted in significant changes in the impact crater characteristics. Systematic evaluation of this effect is necessary. Since the results show that a crater is formed even when sand covers a relatively rigid surface, a more detailed study will be carried out to quantify this effect. The jet and the crater characteristics have to be correlated with the surface porosity, the depth of the submerged rigid surface, the bubble distance above the surface, and the explosive mixture properties. The results of these studies have some obvious practical applications for use as a marker to seek out rigid surfaces (e.g. mines) buried under relatively porous media such as sand.
OFFICE OF NAVAL RESEARCH
PUBLICATIONS/PATENTS/PRESENTATIONS/HONORS REPORT
01 October 1993 through 30 September 1994

R&T Number: 3326839

Contract/Grant Title: N00014-91-J-1963: "UNDERWATER EXPLOSION STUDIES"

Program Officer: DR. RICHARD MILLER

Principal Investigator: SUresh MENON (FORMERLY DR. WARREN STRAHLE, DECEASED)

Mailing Address: GEORGIA INSTITUTE OF TECHNOLOGY, SCHOOL OF AEROSPACE ENG.
ATLANTA, GEORGIA 30332-0150

Phone Number: (404) 853-9160

FAX Number: (404) 894-2760

E-Mail Address: menon@falcon.ae.gatech.edu

a. Number of Papers Submitted to Referred Journal but not yet published: 1

b. Number of Papers Published in Referred Journals:
(List Attached): 1

c. Number of Books or Chapters Submitted but not yet Published:

0

d. Number of Books or Chapters Published (List Attached): 0

e. Number of Printed Technical Reports & Non-Referred Papers (List Attached): 0

f. Number of Patents Filed:

0

g. Number of Patents Granted (List Attached):

0

h. Number of Invited Presentations at Workshops or Professional Society Meeting (List Attached): 0

i. Number of Presentations at Workshops or Professional Society Meetings (List Attached):

1

j. Honors/Awards/Prizes for Contract/Grant Employees:
(List Attached, may include Society Awards/Offices, Promotions, Faculty Awards/Offices, etc.): 0
k. Providing the following information will assist with statistical purposes.

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l. Degrees Granted (List Attached):

* Underrepresented or minority groups include Blacks, Hispanics, and Native Americans. Asians are not considered an underrepresented or minority group in science and engineering.

** Supported at least 25% this year on contract/grant.

+ Originally Dr. Warren Strahle. As of March 1994, Suresh Menon.
++ Starting September 1994.
TECHNOLOGY TRANSFER

Technology transfer is an important measure of the relevance of scientific endeavors. ONR Program Officers need to be aware of any such transfer, and they will use it to the benefit of their programs. Please describe any recent (approximately last three years) direct or indirect interactions you had with Navy, other DoD, or industrial scientists and engineers; describe only those interactions that resulted in their use of methodology, data, software, or other developments produced or directly derived from your ONR grant/contract. Also describe similar technology transfer, if any, that resulted without any such interactions.

None
LIST OF PUBLICATIONS/REPORTS/PATENTS/GRADUATES

1. Papers Published in Referred Journals:
   "Physical and Chemical Observation in Underwater Explosion Bubbles,"
   25th International Symposium on Combustion, The Combustion Institute,
   1994 (in press).

2. Books (and sections thereof) Published:
   None

3. Technical Report, Non-Refereed Papers:
   None

4. Presentations:
   "Physical and Chemical Observation in Underwater Explosion Bubbles,"
   25th International Symposium on Combustion, University of California,

5. Patents Granted:
   None

6. Degrees Granted (name, date, degree):
   None

* List only those funded directly from you ONR grant/contract. Use additional pages, if necessary.

Enclosure (3)
**LIST OF AWARDS/HONORS/PRIZES**

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Enclosure (4)
OTHER SPONSORED RESEARCH
(Include title, sponsors' name, dollar amount and start and end dates for the award)


2) "Premixed Flame Propagation in Microgravity," NASA Lewis Research Center. $449,000, July 1994-June 1998 (Dr. J. Jagoda, co-investigator)


Enclosure (5)
FUNDING BALANCE

A major issue at ONR is expenditure rates. Not meeting Navy required expenditure rates will result in redirection of resources within ONR for FY95 that starts on 01 October 1994! I would like to enlist your help in preventing this from occurring. If your expenditure rate is below that required by your contractual arrangement with ONR, we will be forced to delay funding increments in your contract/grant, even if the delay is due to slow billing from your business office. If the problem is at your business office, please take action to correct it.

Indicate the remaining ONR grant/contract resources you have in your institution as of 30 SEP 94:

$ 36,000.00

Enclosure (6)
UNDERWATER EXPLOSION STUDIES
Report for the Period: October 1994 - September, 1995
Grant No. N00014-91-J-1963

Suresh Menon
School of Aerospace Engineering
Georgia Institute of Technology
Atlanta, Georgia 30332-0150

a. Description of Scientific/Technical Goals:

When an explosion occurs underwater, the resulting energy release creates an expanding gas bubble that undergoes multiple expansion/collapse process losing energy continuously until, finally, it breaks down. The bubble oscillation process, the process of initiation, the source of the instability leading to bubble collapse, and the energy loss mechanism are not completely understood. Another practical issue of interest is the behavior of the bubble near a rigid surface. In this case, the attraction of the bubble to the surface results in the formation of a coherent water jet that impinges on the surface with pressure substantially higher than the explosion pressure. This feature has obvious practical applications, such as, mine targeting and destruction. To address these issues, an experimental and numerical investigation is underway:

[1] to understand the dynamics of isolated and multiple underwater explosion bubble(s),
[2] to investigate bubble collapse near porous/rigid surfaces, and,
[3] to develop scaling rules to quantify the relationship between explosive composition and energy release.

b. Significant Accomplishments in the Past Year

The progress in the experimental and analytical/numerical programs is briefly discussed below.

Experimental Results - Single Bubble Explosion Behavior

Underwater explosion bubbles generated by igniting a gas mixture in a glass globe were studied in a laboratory water tank. The bubbles were suspended using thin wires to mimic free explosions. The pressure inside and outside the bubble was measured using dynamic pressure transducers and a hydrophone was used to monitor the acoustic waves reflected from the wall. High speed imaging of the bubble during its oscillation was used to obtain the bubble shape for analysis. The test program also varied the gas mixture properties such as the fuel-type (e.g., H2 and CO), fuel molecular weight (by adding inert gases such as He and Ar) and the equivalence ratio to determine the correlation between the bubble dynamics and the explosive mixture properties.

A recent paper (Menon and Lal, 1995) discusses the results obtained in this study. Using both geometric and dynamic scaling analysis it has been demonstrated that the characteristic nondimensional parameters for the current experiments are very similar to those measured in realistic deep sea explosions. Thus, the present results have practical implications for understanding the characteristics of high explosive deep sea explosions (which are very difficult to quantify due to measurement problems).

Energy partition analysis of the total energy released during a typical explosion showed that nearly 26% of the total explosion energy is dissipated (or lost) during the first bubble expansion
phase and another 15% of the total available energy is dissipated (or lost) during the first contraction phase. As a result, the bubble size and the oscillation amplitude are decreased significantly after the first pulsation. This rapid decrease (both qualitatively and quantitatively) in the bubble size and amplitude is similar to the observation in deep sea explosions. Past studies attributed this observed decrease partly to the energy dissipated by the initial shock wave and partly to some unaccounted mechanism(s). Various physical processes that can cause this energy loss, such as (i) the turbulence induced in the water, (ii) the mass loss from the gaseous bubble, (iii) Taylor instability at the interface, and, (iv) gas cooling and steam condensation near the interface, have been proposed in the past; however, without any confirmation. The analysis of the present data suggests that the unaccounted dissipated energy is likely to be used up to excite interface instability.

Various sources of the interface instability have been evaluated in the present study using the experimental data and stability analysis. Results suggest that the evaporative Landau-Darrieus instability (which causes mass and heat transfer at the interface) is likely to be first excited near the first bubble maximum and results in a corrugation of the bubble interface (and, as a result, the bubble loses spherical symmetry). The L-D instability is shown to be operative as the bubble collapses; however, near the bubble minimum, the hydrodynamic Rayleigh-Taylor and Birkhoff instabilities are also excited. It has been shown here, that although the Birkhoff instability is excited first, surface tension effects are likely to damp this high wave number, small wavelength instability and that the large wavelength Rayleigh-Taylor instability (with its exponential growth rate) eventually dominates the interface instability. Analysis of the bubble images from the second pulsation shows that the large wavelength interface perturbation continues to persist and severely distorts the bubble shape. Beyond the first pulsation, the buoyancy effect causes an upward migration of the bubble and could result in an effective shear velocity at the bubble interface. However, analysis shows that the Kelvin-Helmholz instability (which requires the presence of interface shear motion) is not likely to become relevant, at least for the current experiments.

This study confirms earlier conjectures that both the evaporative and hydrodynamic instabilities play a major role in the energy loss and the bubble dynamics. Results also show that for fuel-air mixtures with a lower effective molecular weight, the energy loss is decreased. This is possible since the molecular weight appears in the stability equations (through the species density). However, this observation may be limited to the present gas explosions and may not be operative for solid (e.g., TNT) explosives used in real deep sea explosions. This issue needs to be confirmed by carrying out additional experiments using solid propellants.

**Experimental Results - Multiple Bubble Interaction**

The behavior of the two explosions near each other has also been conducted. Two orientations are being studied: vertical and horizontal. These experiments were recently completed and all the data have not yet been analyzed. However, results do show some interesting trends that suggest that interaction process is highly complicated, but perhaps could be controlled by careful (phased) triggering of the explosions. It appears that the key parameters are the spacing between the two bubbles (which is obvious) and the delay between the explosions (since this delay defines the state of the two bubbles prior to their interaction). It has been shown (a paper is currently being written, Lal and Menon, 1995) that by varying the time delay, it is possible to control the bubble motions (towards or away from each other) and to make one bubble penetrate the other. At present, it has been very difficult to completely control the time delay between explosions (due to hardware mismatch which is very difficult to avoid). Fortunately, the bubble image data is capa-
ble of provides good resolution between images and therefore, can be used to estimate the actual delay. This data is currently being analyzed to quantify the effect of delay on bubble-bubble interaction and to understand its effect on bubble instability.

An issue of related interest is to determine how bubble-bubble interaction is modified when it occurs in the vicinity of a solid wall. This issue will be addressed in the next year.

**Numerical Results**

The one-dimensional equation for the bubble dynamics in spherical symmetry was numerically solved including the effects of acoustic radiation losses and by neglecting surface tension effects (based on experimental results). These results were then compared to the current experimental data (Menon and Lal, 1995). Very good agreement between the 1D model and the experimental data is obtained only during the first bubble expansion. As the bubble contracts and approaches the first bubble minimum, the numerical results deviated from the data. This is not surprising since the experimental data showed that the bubble loses spherical symmetry during collapse and that both L-D and R-T instability begins to control the energy loss and bubble dynamics. Therefore, it was decided that more complex simulation model is required for more detailed study.

A three-dimensional solver called ALE-3D (Arbitrary Lagrangian-Eulerian in 3D) was recently acquired from Lawrence Livermore Lab. This code is capable of simulating full 3D bubble behavior and allows for proper resolution of the bubble interface. There are, however, some limitations to this code, namely, ALE-3D does not include mass/heat transfer (although it includes momentum transfer) between two phases (gas and liquid). Since the experimental data clearly shows the importance of L-D instability, this capability needs to be included into the code to allow proper comparison. Preliminary simulations using the basic code has been able to reproduce the behavior of the 1D equation and the computed bubble radius and pressure variation are in good agreement for the first bubble expansion process. However, the code has been unable to mimic the 3D bubble instability of the interface. This result is particularly interesting since it suggests that the mass/heat transfer processes needs to be accounted. Note that the current code is capable of accurately modeling momentum transfer, compressibility effects, density stratification and large scale acoustic/shock energy dissipation. Therefore, the hydrodynamic instabilities can be captured by this code. Since the calculations did not show any interface corrugation suggests that the L-D instability is critical to the interface instability.

**c. Advancement of the State-of-the-Art and/or Response to Navy Requirement**

The current experimental program is in support of Navy's study of underwater explosions. The problems are chosen in close collaboration with the Program Officer. The new understanding of bubble instability process (Menon and Lal, 1995) has provided the knowledge to further study bubble-surface interaction which has applications to, for example, underwater buried mine detection and targeting which is of current interest to the Navy.

**c. Plans For Next Year's Research**

The next year's research will focus on three major areas: (a) the multiple bubble interactions, (b) the bubble-surface interactions, and, (c) numerical simulation of the bubble pulsation. In the following, a brief description of these tasks is given.

**Experimental Study of Multiple Bubble Interaction:** This is continuation of the current work and
is expected to be completed in couple of months. The effect of different bubble sizes on the interaction and the bubble instability will be the principle focus of this study.

**Experimental Study of Bubble-Surface Interaction:** Preliminary study of this phenomenon has been completed (Strahle and Liou, 1994). Of particular interest now is the effect of the surface property such as porosity (e.g., sand, sand/clay) since previous results suggest that the impact behavior are significantly changed by the surface characteristics. For example, it was seen that a crater is formed on a relatively rigid surface such as clay even when it is buried under sand. Systematic evaluation of the effect of surface property will be carried out. The water jet and the crater characteristics will be correlated with the surface porosity, the depth of the submerged rigid surface, the bubble distance above the surface, and the explosive mixture properties. The effect of multiple bubble explosions near the surface will be also be investigated. The results of these studies have some obvious practical applications for use as a marker to seek out rigid surfaces (e.g. mines) buried under relatively porous media such as sand.

**Validation of the Numerical Model:** The ALE-3D will be extended to include heat/mass transfer and will be validated using the single bubble data. Once this is accomplished, a numerical tool will become available to evaluate deep sea scaling rules and to simulate realistic explosions.

**e. Transition to 6.2 Program**

The current research has some obvious applications to practical Navy issues. The results obtained so far have demonstrated that the current experiments are relevant to deep sea explosions and have revealed some of the fundamental physics of the bubble collapse process. The experimental and the numerical studies currently underway should establish a capability to evaluate realistic explosions that currently cannot be quantified in the field. Discussions are underway to evaluate continuation of this research on 6.2 funding.

**References**


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a. Number of Papers Submitted to Refereed Journal but not yet published(*): 2

b. Number of Papers Published in Refereed Journals: (List Attached): 1

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d. Number of Books or Chapters Published (List Attached): 0

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i. Number of Presentations at Workshops or Professional Society Meetings (List Attached): 0
j. Providing the following information will assist with statistical purposes.

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k. Degrees Granted (list follows):

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EXPERIMENTAL AND NUMERICAL STUDIES OF UNDERWATER EXPLOSIONS

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SIMULATIONS OF UNDERWATER EXPLOSIONS

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Abstract

The dynamics of bubbles formed during underwater explosions is numerically investigated using an Arbitrary Lagrangian-Eulerian three-dimensional finite-element code. The expansion and the collapse of a vapor bubble in a water tank is first simulated to compare the predictions with data from a parallel experimental study. Experimental and numerical results show good qualitative and quantitative agreement and suggest that the excitation of Rayleigh-Taylor instability is a major cause of bubble interface instability. This observation is consistent with earlier data and confirms that interface instability plays a significant role in the loss of energy from the explosion. Simulations have also been carried out to investigate bubble-bubble and bubble-wall interactions. Results from the bubble-bubble interaction studies show the formation of a water jet as one bubble collapses into the other, in agreement with recent experimental observation. The collapse of a bubble near a rigid wall and the formation of high velocity re-entrant jet onto the wall has also been successfully simulated. The peak impact pressure and the fluid flow velocity agrees well with the experiments. In addition, the well known vortex ring bubble during the collapse process has been numerically captured. Application of the computational methodology to realistic deep sea explosions and to detonation cords used for mine destruction has also been carried. Results of these studies are also discussed in this report.
1. Introduction

Vapor and gas bubble dynamics are of great practical interest in the prediction and prevention of cavitation erosion of marine propeller and turbine blades (e.g., Rood, 1991). In addition to this effect from collapse of cavitation (small) bubbles, the destructive nature of strong underwater explosions near walls is also well known. Both experimental and numerical studies have been carried out in the past to study the complex flow fields associated with such explosions. Detailed reviews (e.g., Plesset and Prosperetti, 1977; Blake and Gibson, 1987; Prosperetti, 1982; Rood, 1991) have summarized past experimental and numerical results. Experimental studies are too numerous to list completely; however, most studies in the past focused on the dynamics of cavitation (small) bubbles. Controlled experimental studies of large scale explosions have been limited due to difficulties in acquiring the data. Some limited number of studies of large-scale bubbles have been reported (Cole, 1948; Snay et al., 1952). Bubble-bubble interactions have also been studied in the past using experimental and numerical methods (e.g., Warren and Rice, 1964; Chahine et al., 1992; Chahine, 1994). However, in most experimental cases, due to difficulties in acquiring detailed data, only limited information (e.g., pressure outside the bubble and bubble radius variation in time from photographic images) has been obtained. More recently, a series of experiments were carried out to investigate relatively large-scale bubble explosions (Menon and Lal, 1996a, b; Lal and Menon, 1996a, b). These experiments were conducted in shallow water (1 atmosphere ambient pressure) to investigate the dynamics of bubble-bubble and bubble-wall interactions in such flows and to investigate feasibility of targeting buried mines for destruction in beaches. The data obtained from these experiments have been used to validate the numerical model discussed in this paper.

Various numerical studies have also been conducted in the past. These range from very simple 1-D analytic solutions (e.g., Lauterborn, 1976; Plesset, 1971; Prosperetti, 1982) to more complex 2D/3D studies. Many of the past studies employed the Boundary Element Method (BEM) or its variants (e.g., Chahine et al., 1992; Chahine and Duraiswami, 1992; Chahine and Perdue, 1988; Duncan and Zhang, 1991; Blake et al., 1986, Taib et al., 1984; Plesset and Chapman, 1970). The BEM method is computationally very efficient since only the flow outside the bubble surface is computed which allows the reduction of the dimensionality of the problem by one. Results have shown that this method has the ability to capture many aspects of the bubble oscillation and collapse process (e.g., Chahine et al., 1992; Chahine, 1994; Kalumuck et al., 1995). However, due to the simplifications
incorporated into the BEM formulation, this method also has some inherent limitations. For example, compressibility in the gas cannot be included, the gas motion inside the bubble is not computed and in the study of bubble collapse near a surface, BEM can be used only up to the point of jet formation. To model the flow beyond the point of bubble collapse, BEM has been modified by introducing vortex elements (e.g., Zhang and Duncan, 1994; Zhang et al., 1993; Best and Kucera, 1992; Best, 1993). Another limitation of this approach is that to set up the simulation problem, some basic assumptions are required (for example, the gas is assumed to satisfy the polytropic equation of state, see Chahine, 1994). Although for many bubble/flow interaction situations, the assumptions inherent in the BEM method are acceptable, in general, for complex strong explosions the validity of these simplifications become questionable (see further discussion below).

A numerical method that includes both compressibility and an ability to capture the entire bubble collapse in complex configuration is used in this study. This numerical scheme, called the ALE3D, is based on the Arbitrary Lagrangian-Eulerian (ALE) method and was originally developed at the Lawrence Livermore Laboratory to study structural dynamics. Past applications of this method include the 2D ALE (e.g., Tipton, et al., 1992) and the full 3D calculations (Milligan et al., 1995) of bubble collapse. However, it appears that so far, the full capabilities of the ALE3D have not yet been exploited to simulate and investigate the dynamics of underwater explosions. This paper reports some recent results of single and double bubble explosions both in free field and in the vicinity of a rigid wall.

The current numerical approach is also motivated by the observation that many of the assumptions used in past numerical studies are violated by the bubble behavior as observed in the experiments. For example, the bubble oscillation process occurs in a very short time (typically, in a few micro or milliseconds) with significant compressible effects in the collapse phase, especially when the pressures experienced are comparable to the bulk modulus of water, as is the case in deep sea strong explosions. Bubble shape is also known to quickly deviate from sphericity at bubble maximum, thereby, violating axisymmetric assumptions used (e.g., Szymczak et al., 1993; Zhang and Duncan, 1994) in the past. Non-spherical bubble evolution requires full 3D treatment. Thus, simple 1-D or 2-D/axisymmetric analysis or incompressible methods cannot accurately resolve the bubble dynamics. Furthermore, some of the past simplified studies also fail to provide sufficient details of the flow field inside and outside the bubble and cannot account for the interaction between the vapor and the liquid phases. Conventional numerical methods (even using full 3D) such as Lagrangian or Eulerian techniques are also not practical for
bubble oscillation studies, since the expansion and collapse of bubbles create severe fluid motion so that a Lagrangian approach (in which the grid points move with the fluid resulting in severe grid distortion) becomes inappropriate, while a Eulerian approach becomes computationally very expensive since very high (adaptive) grid resolution in the regions of interest (which is essential) is required to resolve the bubble's shape.

The ALE3D employed here combines the lagrangian and eulerian features into one code. This code has been extended to study bubble instability and collapse in this study. In the present paper, section 2 contains a brief description of this code, followed by a discussion of the equations of state in section 3. The results are discussed in section 4 and the conclusions are summarized in section 5.

2. The Numerical Method

ALE3D (Anderson et al., 1994) is an explicit, 3D finite element code that simulates the fluid motion and elastic-plastic response on an unstructured grid. The grid may consist of arbitrarily connected hexahedral shell and beam elements. The ALE algorithm is implemented by carrying out a complete Lagrangian calculation followed by an advection step. After each lagrangian step, a new mesh is created using a finite element based equipotential method to relax the distorted grid. In the eulerian advection step, the fluid variables such as mass, density, energy, momentum and pressure are reevaluated on the new mesh by allowing fluid motion (based on the solution of the euler equations of motion). Following DYNA3D (Hallquist, 1982), the stress gradients and strain rates for the lagrange step are evaluated by a lowest order finite-element method. The equation of state and constitutive models are described elsewhere (e.g., Woodruff, 1976; Steinberg, 1991) and, therefore, are only summarized here for brevity.

The advection step uses methods similar to those developed for 2D ALE code, CALE (Tipton, 1990), and the 3D Eulerian code, JOY (Couch et. al., 1983). For pure zones, a second order, monotonic advection algorithm is used (Van Leer, 1977). This advection step can create mixed material elements (i.e., liquid and vapor). Material interfaces are not explicitly tracked but for the purpose of carrying out mixed element advection, they are inferred from volume fractions. Separate state variables are kept for each component of a mixed element. Further details of this code is given in the above mentioned references and therefore, avoided here for brevity.
To use this code, proper inputs must be provided to first generate the appropriate grid and then to model the fluid and structure properties. Some of the relevant details for the bubble studies are summarized in the next section.

3. Equation of State

A key requirement in employing ALE3D is the proper choice of the equations of state for the various materials in the problem. Only a brief summary is given here primarily to highlight some of the inherent limitations of the original code.

The explosion bubble is assumed to be of noncondensible steam and its equation of state is assumed to be gamma law with a gamma of 1.3. The equation of state is represented as:

\[ p = (\gamma - 1)(1 + \mu)E \]  

(1)

where, \( p \) is the pressure, the relative volume is given as: \( \mu = \rho / \rho_0 - 1 \) and \( E \) is the internal energy per unit volume.

The water is modeled using a Grunenisen form given as:

\[ p = \frac{\rho_0 C^2 (1 + (1 - \gamma_0 / 2)\mu - a / 2\mu^2)}{\left(1 - (S_1 - 1)\mu - S_2 \frac{\mu^2}{\mu + 1} - S_3 \frac{\mu^3}{(\mu + 1)^2}\right)^2} + (\gamma_0 + a\mu)E \]  

(2)

For expanded material the above expression is replaced by

\[ p = \rho_0 C^2 \mu + (\gamma_0 + a\mu)E \]  

(3)

Here, \( C \) is the intercept of the shock velocity-particle velocity curve (the Huguenot curve), \( S_1, S_2 \) and \( S_3 \) are the coefficients of the slope of the shock velocity-particle velocity curve and \( \gamma_0 \) is the Grunenisen gamma and \( a \) is the first order volume correction to \( \gamma \). The common values of these coefficients are tabulated in Table I. Our present interest is in underwater explosions in shallow water. Comparison of the pressure predicted by the above equation of state with the data obtained from NIST (1988) in the appropriate temperature and pressure range of interest showed significant discrepancies. In order to circumvent this problem, water regime data was modified to obtain new coefficients. It
was determined that with these coefficients, the equation of state very closely agreed with the experimental results.

However, the current limitation of the ALE3D input structure is that it requires that the equation of state in the form (3) must be used in the code. Thus, the behavior of a material like water which has a discontinuous transition from steam to water can not be adequately represented by this form. A more general approach would involve actually reading the data directly rather than fitting it to the above form. This has not yet been accomplished but is under investigation.

The high explosive materials in the deep sea explosives are modeled using the Jones-Wilkins-Lee High Explosive equation of state. The equation of state is represented as

\[
p = A \left( 1 - \frac{\omega}{R_1 V} \right) e^{-R_1 V} + B \left( 1 - \frac{\omega}{R_2 V} \right) e^{-R_2 V} + \omega V E
\]

(4)

The coefficients used for the case of pentolite (a representative high explosive) are given in Table II. Again, the current format of the ALE3D input does not allow inclusion of explosion material that do not conform to the state equation (4). Thus, modifications are required to allow study of more complex explosive material.

The results of the numerical studies are reported in this paper. In addition, the earlier experimental results have been included as Appendices to this report. Appendix A summarizes the results of the bubble-wall experiments, Appendix B and Appendix C are copies of papers presented/submitted on the bubble-bubble interaction experiments and Appendix D contains a draft paper submitted for presentation on the numerical studies.
4. Results and Discussion

In this section, the results obtained for the various test cases are summarized and discussed. The present study was first carried out to compare with data obtained in an ongoing experimental study (Menon and Lal, 1996a; 1996b; Lal and Menon, 1996a, 1996b). These studies serve to identify the capabilities and limitations of the ALE3D code and to identify areas for further study. The computations were then extended to realistic deep sea explosions and detonation cord (used for mine destruction) explosions to demonstrate the capability of the present methodology.

4.1 Free Field Single Bubble Oscillation

This simulation employed test and geometrical conditions similar to the experimental set-up of Menon and Lal (1996a). The experimental case is idealized as a freely oscillating bubble placed in the center of a 1.5 m x 1.5 m x 1.5 m tank filled with water. The computational domain is considered an approximation to the actual tank, since, in the experiments, the tank had similar dimensions but the top surface was a free water surface. A current limitation of the ALE3D is that it requires all far field boundaries to be modeled as a rigid reflecting wall. This limitation of the code can be modified by appropriate changes to the source code; again, this is an issue for future investigation.

The initial bubble diameter is 6.34 cm and the initial explosion pressure is 9.34 atmospheres (both identical to the experimental case). The water pressure is 1 atmosphere and hence, these studies are relevant for shallow water explosions. The water and the steam are modeled as described earlier. The ALE mesh treatment is applied to all the elements in the bubble and in the vicinity of the bubble. But away from the bubble where the bubble explosion does not cause much grid distortion, lagrangian mesh treatment is used. The number of elements used to resolve the bubble and the surrounding water was varied to confirm that the results are not grid dependent. In addition, both full bubble and 1/8-th section (using symmetry boundary conditions) were compared to determine if the reduced domain (which is computationally more efficient) could be employed for scaleup studies. Results showed that for most of these cases studied here, the 1/8 size domain computations were in very good agreement with the full 3D bubble results. For a typical 3D simulation, 50,000 elements were used to discretize the domain, while for the 1/8th domain, 10,000 elements were used. Figure 1 shows a representative grid distribution used for the single bubble calculations. Grid independence studies were carried out by repeating
representative simulations using grids as large as 150,000 element. Typical calculation using the full 3D coarser grid required around 3 CPU hours on a single processor Silicon Graphics Power Challenge (MIPS R8000/90 MHz). Although, various cases have been simulated, only characteristic results are discussed below.

The bubble grows after the explosion due to the high vapor pressure inside the bubble. Because of inertia, this results in an over expansion and the pressure inside the bubble falls below the ambient (water) pressure. As a result, the bubble collapses and reaches a bubble minimum at which time the internal pressure again exceeds the external pressure. Thus, a bubble oscillation process is set up and continues as long as there is sufficient energy available. However, energy is continuously lost during the oscillation due to irreversible mechanical work done on water and vapor and due to the onset of various instabilities. In Menon and Lal (1996a), detailed analyses of the energy loss and the instability mechanisms were performed and results suggest that during the collapse process the Rayleigh-Taylor (R-T) instability occurs at the interface. This results in a distortion in of vapor-water interface and a consequent loss of energy. The R-T instability has been captured in the numerical study. For example, Figs. 2a-f show the time sequence of bubble shape during the first bubble oscillation. These figures clearly show the loss of sphericity after the first bubble maximum (figs 2d-e) and, by the bubble minimum, the bubble interface has become distorted with the formation of large-amplitude wave-like disturbance. The deviation from sphericity and the formation of waves on the bubble interface is characteristic of Rayleigh-Taylor instability.

To ensure that this wave-like undulation of the bubble interface is not due to the acoustic reflections from the wall, a series of calculations were carried out (a) by extending the dimensions of the tank by a factor of 4, and (b) by replacing the rectangular domain by a spherical domain. Results (not shown) showed that although, there are changes in the bubble oscillation period (discussed below), in all cases, however, the wave-like interface distortion appears near the bubble minimum. Thus, these results clearly show that the R-T instability does occur (as observed in experimental data, Menon and Lal, 1996a,b) and has been captured in these calculations.

Figures 3a-c and 4a-c show respectively, the pressure field in terms of contours and gray-scale representation inside and outside the bubble during the bubble oscillation. The pressure field around 12 ms (as the bubble is collapsing) clearly shows the lack of spherical symmetry; however, after the bubble minimum (Figs 3c, 4c), the pressure field
again becomes more symmetric except very near the bubble interface. This is consistent with the R-T instability, since interface deformation only occurs during the bubble collapse, however, as the bubble rebounds into the next oscillation, the interface again becomes more spherical since the conditions are such that the R-T stability condition is satisfied.

The 3D nature of the flow field inside and outside the bubble can be visualized more easily in the velocity vector fields, as shown in Figs. 5a-c. Figure 5a shows the outward motion of the bubble just before the bubble maximum, Fig. 5b shows the velocity vectors during the first compression cycle and Fig. 5c shows the outward motion of the bubble just after the first bubble minimum. The magnitude of the velocity vectors also indicates that the acceleration of the fluid is minimum at the beginning and end of compression or expansion phases. These features demonstrate the complex nature of the flow field set up during bubble oscillation (and as discussed later, the flow patterns are even more complex for the bubble-bubble and bubble-wall cases).

The R-T instability can also be inferred by analyzing the variation of the radius with time. For example, Fig. 6 shows that $d^2R/dt^2 > 0$ (which is a necessary condition for R-T instability) occurs as the bubble collapses and reaches its first minimum. During the second expansion process, this criterion is no longer satisfied (not shown), consistent with the earlier observation that the interface becomes more smooth.

Figures 7a and 7b show respectively, the variation of the pressure within the bubble and the bubble radius with time. The computed results are compared with the data obtained in the recent (Menon and Lal, 1996a). The comparison shows that the computed first period of oscillation (around 15 ms), the first peak pressure (at first bubble maximum) and the maximum radius (at bubble maximum) agrees quite well with the experimental data. The slight differences during the contraction phase may be due to differences between the experimental and numerical setups. For example, the experimental set-up (described in Menon and Lal, 1996a) employed a glass globe (which contained the stoichiometric fuel-air mixture) with an insert that contained the pressure transducer and the spark generator while these features were ignored in the numerical model. In addition, the presence of the free water surface on the top boundary and the presence of glass fragments (formed during the glass globe explosion) in the experimental study were also not included in the numerical model. Furthermore, it is also not clear how much of the measured pressure is being modified (or contaminated) due to the present of the transducer in the bubble.
In spite of these differences it is interesting to observe that the numerical and experimental results are in relatively close agreement. This suggests that the experimental artifacts (identified above) are not significantly modifying the dynamics of the bubble oscillation process. Therefore, the present numerical study serves to provide an independent validation of the results described earlier in Menon and Lal (1996a) and in Lal and Menon (1996a, b).

To ensure that the observed discrepancies were not due to any numerically introduced artifacts, a series of parametric studies were carried out to address various numerical issues. For example, to study the effect of grid resolution, simulations employing 50,000 elements and 150,000 elements were compared. The higher resolution grid was clustered in the vicinity of the bubble. Figure 8 compares the results of these simulations. Although, the higher resolution results show a slightly better peak pressure at the first bubble minimum, the overall agreement between both the simulations is quite good. This clearly demonstrates that the results obtained with the relatively coarser grid are not grid dependent.

As mentioned earlier, to reduce the computational expense, the tank and the bubble configuration was also idealized as an octant with three planes of symmetries. This reduces the computational expense by a factor of eight. The results are compared in Fig. 9 and as expected, since the observed loss of sphericity (in the full 3D case) is not allowed in the octant simulation, the octant case deviates slightly from the full tank case especially near the bubble minimum. However, the period of oscillation and pressure peak at the bubble minimum are very close to the full 3D prediction and therefore, it was determined that the octant domain could be used (with substantial savings in the computational time) for parametric studies.

An inherent limitation of laboratory experiments is that the facility set up (e.g., the water tank) can modify the bubble oscillation dynamics making the experimental result not fully representative of a free field underwater explosion. For example, it has been pointed out in the past, that the acoustic reflections from the tank wall may modify the bubble dynamics and also control the formation of the interface instability. Although, the acoustic pressure is significantly lower (by an order of magnitude) than the explosion pressure, the high speed of acoustic waves (e.g., 1500 m/s) makes the waves to bounce back and forth (from the side and bottom walls and also from the free surface) numerous times during a typical
bubble oscillation period of 15 msec. To determine how the walls play a role in the bubble dynamics, simulations were carried by (a) moving the walls considerably further out (by a factor of 5) so that the total time for the acoustic wave to bounce from the wall is increased from around 1 msec to around 5 msec, and, (b) by changing the far field boundary from a rectangular domain to a spherical domain (so that characteristics of the reflected acoustic waves becomes different).

Figures 10a and 10b show respectively, the pressure signature observed near the wall in the experimental study and at a similar location in the computational model. The strength of the pressure fluctuation near the wall is very similar in both studies and the high frequency content of the pressure signature is also similar (although not identical). Spectral analysis of the pressure trace did confirm the presence of a high frequency in both the data. This high frequency is not clearly visible in the numerical data since the very coarse grid used in the far field damps this frequency considerably. Simulation using much higher grid resolution in the far field showed the presence of the high frequency.

Comparison of the results of all these cases demonstrated that the wave-like instability of the bubble interface remains unaffected by the changes in the wall location and shape. Thus, it was determined that the walls were not causing the distortion of the bubble interface and that the formation of wave like distortion on the interface was due to the R-T instability. However, acoustic wall reflections do have some subtle effects on the bubble oscillation period and the peak pressure at the bubble minimum. When the tank dimensions were increased, the period of oscillation increased and the peak pressure at the bubble minimum was decreased. This is shown in Fig. 11. Replacing the rectangular walls by a spherical wall in the far field showed no change in the time period but did show a decrease in the peak bubble pressure at the first minimum (Fig. 11).

An estimate for the time period for a freely oscillating bubble can be made relative to the reference time period $T$ which is given by the relation $T = R_{\text{max}}(\rho/\rho_\infty)^{1/2}$, where $R_{\text{max}}$ is the maximum radius of the bubble, $\rho$ and $\rho_\infty$ are respectively, the density and ambient pressure. Based on our experimental data, $R_{\text{max}}$ is approximately 7.5 cm, which results in an estimate for the reference time period as around 7.5 msec. Thus, the ratio of the actual period of oscillation in our experiment (around 15 msec) to the reference time period $T$ is approximately 2. This ratio is in agreement with the observations in deep sea studies (e.g., Cole, 1948) where it was reported that the scaled time period is also around 2. Other scaling analysis (reported in Menon and Lal, 1996a) also suggests that the present
experiments are a reasonable (although not exact, since both the effects of depth and Froude number are not fully captured) representation of field explosions.

The current results do suggest that the presence of walls is effecting the time period (i.e., it is decreased). In free field, an increase in $T$ will occur since the bubble will be allowed to expand further than the maximum reached in the laboratory tank. However, the maximum radius achieved is also a function of the initial explosion energy and thus, there is an upper limit to $R_{\text{max}}$ for every explosion. This suggests that if the outer boundaries are removed far enough, the numerical results will asymptotically reach the free field case. Free field explosion can also be simulated by replacing the outer wall boundary conditions with outflow boundary conditions. However, as mentioned earlier, the present ALE3D code requires that the far field boundary be modeled as a solid reflecting wall. This limitation of the code can be removed only by modifying the source code. This is will be investigated in the future.

In any event, for the experimental configuration, it appears that the bubble dynamics in the water tank cannot be considered identical to a free field bubble behavior. However, there are no glaring differences in the bubble oscillation dynamics. The interface distortion appears to be independent of the wall location and shape and overall nature of the bubble dynamics is captured relatively accurately with only differences in the exact value of the time period and peak bubble pressure.

Another issue that was addressed was the effect of pressure variation with depth. In shallow water explosions of small bubbles it is commonly assumed that the variation of pressure with depth will have very negligible effect on the bubble dynamics. However, since the present study is focused on relatively large bubbles (the radius at the bubble maximum is around 7.5 cm), the effect of pressure variation with depth may play a role. Therefore, simulations were carried out with and without pressure variation with depth and the results are shown in Figure 12. It can be seen that an increase (although not very significant) in the pressure peak at the first bubble minimum occurs when the pressure-depth variation was included.

The above studies clearly demonstrate the general capability of the ALE3D code. Comparison with the experimental data also provides confidence on the accuracy of the numerical predictions. With the basic code validated against our current experiments, the ability of the code to simulate realistic deep sea explosions can be evaluated by simple
modifications to the input conditions. Although detailed calculations of the deep sea explosions have not yet been completed, a sample calculation using a pentolite explosive at depth of 1000 m was simulated using this code. Figure 13 shows the pressure trace of the deep sea explosion. It can be seen that the deep sea explosion dynamics is qualitatively similar to that observed in the shallow water studies. The peak pressure at the first bubble minimum (when normalized by the explosion pressure) is approximately 0.83 in the deep sea case while in the shallow water experiment it is 0.94. Furthermore, when the time period is scaled with $R_{\text{max}} \sqrt{\rho / \rho_w}$, both the deep sea and the laboratory study predict a ratio of around 2 which is in agreement with earlier deep sea data (e.g., Cole, 1948).

Further simulations are planned to investigate deep sea explosions using actual explosive materials and configurations. These studies will be reported in the future.

4.2 Bubble-Bubble Interactions

To investigate bubble-bubble interactions, a series of studies were carried out using bubbles of various sizes. A limitation of the ALE3D code current setup is that it does not allow the effect of phase difference (which was explored in the experiments, see Lal and Menon, 1996a, b) to be incorporated into the model. Thus, for all simulations, both bubbles explode simultaneously and are therefore, in phase. However, by using different bubble sizes the effective volumetric energy release from each bubble was varied. The explosion energy per unit volume in each bubble was also varied for the same bubble size to determine interaction between two equal size but disparate explosions. The effect of inter-bubble distance on the interaction process was also studied using the current ALE3D setup. Some characteristic results are discussed below.

As for the free bubble case, the mesh within and in the vicinity of the bubbles is treated using the ALE algorithm while the rest of the field is treated using Lagrangian approach. For a typical simulation, 25,000 elements are used to resolve the bubble regions with another 38,000 elements were used to resolve the surrounding and far field. Again, grid independence studies were carried out to ensure that the results are not grid dependent. The water and steam equations of state are modeled as described earlier.

Consider first the situation of two identical bubbles of radii 3.17 cm placed approximately 8 cm apart in the water tank (similar to an experimental case). The explosion energy in both bubbles were identical and hence, this scenario essentially models (by virtue of the
method of images) a single bubble explosion near a reflective wall. The bubbles expand and then collapse onto each other and a reentrant water jet with a high speed (30 m/s) is formed in both vertical and horizontal directions. Figures 14a-f shows a time sequence of the bubble-bubble interaction and also views from different angles. The corresponding pressure contours and the velocity vector field are shown in Figs. 15 and 16, respectively. The jet directed towards the adjacent bubble impinges on its counterpart as in a stagnation point flow. As the bubble-bubble process continues, two counter vortex rings are formed with the velocity between the bubble increasing to as high as 50 m/s (Fig. 16c). Formation of this type of vortex ring bubble has been observed in experimental studies of bubble collapse near a rigid wall.

For comparison, the images for a similar study carried out in our experimental facility is shown in Figs. 17a-c. Comparison between Figs. 14 and 17 show remarkable similarity in the bubble-bubble interaction and the collapse stage.

The pressure time trace for this bubble-bubble interaction is shown in Fig. 18. Since both bubbles are identical, their oscillation period and peak pressure at the first minimum are also identical. The pressure in-between the two bubbles (which can be considered the impact location) is higher than the pressure in the bubble itself at the bubble minimum. This increase in peak impact pressure is similar to that seen when a bubble collapses near a wall (described in the next section) and is due to the impingement of the water jet.

When same size bubbles were exploded at the same distance as before, but with one bubble containing four time more energy than the other, a similar result was obtained except that in this case, the weaker bubble is sucked into the other bubble with a velocity reaching a maximum of around 85 m/s. Figures 19a-h show this interaction process along with different views of the collapse of one bubble into the other. The reentrant water jet is first formed in the weaker bubble during the first oscillation (Fig. 19a) and the vortex ring thus formed merges into the (still coherent) stronger bubble (Fig. 19b). The shapes of the two bubbles (bubbles can be identified in the figures by the colors: the cyan bubble energy is less than the blue colored bubble) at the instant of jet formation in the weaker bubble (Figs. 19c and 19d) and at the merging of the two bubbles (Figs 19e-h) are shown in these figures. The jet formation in the stronger bubble is delayed till the second oscillation at which time the second bubble also collapses.
Figure 20 shows the pressure time trace for this case. The general behavior is similar to the earlier (identical explosion case) except that here, the peak pressure in the bubbles at the bubble minimum is different due to different energy content. The pressure in-between the bubbles is still somewhat higher than even the stronger bubble pressure peak due to the high reentrant water jet velocity (85 m/sec compared to 30 m/sec for the case described in Figs. 14-16).

When two bubble of different sizes, e.g., of radii 3.17 cm and 2.17 cm (and thus, with different total explosion energy) are exploded, the results are quite similar to the case discussed above. Figs. 21a-f shows the time sequence of this bubble-bubble interaction and Figs 22a-c shows the corresponding velocity vector field (the time index in the figures can be used to correlate these two figure sets. In Fig. 21a, the bubbles are expanding but because of the greater inertia and explosion strength of the bigger bubble, the smaller bubble is inhibited as seen at their first maximum, (Figs. 21b-c). During the collapse, the pressure drop in-between the bubbles is more than on the other sides and as result, this pressure differential causes the smaller bubble to be engulfed into the larger bubble. The center of motion of the water jet directed towards the bubbles does not immediately adjust to the motion of the bubbles and thus, the water motion is directed off center of the bubble. This creates a very high pressure on the side of the smaller bubble away from the larger bubble. This high pressure and the low pressure in-between the bubbles creates enough momentum to form a water jet through the bubbles which penetrates to the other side of the bubble as shown in Figs. 21d-f

The velocity vector field (Figs. 22a-c) shows very clearly the formation of the water jet. Experimental study also showed a similar behavior when two bubbles of unequal sizes were exploded simultaneously (Lal and Menon, 1996a,b). Figure 23 shows a characteristic image obtained from the experimental study. Formation of the water jet was also observed in the experiments when two identical bubbles were exploded out-of-phase. Out-of-phase explosion essentially changes the relative strength of the bubble explosion during interaction and is therefore, somewhat similar in the dynamics to the present case with two unequal bubbles exploding simultaneously. However, as mentioned earlier, since the current ALE3D code cannot simulate phase difference between the adjacent explosions, it is premature to correlate these different types of initial conditions. Once the ability to include phase difference in the simulation code is incorporated, more details investigation of the interaction can be carried out.
Finally, Fig. 24 compares the pressure between the two bubbles for the various test cases simulated so far. As can be seen, all cases demonstrate the same period of oscillation. However, the case with increased energy content shows the strongest water jet formation (around 85 m/s) and as a result, causes the largest impact pressure at the first bubble minimum.

4.3 Bubble-Wall Interactions

Bubble collapse near a rigid wall is of significant interest due to a variety of reasons related to its ability to cause serious damage to the structure. This is because when the bubble collapses near a rigid surface, a strong reentrant water jet is formed that is directed towards the wall. The peak impact pressure on the wall due to this water jet can be substantially higher that the explosion pressure especially when the initial explosion energy is very large. The dynamics of this collapse process has been under investigation for some time; however, experimental capability to record all the effects of the interaction process is limited due to the difficulty in accessing the interaction zone. Past numerical studies have been able to capture the collapse process, but as noted before, such calculations resorted to obtaining information from experiments to ensure that the simulation initial conditions agreed with experimental data. The ALE3D approach employed here allows for the entire process to be captured without making any adjustments. For a typical simulation, a total of 22,000 elements were used to resolve the bubble and the wall region and another 35,000 elements were used for the rest of the domain. Various simulations were performed by varying the explosion strength and distance of the bubble from the rigid plate. However, only characteristic results are discussed here to highlight the pertinent observations.

Two cases are discussed here with bubble placed 5 cm above (buoyancy inhibiting jet formation) and 5 cm below (buoyancy aiding the jet formation) the wall. Figures 25a-i show the time sequence of the expansion and collapse of a bubble near a rigid wall, for the first case. Different views of the bubble collapse process is also shown in this figure. Figures 26a-i and 27a-i show respectively, the corresponding pressure gray scale and contours and Figs. 28a-i show the velocity vector field. Figure 25a shows that the bubble is almost spherical at the first maximum. As observed earlier, the bubble begins to distort as it collapse (Figs. 25b-f). As the bubble collapses, the differential pressure forces a reentrant water jet to be formed as shown in figs. 25b-d (and also seen more clearly in Figs. 28b-d). As this jet impacts on the rigid plate, a ring bubble vortex is formed as
shown in Figs 28g-i. The physics of the re-entrant jet formation is quite similar to the bubble-bubble case. The bubble expands until it touches the wall. Since there is less volume of water between the wall and the bubble during the collapse, the pressure drop is quite large relative to the pressure on other sides of the bubble. This pressure differential further forces the bubble towards the wall as can be seen in Figs. 26-28. Since steam is lighter, the bubble tends to move further away from the wall (due to buoyancy) for the case where the gravitational force is inhibiting the jet formation (Figs 26-28) while for the second case, the bubble is further accelerated towards the wall (not shown since the general characteristics are qualitatively similar to the case described in Figs. 25-28).

The effect of buoyancy in the formation of jet is very evident in Fig. 29a where the impact pressure on the wall is plotted versus time. The buoyancy-aided case almost doubles the impact pressure when compared to the buoyancy-inhibited case. The impact pressure for even the low explosion bubbles is as much as two-and-half time that of the peak explosion pressure for the buoyancy aided case.

The current methodology is also able to capture the vortex ring bubble as shown in the above figures. This vortex ring bubbles after the jet impact has been also observed both in experiments (Tomita and Shima, 1986 and Vogel et al., 1989) and in recent numerical work (Best, 1993; Szymczak, et al., 1993; Zhang and Duncan 1994).

Finally, Fig. 29b compares the peak impact pressure measured at the flat plate in our experiments (see Appendix A) with the current numerical prediction. The experimental data suggested that there is an optimum distance \(d/R_e\), \(d\) is the distance above the plate where the initial bubble is located. Data suggests that for \(d/R_e = 2\), the impact pressure (normalized by the explosion pressure) is a maximum with the impact pressure more that 4 times the explosion pressure. The numerical study shows a similar trend, i.e., there is an optimum distance for peak impact pressure. However, the numerical data shows that this optimum distance is smaller that observed in the experiment but interestingly enough, the predicted peak pressure at the impact location is very similar to that observed in the experiment. The discrepancy between the optimum location measured and predicted may be related to the limitations of the experimental set up, as noted earlier, (such as, the presence of the transducer and spark igniter inside the bubble which are not included in the simulation model. More detailed analysis is currently underway to understand the physics of this interaction that leads to such an optimum location. This will be reported in the near future.
4.4 Detonation Cord Explosions

Currently, there is an ongoing study of underwater detonation using explosive cord at NSWC, Indian Head Division. Such explosives are long cylindrical cord made of explosive material that is placed next to surfaces targeted for destruction and then ignited. Experiments have shown some contradictory results as related to the explosion bubble behavior. To demonstrate the capability of the ALE3D code to simulate realistic underwater detonation of currently employed munitions, a series of calculations were recently performed to study the behavior of detonation cord in shallow water. Three cases are discussed below: (a) a single 30 cm long cord of 0.5 cm diameter exploding in free field, (b) a 30 cm long cord of 0.5 cm diameter exploding 1 cm above a rigid wall and (c) a rectangular mesh of 20 cm x 20 cm square of cord of 0.5 cm diameter exploding 1 cm above a rigid wall. The last configuration is a very close idealization of the cord used for actual field studies. The explosive material was assumed to be pentolite which is strictly incorrect, however, the present study is more of demonstration nature. Realistic cord material properties can be easily included at a later stage. Some characteristic results are discussed below to demonstrate the ability of the ALE3D code to capture these types of realistic detonation.

Figures 30a-i show the bubble expansion and collapse process for the foot long (30 cm) cord exploding in free field. Figures 31a-i show the corresponding velocity vector field for the expansion and collapse process and Fig. 32a and 32b show two other perspective of the velocity field. As can be see, initially the bubble expands almost cylindrical, however, as it begins to collapse, the collapse process is faster along its major axis (from the ends) than along its minor axis. This is seen in Figs. 30b, c and 31b, c. This differential collapse results in the surrounding water rushing into the bubble at a higher velocity along the major axis when compared to the fluid motion along the minor axis. Thus, the bubble collapses along the major axis while it bulges along the major axis (in the central portion of the cord). The water jet formation due to this collapse process is shown in Figs. 30d-f, 31d and 32a, b. After the jet is formed, the bubble rebounds (Figs. 30g-i, 31e-f) and a double vortex ring bubble is formed (similar to the ring bubble seen earlier for the bubble-bubble case).

Figures 33a-i show the time sequence for the cord explosion near the rigid wall. Figures 33a-f shows the bubble shape during collapse, Fig. 33g shows the shape during jet
formation and Figs. 33h-i show the shape during the rebound stage. Figures 34a-f show
the corresponding velocity vector field and Figs. 35a and 35b shows different perspective
of the flow field during collapse process. Additional visualization of the flow field are
shown in Figs. 36a, b (velocity field at 6.8 ms - see Fig. 33), Figs. 37a-c (pressure
contours) and Figs. 38a-c (bubble shape).

Figures 33a-i show the oscillation of the bubble formed during cord explosion near the
wall and shows a picture quite different from the spherical bubble collapse studied earlier.
It appears that unlike the spherical bubble case, where the differences between the
pressure on the side of the wall and away from the wall drives the jet formation, in the
cord case, the bubble experiences (in addition to this pressure differential) another
pressure differential due to the asymmetric bubble expansion along the major and minor
axis. As shown in Figs. 34a and 35a,b, this causes the water to rush into the bubble at an
angle (while in the spherical bubble case the jet motion is perpendicular). As a result, the
jet formation for the cord-wall case is much more complicated, as shown in Figs. 33g, 34d
and 36a, b. Finally, the rebound of the bubble is also more complicated as shown in Figs.
33h, i and 34e, f.

Finally, a simulation was carried out for a cord mesh adjacent to the wall. As mentioned
above, this is a close approximation to the actual field device. Figure 39 shows two images
of the grid domain just after the initial explosion. The grid is clustered near the cord region
and near the wall to resolve the bubble dynamics. The cord region and the surrounding
water is treated as ALE materials while the far field water is treated as lagrangian. Figures
40a-i show the bubble shape at various stages of the expansion and collapse process and
Figs. 41 and 42 show different views of the bubble shape at 5.8 ms and 6 ms (by which
time the collapse process is almost completed). The fluid motion during this interaction
process is shown in Figs. 43a-f and the pressure contours corresponding to Figs. 42b and
42e are shown in Figs. 44a and 44b, respectively.

The bubble expansion and collapse process is even more complex for this cord-mesh case.
However, the general collapse process is similar to the cord-wall case with the major
difference being related to the uneven expansion of the bubble. During the collapse
process the diagonal regions travel at a different speed compared to the normal direction.
As a result, the jet formation is also modified by this differential collapse and the jet is
formed at an angle to the wall as can be seen in Figs. 43b and 43e.
5. Conclusions

These studies show that the present implementation of the ALE3D for underwater explosion studies has the potential for addressing fundamental issues related to the highly complex process of bubble oscillation and collapse under various conditions. The basic code has been validated using shallow water explosion data obtained in recent experiments. In addition to isolated bubbles exploding in (an approximate) free field, bubble-bubble and bubble-wall interaction studies were also performed. It has been shown that all the features observed in past experiments have been captured in these studies. The formation of reentrant water jet as the bubble collapses near a rigid surface and the formation of ring vortex bubble has been captured in the simulation. These features are in good qualitative agreement with experimental data. Furthermore, the ability of the ALE3D code to study realistic deep sea explosions or detonation of realistic explosives such as detonation cord (used for mine destruction) has been demonstrated in these studies.

Limitations of the current ALE3D code have also been identified during these studies. For example, the current code is unable to simulate bubble-bubble interaction with a phase difference between the explosions. The far field boundary conditions currently cannot be changed from reflecting wall. Time dependent spatially moving detonations are also not feasible using the present code. However, such limitations can be removed by proper modifications to the code. These issues are still under investigation.

Acknowledgments

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References


TABLE I

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Coefficients of the Grunesein Form used for equation of state of water.

TABLE II

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Coefficients used to model EOS of Pentolite using JWHL High Explosive form.
Fig. 1: Figure showing the grid used in ALE3D freely oscillating runs. 
a) The View of the whole domain & b) View of the domain in the proximity of the bubble.
Fig. 5: Time sequence of the collapse and rebound of a freely oscillating bubble for a complete cycle.
Fig. 10a) : Pressure fluctuation measured near the side wall of the experimental facility (Menon and Lal 1996a)

Figure 10b) : Computed pressure near the side wall in the computational domain.
Figure 11: Effect of wall location and shape on the bubble oscillation.
Figure 12: Effect of pressure variation with depth on the bubble oscillation.
Figure 13: Pentolite explosion at a depth of 1000 meters.
(The init. bubble radius was .8cm and the maximum radius at bubble max. was around 5cm)
Fig. 14 The time sequence of interaction between bubbles of same size. a) Side view at Bubble Max. b) Side View just before the jet formation. c) Side view just after the jet formation. d), e) and f) are different angles of the bubbles after the jet is formed.
Fig. 3 Time Sequence of the smooth pressure contours around a freely oscillating bubble.

a) 7 ms

b) 12 ms

c) 15 ms
Fig. 4 Time Sequence of the smooth pressure contours around a freely oscillating bubble.
Fig. 5 Time Sequence of the velocity contours around a freely oscillating bubble.

a) 7 ms

b) 12 ms

c) 15 ms
Fig. 6: Variation of $\frac{d^2 R}{dt^2}$ with time showing that R-T instability criteria is satisfied near bubble minimum.
Figure 7a: Variation of bubble pressure with time.

Figure 7b: Variation of bubble radius with time.
Figure 8. Effect of grid resolution on the pressure in the bubble.
Figure 9: Comparison of pressure in the bubble for full 3D domain and Octant(1/8th) domain.
Figure 15

Fig. Time sequence of pressure contours around two interacting bubbles of same size.
Figure 16

Fig: Time Sequence of velocity vectors around two interacting bubbles of same size.
Figure 17: Experimental (Lal and Menon, 1996) pictures of two bubbles of the same size interacting after they are exploded simultaneously.
Figure 18: Pressure variation when two identically sized bubbles with identical energy are exploded near each other.
Fig. 19 Interaction of bubbles of different energy but of same volume. a) Velocity vectors after the jet impinges the smaller bubble, b) Velocity vectors during the rebound at the time the jet pierces the second bubble, c) & d) are two different views when the jet pierces the smaller bubble, e) during rebound & f), g) & h) are three different views after the jet pierces the bigger bubble.
Figure 20: Pressure variation when two bubbles of identical size but of different energies (one four times the other) are exploded.
Time sequence of the interaction of a bubble with a bubble 30% its volume but with the same energy density. a) Just before the jet formation, b) & c) Two different views after the jet passed through the small bubble, d), e) & f) are three different views at the time of jet formation through the bigger bubble.
Fig. 22 Velocity vectors around the interacting bubbles of different volumes. a) At the time of jet formation through smaller bubble b) During rebound & c) At the time of jet formation through the bigger bubble.
Figure 23: Experimental (Lal and Menon, 1996) picture showing the merging of two out of phase exploded bubbles.
Figure 24: Pressure in between the interacting bubbles for four different cases.
Fig. 25 Time sequence of the bubble expansion and collapse near a wall (View from the side away from the wall, unless specified)
a) 9 ms

b) 15 ms

c) 15.6 ms
Fig. 25 Smooth pressure contours of bubble collapsing near a wall
d) 15.9 ms

e) 16.1 ms

f) 16.4 ms
Fig 2.7 Pressure Contours for the collapse of bubble near a wall.
d) 15.9 ms

e) 16.1 ms

f) 16.4 ms
Figure 28
Fig. Velocity Vectors Showing the development of jet during the collapse and rebound of a bubble near a wall.
Figure 29a: Impact pressure computed on the wall for the gravity aided and inhibited cases.
Fig. 29b Variation of impact pressure on the wall with distance of the bubble from wall.
Fig. 30 Bubble formed by a explosive cord in a free field. a) At bubble max when it is almost cylindrical, b) & c) the collapse along the axis of the cylinder and expansion in the horizontal direction, d) , e) and f) show the different views of the bubble after the jet is formed, g) during rebound and h) & i) show the different views at the maximum of the second oscillation.
Fig. 34 Velocity vector field around a freely oscillating bubble formed by a cord explosive (with slice parallel to the axis of the cylinder).
Figure 32

Fig. Velocity vector field around the bubble formed by a cord explosive in free field. a) Slice perpendicular to the axis of the cord and b) Slice parallel to the axis of the cord.
Fig. 33 Time sequence of the bubble generated by a cord exploding near a rigid wall. a)-f) During Collapse, g) Jet formation and h)&i) During Rebound.
g. Velocity vector field around the cord collapsing near the
    illus. a), b) During collapse, c) Just before the jet formation,
    After jet formation and e) f) During Rebound. (Plan View with
    illus. at the bottom and in the plane of paper.)
Fig. 35 Different side views corresponding to Fig. 34: a) Slice along the cord with wall at the bottom and perpendicular to the plane of the paper and b) Slice along the axis of the cord with the wall towards the left and perpendicular to the plane of the paper.
Figure 35: Different side views corresponding to Fig. 34d. a) Slice along the cord with wall at the bottom and perpendicular to the plane of the paper and b) Slice along the axis of the cord with the wall towards the left and perpendicular to the plane of the paper.
Fig. Pressure contours corresponding to Fig. 36.
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a) The View of the whole domain & b) View of the domain in the proximity of the explosive.
Fig. 4D Time sequence of the bubble generated by a square explosive grid near a rigid wall. a) Before max., b) At max., c), d), e) & f) During collapse, g) & h) during the jet formation and i) after the jet formation.
Different views of the Square grid generated bubble at time 5.8ms
42 Different views of the Square grid generated bubble at time 6ms
Velocity Vectors around a square grid explosive bubble collapsing on a rigid wall. a), b) and c) are top views at different instances and e) and f) are the corresponding side views.
Fig. 44: Pressure contours corresponding to the a) Fig. 43b & b) Fig. 43c.
APPENDIX A

Underwater Explosions Near Exposed
And Buried Rigid Surfaces
Underwater Explosions near Exposed and Buried Rigid Surface

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Introduction

Vapor and gas bubble dynamics are of great practical interest in the prediction and prevention of cavitation erosion of marine propeller and turbine blades. It is well known that when an explosion bubble collapses near a rigid surface, a strong re-entrant water jet is directed towards the wall. The impact of this jet on the plate results in impact pressure that can be much larger than the explosion pressure. This impact on the surface can cause structural damage especially when the explosion energy (and hence the bubble size) is large. Both experimental and numerical studies have been carried out in the past to study the complex flow fields associated with such explosions. Detailed reviews have summarized many of the pertinent results obtained in past studies. Experimental studies are too numerous to list completely; however, most studies in the past focused on cavitation (small) bubbles. Many studies have focused on explosions near structures. See references cited and the discussion in the main text (and therefore, not included here). However, in most of these studies, the primary focus has been on small bubbles.

Recently, a series of experiments were carried out to investigate underwater explosions in shallow water (1 atmosphere ambient pressure) to understand the dynamics of bubble-wall interaction in such flows and to investigate feasibility of targeting and destroying mines buried in beaches. In this configuration (shown in Fig. 1), the free water surface is close enough to the bubble-wall interaction region to play a role in modifying the dynamics of the bubble collapse. The free surface provides a constant pressure boundary in close proximity to the wall. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration. Therefore, the presence of the free surface above the bubble collapse region is likely to increase the net impact pressure on the wall.

Another issue that was investigated is the behavior of the impact process when the rigid surface is buried below a layer of sand as would be the case for buried mines. This study was motivated by some earlier observations that when the rigid flat plate was buried below a layer of clay, the impact pressure felt on the plate was still significant. On the
other hand, when a thick layer of sand was used (without the presence of the rigid plate), no significant impact process was observed. To resolve these issues, experiments were carried out to investigate what parameters control the impact process. Obviously, the distance of the bubble from the plate and the sand layer thickness are two important parameters for given explosion. Scaling analysis of this data could shed light on the interaction process when the upper layer is made up of loosely packed material that has a tendency to deform under pressure. Some interesting results have been obtained and summarized in this communication.

Results and Discussion

Underwater explosion experiments near a solid boundary were conducted by exploding fuel (Carbon Monoxide) and oxidizer (Oxygen) mixture contained in a glass globe of average volume of 90 ml (6.35 cm diameter) over a steel plate (shown in Fig. 1). The premixed fuel-air stoichiometric mixture is ignited by an electric spark and the explosion takes place at a constant volume until the globe bursts. A pressure transducer is placed in the bubble holder to record the explosion pressure and the pressure in the bubble during expansion/collapse process. Since the experiments were conducted in a laboratory shallow water setup using a gaseous explosive mixture, the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep sea explosions. To determine the impact pressure, eight KISTLER pressure transducers were mounted on the plate, as shown in Fig. 2, to obtain a surface distribution of the impact pressure field. Optical record of the bubble wall interaction was also obtained using a CCD enhanced digital video camera. Since the viewable picture size is inversely proportional to the recording speed of the camera, the maximum speed was limited to 2000 frames per second in order to obtain a full screen image.

Parameteric studies of the impact process was carried out by systematically varying the distance between the globe and the plate to investigate the effect of solid wall location relative to the explosion. The layer thickness of the sand above the instrumented plate was also varied to determine how the impact pressure is effected by the porous material above the plate. All the experimental data have not been fully analyzed as yet. However, the results obtained so far do suggest some interesting trends and also provide an estimate of the effect of the sand layer on the impact pressure. For all studies discussed here, the
explosion strength and the initial bubble size were held constant. Only characteristic results of this study are summarized here.

**Bubble-Wall Interactions**

The collapse process near the wall (with and without the sand layer) was dynamically similar. The bubble expands subsequent to the explosion, however, the extent of the expansion (for a free field explosion, these bubbles were found to expand up to three times the initial diameter) depends on the relative position of the free surface and the rigid plate. Figure 3 shows a time sequence of the bubble expansion and collapse for a given set of parameters. For this case the bubble does not fully expand as in the free field case due to the presence of the wall. By the time the bubble has expanded, it already is interacting with the plate. The collapse of the bubble, the formation and the impingement of the water jet on the plate can be seen in these figures. The time period for the expansion/collapse process is found to be 19 ms (for this configuration), while it is around 15 ms in free field configuration. Therefore, as the bubble is brought close to the surface, an increase in the time period of oscillation is observed. If the bubble is brought further close to the surface beyond the optimum distance (defined below), the time period reduces slightly to 18 ms.

The pressure data recorded on the plate was found to be a strong function of the initial location of the bubble relative to the plate. The impact pressure recorded by the eight pressure transducers were not the same suggesting that the impact jet is highly coherent and focused. Figures 4a-d show the pressure data from the plug transducer (Fig. 4a) and three of the eight transducers (4b-d; see Fig. 2 for their locations). It can be seen that the transducer no. 8 (which is located right below the bubble center) records the highest impact pressure while the transducers away from this location records much lower pressures. This behavior was seen for all cases studied so far and suggests that the impinging water jet is highly focused in a narrow jet.

The impact pressure recorded on the plate for all the test cases are summarized in Figs. 5a and 5b. Figure 5a shows that impact pressure (normalized by the explosion pressure) measured by the transducer No. 8 as a function of the initial distance of the bulb above the plate (normalized by the initial bulb size). Figure 5b shows the impact pressure data from the other two transducers (shown in Fig. 4c, d). As the bulb is brought closer to the plate, up to a certain distance, an increase in the impact pressure is recorded by all.
transducers. If the distance between the bulb and the plate is further reduced beyond the certain distance, a reduction in the impact pressure is noted. The optimum distance determined from the current data set appears to be \( d/R_o = 2 \); where \( d \) is the distance between the bulb and plate, and \( R_o \) is the initial globe radius. The impact pressure ratio for this configuration was \( P_{imp}/P_o = 4.19 \). Here, \( P_{imp} \) is the impact pressure on the plate at the center and \( P_o \) is the explosion pressure inside the bubble.

It is worthwhile noting here that the recent numerical studies described in the main text also reproduced the impact process (see Figs. 29). It was determined that there is an optimum location for peak impact as seen in the experiments; however, the optimum location was found to be smaller than that seen in these experiments. There are many artifacts in the experiment such as, the presence of the pressure transducer and the igniter in the bulb holder, the presence of the glass bulb and the formation of glass fragments after the explosion. These features were not included in the numerical studies. However, the relative agreement between the experimental and numerical data do suggest that the present experiments are dynamically correct.

Additional data are still being analyzed. Numerical studies for these cases and also for realistic deep sea explosions near rigid surfaces are planned to determine how the impact pressure scales with the parameters of this problem. These results will be reported later.

**Bubble-Sand-Wall Interactions**

Experiments were also conducted to simulate underwater explosion over a buried surface near a beach by covering the plate with varying depths of sand on the top. The typical parameters are identified in Fig. 1. The bubble collapse process optically obtained for this case is shown in Fig. 6. The bubble is once again attracted towards the plate and a reentrant jet is formed in the bubble in the direction of its migration. It can be seen from this figure that the bulb is almost touching the sand. The effect of covering the plate with sand is to reduce the impact pressure on the plate. Figure 7 shows the impact pressure at the center of the plate for various sand depths. When the plate is covered with the sand while maintaining the same distance between the plate and the bulb, a reduction in the impact pressure at the center of the plate is observed. When the sand depth is further increased so as to bring the bulb closer to the sand, a partial recovery of the impact pressure occurs as shown in Fig. 7.
In order to simulate the explosion near a beach and to investigate the effect of the proximity of the free surface, the water depth, \(d_w\), was lowered. Since water free surface is known to repel the bubble, the free surface should aid in the impact process. This was confirmed in the experiments. Figure 8 shows the impact pressure on the plate for cases with sand covering as a function of varying water depth. A lower water depth increases the impact pressure and decreases the time period of oscillation. The time period of oscillation of the bubble shown in Fig. 6 is about 16 ms. When the water depth is decreased for this experiment such that the bulb center is only about 10 cm below the free surface, the period of oscillation reduces to 14 ms.

Other studies for varying sand depths showed qualitatively similar results. In some cases, when the bulb was placed much above the plate, the impact process is completely inhibited and in some cases, the bubble collapse occurred after multiple oscillations. Further analysis of this data will be carried out and reported in the near future.

Conclusions

The results obtained so far suggest that the impact pressure resulting from the impinging water jet can be more than 4 times the explosion pressure for the laboratory test cases. Furthermore, the analyses of the pressure distribution on the plate show that the peak impact is felt only in a narrow region right below the center of the bubble. Further away from the central region the peak pressure felt on the plate is smaller by an order of magnitude. This clearly shows that the impinging jet is highly focused and does not spread at all. The region where peak impact is felt appears to be smaller than the radius of the initial bubble size.

Studies using a layer of sand above the plate showed that both the locations of the initial bubble and the sand layer thickness control the pressure impact process. In general, the peak impact pressure recorded on the buried plate is lower than the exposed plate case. However, the reduction of the impact pressure by the damping effect of the sand can be alleviated by bringing the bubble closer to the surface. The effect of free surface above the interaction zone was to increase the impact pressure consistent with the notion that the free surface repels the bubble.
Fig. 1. Experimental setup for simulating explosion near an exposed or buried object. For exposed object, $d_s = 0$.

Fig. 2. Layout of pressure transducers on the plate.
Fig. 3. Time sequence of bubble collapse near an exposed plate for $d/R_o = 2$. 
Fig. 3. (Continued)
Fig. 4. Pressure time trace recorded by (a) the plug transducer; and (b-d) the plate transducers for the exposed plate case with \( d/R_a = 2 \).
Fig. 5. Normalized impact pressure measured by (a) transducer no. 8; and (b) transducer no. 6 and 7 (away from the center) as a function of distance of the initial bulb from the plate.
Fig. 6. Bubble collapse near a buried object. $d/R_o = 2.4$, $d_y/R_o = 1.1$ and $d_w/R_o = 24$. 
Fig. 7. Normalized impact pressure at the center as a function of sand layer thickness.
Fig. 8. Normalized impact pressure at the center as a function of the location of free water surface above the bubble.
APPENDIX B

Dynamics of Interaction between Two
Underwater Explosion Bubbles
DYNAMICS OF INTERACTION BETWEEN TWO UNDERWATER EXPLOSION BUBBLES

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Dynamics of Interaction between Two Underwater Explosion Bubbles

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ABSTRACT
Underwater explosion bubbles are created by detonating a mixture of oxygen and Carbon Monoxide or Hydrogen in glass globes submerged in a water tank. A cinematographic technique is employed to capture entire interaction process in both horizontal and vertical configurations. Instrumented tubes and plugs measure pressure inside and outside the bubbles. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or they may violently repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. When a coalesced bubble is formed by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble. Experiments were also conducted to qualitatively and quantitatively study the interaction of a free surface with the explosion bubble(s).

Introduction
Much of the research activities in the area of underwater bubble dynamics has been focused on the behavior of cavitation bubbles. Cavitation bubble dynamics play very important role in underwater acoustics and in predicting and preventing propeller and turbine blade damage. These bubbles, however, seldom occur singly. Actual cavitation fields contain several thousands of
oscillating and translating microbubbles. Study of the behavior of a cloud of bubbles thus becomes inevitable and experimental (e.g., Chahine and Sirian, 1985; Tomita et al., 1984), numerical (e.g., Chahine and Liu, 1985; Chahine, 1991; Chahine and Duraiswami, 1992; Wang and Brennen, 1994) and analytical (e.g., Van Wijngaarden, 1972) techniques have all been developed. The simplest model that has been studied by the researchers is the interaction between two bubbles. Theoretical and numerical studies of the interaction of two spherical or nonspherical bubbles of same or different sizes (e.g., Fujikawa et al., 1985; Fujikawa and Takahira, 1986; Fujikawa and Takahira, 1988; Morioka, 1974; Serebryakov, 1992; Shima, 1971; Takahira, 1988) have also been carried out. Interesting experimental observations of the interaction of a gas bubble with a pressure wave (Shima and Tomita, 1988) or with a vapor bubble (Smith and Mesler, 1972) have also been made. However, most of these observations are for microbubbles and find applications in cavitation, erosion and related topics.

Large bubbles, such as those created by underwater explosion, owing to their tremendous inherent destructive capabilities upon collapse near a rigid boundary, find practical applications in underwater weaponry. Detailed measurements and imaging of pulsating bubbles formed during deep sea explosions are very difficult due to a variety of obvious reasons (e.g., Arons et al., 1948) and therefore, there is insufficient data available to analyze the dynamics of interaction of bubbles. Controlled experiments described here, are required to investigate the physical processes that contribute to the interaction of bubbles. These experiments were conducted in a laboratory shallow water setup using a gaseous explosive mixture. The observations reported here are of practical significance for buried mines detection in shallow water beaches, where the interaction of an explosion bubble with a solid boundary and water free surface is anticipated.

The interaction of two underwater explosion bubbles is a very interesting and complex phenomenon, because of the fact that one bubble is influenced by the time-delayed pressure or shock wave radiated from the adjacent bubble. Radial motion of the bubble may be greatly excited
or subdued due to the interaction depending on their temporal and spatial separations. Though some experimental work has been done on the interaction of gas bubbles with two adjacent underwater explosion bubbles, and it has been shown that strong and complicated interactions ensue, it appears that no detailed results on the interaction of two underwater explosion bubbles have been published in the public domain literature.

This paper reports the results of the experiments carried out in a laboratory water tank to study the interaction between two adjacent bubbles created by underwater explosion of flammable gas contained in glass globes. The globes were placed side-by-side either in a horizontal or a vertical plane. The distance between the two globes and their sizes were both varied. This paper also discusses the interaction of single and double explosion bubble(s) with the water free surface.

**Experimental Procedures**

Underwater explosion experiments were conducted in a wooden tank of dimension 2 m × 1.5 m × 1.5 m, coated with fiberglass resin from inside. The tank, as shown in Fig. 1, has three windows on three sides for optical imaging. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe. Three different sizes of glass globes were used for present experiments with average radii of 2.9 cm, 3.2 cm, and 3.8 cm. The glass globe, as shown in Fig. 2, has an electric spark ignition system connected to a 3000V DC power supply. The explosion takes place at a constant volume until the globe bursts. Since the experiments were conducted in a laboratory shallow water setup and using a gaseous explosive mixture, the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep sea explosions. Using both geometric and dynamic (based on Froude number) scaling analysis (e.g., Shepherd, 1988), it is shown that the present experiments approximately simulate deep sea bubble dynamics. The explosion bubble thus formed is a reasonable subscale approximation of the deep sea underwater explosion bubble.
The pressure response in the water around the bubbles were recorded during the experiments by means of 4 KISTLER dynamic piezoelectric pressure transducers fitted at the ends of 4 stainless steel (1.27 cm diameter) tubes bent at right angle, as shown in Fig. 1. A hydrophone is also mounted in the tank and is used for measuring acoustic pressure. Pressure inside the bubble during its oscillation is measured by another KISTLER transducer which is mounted inside the plug, as shown in Fig. 2. Signals from these six pressure transducers and the hydrophone were digitized using National Instrument's AT-MIO-16X analog-to-digital conversion board, and were recorded into a microcomputer. Eight channel data recording was performed with a sustained sampling rate of 10,000 samples per second per channel.

The tank was illuminated by either direct overhead flood lights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis. The optical recording of the bubble motion was performed by a CCD enhanced digital video camera with a maximum speed of 6000 frames per second. Since the viewable picture size is inversely proportional to the recording speed of the camera, the maximum speed was limited to 2000 frames per second as the image size reduces to half at this speed. Many of the experiments, however, were performed at a lower speed of 1000 frames per second in order to obtain a full screen image.

The two glass globes were supported inside the tank by means of a modified sting which made the pressure transducers mounted inside the two globes to face each other (Fig. 1). This facilitated direct measurement of the fluctuation in the pressure inside one bubble due to its interaction with the other. Six holes were drilled in the supporting copper pipe (1.6 cm diameter) of Fig. 1 at equal spatial separation from either ends in order to provide a means for altering the distance between the two bubbles. Experiments were conducted in primarily two configurations; a horizontal configuration, when the supporting pipe was horizontal, and a vertical configuration,
when it was vertical. The former configuration prohibited the use of laser light sheet and only the flood lights were used for imaging, while the latter allowed the use of laser light sheet.

Experiments were also conducted to study the interaction of water free surface with the explosion bubble(s). The motivation for these experiments has obvious reasons. The free surface provides a constant pressure boundary in close proximity of the oscillating bubble. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration (e.g., Birkhoff, 1957; Blake and Gibson, 1981; Chahine, 1977; Chahine, 1982; Wilkerson, 1989). Wilkerson (1989) developed a boundary integral technique for the analysis of expansion and collapse of an explosion bubble in free field, near a rigid surface, or near a free surface. To verify the accuracy of his method for predicting reentrant water jet tip velocities for a bubble near a free surface, he compared his results with a PISCES code calculation and expressed his inability to predict an accurate estimation of the error involved in his method because of the unavailability of any such experimental data. This observed lack of data motivated the current experiments to study the interaction of a free surface with an explosion bubble.

Results and Discussion

The interaction process is highly dependent on the time delay between the two explosions. This time delay is related to a variety of hardware and bubble response characteristics. The two globes have independent power supplies and there is always a time delay between the occurrence of the spark and when the bubble(s) starts expanding. The bubble expansion occurs immediately after the glass globe bursts. The time delay depends primarily on the gas volume (globe size) and the nature of gas mixture inside the globe. A bigger globe size will create a larger time delay. Similarly, fuel-rich or fuel-lean mixtures will also create larger delays as compared to the one associated with a stoichiometric mixture. In addition to this delay, there is another time delay which is associated with the spark system itself. This delay is between the instant when the power
is turned on and when the spark actually fires. This delay primarily depends on the actual gap between the two spark wires since a bigger gap creates a larger delay. The actual delay (temporal separation) between two explosions is therefore measured from the recorded video images as the time elapsed between the instant when the individual globe bursts. It was therefore deemed necessary to conduct several experiments to collect statistical information about the range of bubble behavior with respect to the delay between two explosions.

The entire spectrum of delay can be classified into two broad regions: in-phase oscillation and out-of-phase oscillation (e.g., Morioka, 1974; Shima, 1971; Smith and Mesler, 1972). In most of the past analytical, numerical or even experimental work on the interaction of two cavitation bubbles, interest has been focused primarily on the contraction phase of the bubble oscillation. This yields an in-phase oscillation of identical bubbles as they both start collapsing at the same time following a sudden change in the ambient pressure. In-phase oscillation is obtained when there is strictly no delay between the explosions; both the bubbles start their oscillation cycle at the same time, both have identical time period of oscillation and are in phase at any instant throughout their oscillation cycle. This is the simplest scenario and has been analyzed comprehensively.

Another interesting scenario is a 180° out-of-phase oscillation and it can be best understood in the interaction of two identical explosion bubbles as a case where one bubble starts its oscillation cycle when the other has already reached its maximum radius. In fact, this is one of the two scenarios predicted by Morioka's (1974) theoretical analysis of natural frequencies of two pulsating bubbles which predicts the existence of two natural frequencies corresponding to in-phase and 180° out-of-phase oscillations, respectively. Of course, in an experimental setup one can have any amount of delay between zero to 180°, or even beyond 180°. Two bubbles oscillating in-phase behave in a nearly identical manner as a single bubble near a rigid boundary and therefore, are of considerable practical interest since it has been shown that the collapse of a
bubble near a wall can cause significant damage. The bubbles have an increasing repulsive effect as the delay between two explosions increases, up to the point when they oscillate 180° out-of-phase (Smith and Mesler, 1972).

Figure 3 shows an example for two underwater explosion bubbles oscillating in-phase. The initial volume of the right glass globe is 94 ml and that of the left glass globe is 90.5 ml. They are in a horizontal configuration and are initially separated by a distance d, where d/R_o = 2.32. Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. There is virtually no delay between two explosions (determined from the image data) and since the two globes have almost same volume, they burst also at the same time (time, t = 0 ms). Since the initial spatial separation between the globes was intentionally kept to a very small value so that a violent interaction can ensue, the bubbles soon come in contact with each other. They deviate more and more from sphericity as they expand with time. The bubbles merge together to form a single coalesced bubble.

The surface where the two bubbles come in contact in Fig. 3 grows in size as the bubbles grow and is initially curved. The contact surface slowly becomes a plane surface and remains so thereafter until the bubbles collapse on to each other. This plane surface of contact may be considered as a rigid surface in an equivalent single bubble analogy. Figure 4 shows the relative position of this surface with respect to the initial globes’ centers. Notice that this surface is almost perpendicular to the line joining the initial globes' centers and is located almost midway. The time period of oscillation of the bubbles shown in Fig. 3 is about 21 ms, while that of an identical bubble in free-field is less than 15 ms. Therefore, for two identical bubbles oscillating in-phase, an increase in the bubble period is observed. A similar observation was made by Chahine (1991).

The pressure traces measured around the bubbles show that the bubble behavior is symmetrical. A pressure fluctuation of about 700 kPa exhibited by the plug transducers near
bubble minimum demonstrates the severity of collapse of the jets formed in two bubbles on to each other. The pressure drops exponentially as one moves away from the bubble (Cole, 1948). A pressure drop of 70% (from 1000 kPa to 300 kPa) over a distance of 14 cm, and that of 80% (from 1000 kPa to 200 kPa) over a distance of 34 cm from the bubble center have been recorded by tank transducers.

The coalesced bubble quickly breaks into cloud of smaller bubbles which migrate upward due to buoyancy effect. The bubble contour is traced and 360 bubble radii are obtained at equal azimuth locations. A mean radius is obtained from this data, which is used to normalize the radii data. These data are then Fourier analyzed and the results are shown in Fig. 5, which shows the power spectral density of the coalesced bubble's interface at three instants: just after it is formed and 1 and 5 ms after it collapses. Here \( c \) is the bubble circumference and \( \lambda \) is the wavelength. It can be seen that the coalesced bubble starts exhibiting pronounced and distributed peaks in power spectral density soon after collapse. A peak is actually the square root of the sum of the squares of mode amplitude coefficients and occurs at integral fractions of bubble circumference because a periodic trace is being analyzed. The coalesced bubble is therefore very unstable.

A single coalesced bubble does not form only in an in-phase oscillation. Another situation where the formation of a single coalesced bubble has been observed repeatedly and most surprisingly, is associated with a nonzero time delay and a very short inter-bubble distance. This case is shown in Fig. 6. The initial volume of the right glass globe is 125 ml and that of the left glass globe is 127 ml. They are almost touching each other such that the initial separation distance between them, \( d \), is given by \( d/R_o = 2.1 \). Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. The right globe explodes first (at time \( t = 0 \) ms) and tries to encompass the left globe as the bubble grows. When the left globe explodes (at time \( t = 8 \) ms), the shock wave emitted by this bubble travels through the right bubble as is evident by its protruding pieces. The right bubble, however, maintains its coherence and sphericity. It seems that the
energy of the left bubble is substantially transferred to the right bubble, and it does not even get a chance to expand to its maximum radius. A force field is generated such that when the right bubble starts to collapse, the left bubble just merges into its predecessor to form a single coalesced bubble, which continues the oscillation cycle as a single explosion bubble. Since no jets are formed and the coalesced bubble is formed by merging the energies of two bubbles, it has more residual energy than that of the previous example and does not disintegrate into smaller bubbles so quickly. In fact, it is even more stable than a single explosion bubble in free field and can, therefore, be used for focusing bubble energy for enhancement of its destructive capabilities.

The available energy for successive pulsations of the coalesced bubble can be calculated using Vokurka's (1986 and 1987) energy balance analysis. The various formulae for this analysis are given in the cited references and are, therefore, avoided here for the sake of brevity. The energy, $E_0 = 4\pi R_0^3 P_I / 3$, where $R_0$ is the initial globe radius and $P_I$ is the ambient hydrostatic pressure at the explosion depth, is used to nondimensionalize energy and the heat release of stoichiometric carbon monoxide is taken to be 284 KJ/mole (Strahle 1994). The total nondimensional energy available for oscillation of the right bubble at time $t = 0$ can be given by $\bar{E}_0 = 78.439$ for an explosion depth of 0.65 m. The nondimensional internal and the potential energies of the right bubble at its maximum radius for an expansion ratio of $R_{\text{max}} / R_0 = 2.738$, are estimated to be 38.043 and 19.528, respectively. The energy dissipated into the surrounding water by a shock wave thus equals $20.868E_0$. Therefore, the energy available for successive oscillation of the right bubble at its maximum radius equals $57.571E_0$. A similar analysis for the left bubble for a smaller explosion pressure and expansion ratio of only 1.095 yields the value of the available energy at its maximum radius to be $11.916E_0$. The coalesced bubble should apparently have available energy of $69.487E_0$ for its successive oscillation, which is roughly 20.7% more than what a single explosion bubble should have in a free field condition. The coalesced bubble is, in fact, observed to oscillate for an extended period of time.
The effect of the shock wave generated during the formation of the right bulb is reflected by a 150 kPa peak in the left plug transducer pressure trace (not shown). On the other hand, the plug transducer inside the expanding right bubble records only a tiny fluctuation of 10 kPa when the left bulb explodes. It is interesting to note that the explosion pressure for the left bubble (200 kPa) is only about 20% of what it would have been in a free field case. Thus, an expanding bubble appears to inhibit the formation of another explosion bubble in its close proximity by reducing its explosion pressure. This may be the reason why the left bubble does not have sufficient energy to expand to its maximum radius. But, it certainly aids its predecessor to form a coalesced bubble with a greater energy to collapse violently and this fact is captured by all the transducers and hydrophone in the form of elevated collapse peaks. This time, the right plug transducer lies inside the coalesced bubble as it collapses. The collapse pressure recorded by this transducer is very high (2500 kPa). In addition to a large peak near bubble collapse, the hydrophone pressure also shows a larger acoustic pressure oscillation following the collapse. Except for the plug transducers, the pressure traces recorded by the right and left transducers are once again almost identical, indicating a symmetrical bubble behavior.

Similar dynamic behavior is exhibited by the interaction of bubbles of initially different sizes. It is not possible, however, to obtain an in-phase oscillation because the two bubbles have different time period of oscillation. On the other hand, the formation of a coalesced bubble by merger of two bubbles, have also been exhibited by bubbles of different sizes when the smaller bubble has been absorbed into the larger bubble. This kind of bubble dynamics is not feasible for large inter-bubble distance.

When the initial separation distance is sufficiently large, the bubbles start repelling each other for a nonzero time delay. The repulsion force increases with the delay between explosions, up to the point when they are oscillating 180° out-of-phase. In this case, when the predecessor bubble collapses, the successor bubble reaches its maximum radius. At this point, the pressure
field is abruptly reversed and this causes the formation of a strong reentrant water jet in the successor bubble. Figure 7 shows how this reentrant jet travels with time for a case of identical bubbles oscillating 180° out-of-phase with each other. Here, x denotes the location of the jet tip relative to the inertial frame (the tank) and x = 0 corresponds to the instant when the jet tip becomes visible for the first time. The formation of a strong reentrant water jet has also been observed for two bubbles of different sizes.

As the phase delay between two explosions increases beyond 180°, the repulsion force as well as the water jet velocity decrease in magnitude. If the phase delay between two explosions increases beyond 360° (i.e., if one bubble has already completed one oscillation cycle when the other bubble forms), the resulting interaction is very weak. In this case, even though the predecessor bubble manages to create a depression in successor bubble at its maximum radius, formation of a jet is not observed.

The horizontal configuration is very important from a practical standpoint, as it can set a catastrophic bending vibration to a nearby rigid body if tuned properly. The vertical configuration is also equally important, as it can dramatically enhance the impact pressure of a single bubble when collapsed near a rigid body. If two bubbles are placed close to each other along an axis perpendicular to a nearby rigid body, and if these two bubbles are tuned to oscillate 180° out-of-phase with each other, a water jet will be formed directed towards the rigid surface with a velocity which will be higher than that formed by the collapse of a single bubble under similar circumstances. The bubble dynamics in vertical configuration in the present study (in the absence of any nearby rigid body), however, do not show a remarkable difference from that of horizontal counterpart. This is because of the fact that only the orientation of the gravitational force changes between two configurations. In practical high explosive cases, the force field generated by the interaction between bubbles is much more stronger to be affected negligibly by the gravitational force.
Experiments were also carried out to study the interaction of a free surface with explosion bubble(s). A simple sting mount was used to support a glass globe and the depth of water in the tank was decreased in an increment of 2.54 cm. It was found that the bubble migration velocity smoothly changes its direction as well as its magnitude. The transition point determines the maximum inter-bubble distance for which the two identical bubbles will start interacting while oscillating 180° out-of-phase with each other. Figure 8 shows the transition of bubble migration velocity with water depth. The maximum center-to-center distance between two identical interacting bubbles (of approximate volume of 230 ml) oscillating 180° out-of-phase with each other was found to be $6.8R_0$, $R_0$ being the initial radius.

Both out-of-phase and in-phase oscillations of two explosion bubbles in a horizontal configuration near a free surface were also studied. The bubbles oscillating out-of-phase with each other repel each other and the effects of the free surface become apparent only after they have been repelled by each other. This once again indicates that the force field generated by the interaction between two bubbles is much stronger than that of a free surface and there have been instances where one of the bubbles actually migrates upward. Experiments were also conducted with two globes in a vertical configuration near a free surface. Out-of-phase oscillation was obtained and the consequences are not hard to imagine. Since the force field generated by the interaction between two bubbles is much stronger than that of a free surface, one bubble rapidly migrated downward and the other undoubtedly broke through the free surface venting its high pressure gases to the atmosphere.

**Conclusions**

Underwater explosion experiments were conducted in a water tank using flammable gases in glass globes to study the dynamics of interaction of two explosion bubbles in both horizontal and vertical configurations. The former configuration can excite a nearby submerged structure in
bending vibration mode, while the latter can easily be tailored for the directionality and enhancement of the impact pressure resulting from the collapse of an underwater explosion bubble near a solid boundary. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or they may violently repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. If a coalesced bubble is formed by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble in the direction of its migration. Water free surface repels the bubble and the transition point of bubble migration velocity determines the maximum inter-bubble distance required for the initiation of interaction between two identical out-of-phase pulsating bubbles. These observations are of considerable practical significance for buried mines detection in shallow water beaches, where the interaction of an explosion bubble with a solid boundary and water free surface is anticipated.

Acknowledgment

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Fig. 1.
Fig. 2

Top view of the test plug

Glass globe
Test Plug 3000 V DC
RTV coating
Pressure transducer
Pressure transducer protected by RTV coating
Gas introduction
Spark wires
\[ \frac{d}{R_0} = 2.32 \]

\[ \frac{d_1}{R_0} = 1.1; \quad \frac{d_2}{R_0} = 1.22 \]
Fig 5
Migration Velocity (cm/sec.) vs. $d/R_o$
APPENDIX C

Interaction of Two Underwater Explosion Bubbles
INTERACTION OF TWO UNDERWATER EXPLOSION BUBBLES

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ABSTRACT

Underwater explosion bubbles are created by detonating a store of oxygen and Carbon Monoxide or Hydrogen in glass vases submerged in a water tank. A cinematographic technique is employed to capture entire interaction process in both horizontal and vertical configurations. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either act each other to form a single coalesced bubble, or they may deflect each other. A violent interaction leads to an increased instability of the bubbles. When a coalesced bubble is produced by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive illusions. An out-of-phase oscillation generates a reentrant jet which pierces the bubble. Water free surface repels the bubble and the bubble migration speed and direction change as the explosion depth is continuously decreased.

INTRODUCTION

Much of the research activities in the area of underwater bubble dynamics has been focused on the behavior of cavitation bubbles. Cavitation bubble dynamics play a very important role in underwater acoustics and in predicting and preventing propeller and shaft blade damage. These bubbles, however, seldom occur singly. Actual cavitation fields contain several thousands of oscillating and translating microbubbles. Study of the behavior of a large number of bubbles thus becomes inevitable and experimental (Ahine and Sirian, 1985), numerical (Chahine and Liu, 1985; Chahine, 1991; Chahine and Duraiswami, 1992) and analytical (Vijgjarden, 1972) techniques have all been developed. The simplest model that has been studied by the researchers is the interaction between two bubbles. Theoretical and numerical studies of the interaction between two spherical or nonspherical bubbles of the same size (Fujikawa et al., 1985; Fujikawa and Takahira, 1985; Fujikawa and Takahira, 1988; Morrioka, 1974; Shima, 1971) have been carried out. Interesting experimental observations of the interaction of a gas bubble with a pressure wave (Shima and Tomita, 1988) or with a vapor bubble (Smith and Mesler, 1972) have also been made.

Large bubbles, such as those formed during underwater explosion, owing to their tremendous inherent destructive capabilities upon collapse near a rigid boundary, find practical applications in underwater weaponry. These bubbles were recently simulated experimentally on a laboratory scale in a free field configuration (Menon and Lal, 1996).

The interaction of two underwater explosion bubbles is a very interesting and complex phenomenon, because of the fact that one bubble is influenced by time-delayed pressure or shock wave radiated from the adjacent bubble. Radial motion of the bubble may be greatly excited or subdued due to the interaction depending on their spatial and temporal separations. Though some experimental work has been done on the interaction of gas bubbles with two adjacent underwater explosion bubbles, and it has been shown that strong and complicated interactions ensue, it appears that no detailed results on the interaction of two underwater explosion bubbles have been published in the public domain literature.

This paper reports the results of the experiments carried out in a laboratory water tank to study the interaction between two adjacent bubbles created by underwater explosion of flammable gas contained in glass globes. The globes were placed side-by-side either in a horizontal or a vertical plane. The distance between the two globes and their sizes were both varied. This paper will also deal with the interaction of single and double explosion bubble(s) with the water free surface.

EXPERIMENTAL PROCEDURES

Underwater explosion experiments were conducted in a wooden tank of dimension 2 m x 1.5 m x 1.5 m, coated with fiberglass resin from inside. The tank, as shown in Fig. 1, has three windows on three sides. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe. Three different sizes of glass globes have been used for present experiments with average radii of 2.9 cm, 3.2 cm, and 3.8 cm.
The glass globe, as shown in Fig. 2, has an electric spark detonation system connected to a 3000V DC power supply. The explosion takes place at a constant volume until the globe bursts. It has been shown (Menon and Lal, 1996) with the help of geometric and dynamic scaling rules that the explosion bubble formed is a reasonable subscale approximation of the deep-sea underwater explosion bubble.

The pressure responses in the water around the bubbles are recorded during the experiments by means of 4 KISTLER dynamic pressure transducers fitted at the ends of 4 stainless (1.27 cm diameter) tubes bent at a right angle, as shown in Fig. A hydrophone is also mounted in the tank and is used for measuring acoustic pressure. Pressure inside the bubble during its oscillation is measured by another KISTLER transducer which is placed inside the plug, as shown in Fig. 2. Signals from these pressure transducers and the hydrophone are digitized using a 16-channel AT-MIO-16X analog-to-digital converter, and are recorded into a microcomputer. Eight-channel data logging is performed with a sustained sampling rate of 10,000 frames per second per channel.

The tank is illuminated by either direct overhead flood lights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis. The optical recording of the bubble motion is performed by a CCD enhanced digital video camera with a maximum speed of 6000 frames per second. Since the image size is inversely proportional to the recording speed, the maximum speed was limited to 2000 frames per second as image size reduces to half at this speed. Most of the experiments, however, were performed at the speed of 1000 frames per second (full screen image).

The two glass globes are supported inside the tank by means of a modified sting which makes the pressure transducers mounted inside the two globes to face each other. This facilitates the direct measurement of the fluctuation in the pressure inside one bubble due to its interaction with the other. In order to provide a means for altering the distance between the two bubbles, six holes were drilled in the supporting copper pipe (1.6 cm diameter) of Fig. 1 at equal spatial separation from either ends. Experiments were conducted in primarily two configurations; a horizontal configuration, when the supporting pipe was horizontal, and a vertical configuration, when it was vertical. The former configuration prohibited the use of laser light sheet and only the flood lights were used for imaging, while the latter allowed the use of laser light sheet.

Experiments were also conducted to study the interaction of water free surface with the explosion bubble(s). The motivation for this kind of experiments has obvious reasons as detailed in the next section. The free surface provides a constant pressure boundary in close proximity of the oscillating bubble. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration (Birkhoff, 1957; Blake and Gibson, 1981; Chahine, 1982; Wilkerson, 1989). Wilkerson (1989) developed a boundary integral technique for the analysis of expansion and collapse of an explosion bubble in free field, near a rigid surface, or near a free surface. To verify the accuracy of his method for predicting reentrant water jet up velocities for a bubble near a free surface, he compared his results with a PISCES code calculation and expressed his inability to predict an accurate estimation of the error involved in his method because of the unavailability of any such experimental data. This observed lack of data motivated the current experiment to study the interaction of a free surface with an explosion bubble.
RESULTS AND DISCUSSION

The interaction process is highly dependent on the time delays between the two explosions. These time delays are related to a variety of hardware and bubble response characteristics. The two devices have independent power supplies and there is always a time delay between the occurrence of the spark and when the bubble arts expanding. The bubble expansion occurs immediately after gas globe bursts. The time delay depends primarily on the gas slimes (globe size) and the nature of gas mixture inside the globe. Bigger globe size will create a larger time delay. Similarly, fuel-rich or fuel-lean mixtures will also create larger delays as compared to the one associated with a stoichiometric mixture. In addition to this delay, there is another time delay which is associated with the arn system itself. This delay is between the instant when the arn is turned on and when the spark actually fires. This delay uniquely depends on the actual gap between the two spark wires as a bigger gap creates a larger delay. The actual delay (temporal separation) between two explosions is therefore measured from the recorded video images as the time elapsed between the instant when individual globe bursts. It was therefore deemed necessary to conduct several experiments to collect statistical information about a range of bubble behavior with respect to the delay between two explosions.

The entire spectrum of delay can be classified into three broad types and they are called in-phase oscillation and out-of-phase oscillation (Monoka, 1974; Shima, 1971; Smith and Mesler, '72). In most of the past analytical, numerical or even experimental work on the interaction of cavitating bubbles, stress has been focused primarily on the contraction phase of the bubble oscillation. This yields an in-phase oscillation of the bubbles as they both start collapsing at the same time following a sudden change in the ambient pressure. In-phase oscillation is aimed when there is strictly no delay between the explosions and the bubbles start their oscillation cycle at the same time. Of course, it is assumed that the two bubbles have identical time period oscillation and they are in identical phase at any instant throughout their oscillation cycle. This is the simplest scenario that has been analyzed comprehensively.

Another interesting scenario is $180^\circ$ out-of-phase oscillation. It can be best understood in the interaction of two identical injection bubbles as a case where one bubble starts its oscillation when the other has already reached its maximum radius. In this case, these are the two scenarios predicted by the analytical theories (Monoka, 1974). Monoka's (1974) theoretical analysis of natural phenomena of two pulsating bubbles predicts the existence of two natural frequencies corresponding to in-phase and $180^\circ$ out-of-phase oscillations, respectively. Of course, in an experimental plane one can have any amount of delay between zero to $180^\circ$ or beyond.

The behavior of explosion bubbles under these two scenarios has been long predicted (Birkhoff, 1957; Bjerknes, 1906; Cole, '74; Young, 1989) and it is called the laws of Bjerknes. Bjerknes proposed in 1868 that two spheres pulsating in-phase attract each other, and those pulsating $180^\circ$ out-of-phase repel each other. Two bubbles collapsing in-phase are equivalent to a single sphere collapsing near a rigid surface at a distance which is exactly equal to half of the distance between two spheres. Similarly, two bubbles oscillating $180^\circ$ out-of-phase are equivalent to a single bubble repulsing near a free surface at a distance equal to half of the distance between two bubbles. It has been shown (Birkhoff, 1957) that the migration speed of the bubble towards a rigid surface or from a free surface is inversely proportional to $r^2$, where $r$ is instantaneous bubble radius. Therefore, most of the migration would take place when the bubble radius is small (i.e., when the bubble is collapsing and approaching its minimum radius). Also, Bjerknes (Bjerknes, 1906; Young, 1989) showed as an analogy with gravitational and electromagnetic forces that the attractive force, $F$, between two bubbles of volumes $V_1$ and $V_2$ a distance $d$ apart in a pressure field is given by $F \propto \frac{V_1 V_2}{d^2}$. Since the bubble migration velocity is directly proportional to the attractive force, its magnitude is expected to increase for larger bubbles pulsating out-of-phase at a shorter inter-bubble distance, and decrease for smaller bubbles at a larger distance.

Fig. 3 shows a perfect example for two underwater explosion bubbles oscillating in-phase. The numbers in the parentheses denote the frame numbers, with frame number 1 corresponding to the instant when the sparks are visible for the first time. The initial volume of the right glass globe is 94 ml, and that of the left glass globe is 90.5 ml. They are in a horizontal configuration and are initially separated by a distance $d$, where $d/R_0 = 2.32$. Here $R_0$ is the initial bubble radius, which is taken to be the radius of the glass globe. Both are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. It is seen in Fig. 3 that there is virtually no delay between two explosions. Since the initial spatial separation between the globes was intentionally kept to a very small value so that a violent interaction can ensue, the bubbles soon come in contact with each other. They deviate more and more from sphericity as they expand with time. Fig. 4(a) shows how different radii of the left bubble change with time. A similar behavior is exhibited by the right bubble. The deviation from sphericity is more clearly demonstrated in Fig. 4(b), where the time history of the ratio of major and minor radius is plotted. As is evident from this figure, the bubbles quickly deviate from sphericity and the maximum deviation is attained at around frame no. 11. This deviation slowly diminishes and the bubbles become nearly spherical at around frame no. 21. When the bubbles merge together to form a single coalesced bubble, it becomes difficult to isolate them individually.

![Fig. 3. In-phase oscillation](image-url)

The surface where the two bubbles come in contact in Fig. 3 grows in size as the bubbles grow and is initially curved. The bubbles reach their maximum radii at around frame no. 18 and start collapsing thereafter. The surface of contact slowly becomes a plane surface around frame no. 21 and remains so thereafter until the bubbles collapse near frame no. 28. This plane surface of contact may be considered as a rigid surface in an equivalent single bubble.
The collapse is very violent (as recorded by two plug transducers) and the coalesced bubble quickly disintegrates into a d of bubbles. The bubble contour is traced and 360 bubble radii obtained at equal azimuth locations. A mean radius is obtained from this data, which is used to normalize the radii data. These data then Fourier analyzed and the results are shown in Fig. 5, which shows the power spectral density of the coalesced bubble’s interface over time. These data are taken from the time interval immediately after the bubble formation. The interface length is taken as the bubble circumference and λ is the wavelength. The coalesced bubble is therefore very unstable.

The pressure traces measured around the bubbles show that the late behavior is symmetrical. A pressure fluctuation of about 150 kPa exhibited by the plug transducers near bubble minimum constrains the severity of the collapse of the jets formed in the two bubbles onto each other. The pressure drops exponentially as one moves away from the bubble (Cole, 1948). A pressure drop of 70% (from 1000 kPa to 300 kPa) over a distance of 14 cm, and that of 80% (from 1000 kPa to 200 kPa) over a distance of 34 cm from the bubble center have been recorded by tank transducers.

Fig. 5. Power spectral density of the bubble.

A single coalesced bubble does not form only in an in-phase oscillation. Another situation where the formation of a single coalesced bubble has been observed repeatedly and most surprisingly, is associated with a nonzero time delay and a very short inter-bubble distance. This case is shown in Fig. 6. The initial volume of the right glass bubble is 125 ml and that of the left glass globe is 127 ml. They are almost touching each other such that the initial separation distance between them, d, is given by d/Ro = 2.1. Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. The right globe explodes first (around frame no. 11) and tries to encompass the left globe as the bubble grows. The left globe explodes near frame no. 19 and the shock wave emitted by this bubble travels through the right bubble as is evident by its protruding pieces (frames 21, 24). The right bubble, however, maintains its coherence and sphericity. It seems that the energy of the left bubble is substantially transferred to the right bubble. A force field is generated such that when the right bubble starts to collapse, the left bubble just merges into its predecessor to form a single coalesced bubble, which continues the oscillation cycle as a single explosion bubble. Since no jets are formed and the coalesced bubble is formed by merging the energies of the two bubbles, it has more residual energy than that of the previous example and does not disintegrate into smaller bubbles so quickly. In fact, it is even more stable than a single explosion bubble in free field.

The effect of the shock wave generated during the formation of the right bulb is reflected by a 150 kPa peak in the left plug transducer pressure trace. On the other hand, the expanding right bubble acts as a screen for the propagation of the shock wave generated during the formation of the left bubble, and this is reflected by a tiny fluctuation of 10 kPa in the right plug transducer pressure trace. It is very interesting to note that the explosion pressure for the left bubble (200 kPa) is only about 20% of what it would have been in a free field case. Thus, an expanding bubble inhibits the formation of another explosion bubble in its close proximity by reducing its explosion pressure. This is the reason...
by the left bubble does not have sufficient energy to expand to its maximum radius. But, it certainly aids its predecessor to form a saucered bubble with a greater energy to collapse violently and its fate is captured by all the transducers and hydrophone in the form of elevated collapse peaks. This time, the right plug transducer lies inside the coalesced bubble as it collapses (see Fig. frame no. 27). The collapse pressure recorded by this transducer is very high (2500 kPa). The pressure traces recorded by the right and left transducers are once again almost identical, indicating a symmetrical bubble behavior.

Fig. 6. Bubble formation by merging energies.

Similar dynamic behavior is exhibited by the interaction of bubbles formed by glass globes of initially different sizes. It is not visible, however, to obtain an in-phase oscillation because of a simple reason that the two bubbles have different time periods of oscillation. On the other hand, the bubble dynamics observed were the coalesced bubble is formed by merging the energies of two bubbles that have been exhibited by the bubbles of different sizes. The small bubble has been gobbled up by its predecessor, i.e., bubble. This kind of bubble dynamics is not feasible for large air-bubble distance).

When the inter-globe distance is sufficiently large, the bubbles start repelling each other for a nonzero time delay. The action force increases with the delay between the explosions, up to the point when they are oscillating 180° out-of-phase. In such a case, the formation of a strong reentrant water jet in the successor bubble is observed with increasing magnitude. Figure 7 shows how reentrant jet travels with time for a case of 180° out-of-phase condition. Here x denotes the location of the jet tip relative to the initial frame (the tank) and y = 0 corresponds to the instant when jet tip becomes visible for the first time. As the phase delay between two explosions increase beyond 180°, the repulsion force as well as the water jet velocity decrease in magnitude. If the phase delay between two explosions increase beyond 360° (i.e., if one bubble has already completed one oscillation cycle when the other bubble forms), the resulting interaction is very weak. In this case, the predecessor bubble manages to create a depression in a successor bubble at its maximum radius, neither a jet nor a jet formation is formed.

The horizontal configuration is very important from a practical standpoint, as it can set a catastrophic bending vibration of the nearby rigid body if tuned properly. The research effort is not pursued in this direction. The vertical configuration is also equally important, as it can dramatically enhance the impact pressure of a single bubble when collapsed near a rigid body. If two bubbles are placed close to each other along an axis perpendicular to a nearby rigid body, and if these two bubbles are tuned to oscillate 180° out-of-phase with each other, a water jet will be formed and directed towards the rigid surface with a velocity which will be higher than that formed by the collapse of a single bubble under similar circumstances. This is also being studied.

The bubble dynamics in vertical configuration in the present study (in the absence of any nearby rigid body), however, does not show a remarkable difference from that of horizontal counterpart. This is because of the fact that only the orientation of the gravitational force changes between two configurations. In practical cases, the force field generated by the bubbles is much more stronger to be affected negligibly by the gravitational force.

Fig. 7. Water jet tip location and velocity

**Bubble(s)-Free Surface Interaction**

Since it was shown earlier that two bubbles oscillating 180° out-of-phase are equivalent to a single bubble oscillating near a free surface, experiments were carried out to study the interaction of a free surface with explosion bubble(s). Figure 8 shows the transition of bubble migration velocity with water depth.

Fig. 8. Bubble migration velocity with water depth.
A simple sting mount was used to support a glass globe and the depth of water in the tank was decreased in a step of 2.54 cm. It was found that the bubble migration velocity smoothly changes its direction as well as its magnitude. The transition point determines the maximum inter-bubble distance for which two identical bubbles will start interacting while oscillating 180° out-of-phase with each other. It is found that \[ d = 3.4R_0. \]

Both out-of-phase and in-phase oscillation of two explosion bubbles in a horizontal configuration near a free surface were also studied. The bubbles oscillating out-of-phase with each other repel each other and the effect of the free surface becomes apparent only after they have been repelled by each other. This once again indicates that the force field generated by the interaction between two bubbles is much stronger than that of a free surface and there have been instances where one of the bubbles actually migrates inward.

**Conclusions**

This paper discusses results obtained in an experimental investigation of the dynamical interaction of two underwater explosion bubbles in both horizontal and vertical configurations. The former configuration can excite a nearly submerged structure in a normal mode, while the latter can easily be tailored for directional and enhancement of the impact pressure resulting from the collapse of an underwater explosion bubble near a solid boundary. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or may violently repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. If a coalesced bubble is formed, merging the energies of two bubbles, the resulting bubble has a greater residual energy and is more stable for successive oscillations. The out-of-phase oscillation generates a reentrant water jet which scoops the bubble. Water free surface repels the bubble and the repulsion of bubble migration velocity determines the minimum inter-bubble distance required for the initiation of interaction between two out-of-phase pulsating bubbles.

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**References**


APPENDIX D

Simulations of Underwater Explosion Bubble Dynamics using an Arbitrarily Lagrangian-Eulerian Formulation
SIMULATIONS OF UNDERWATER EXPLOSION BUBBLE DYNAMICS USING AN ARBITRARY LAGRANGIAN-EULERIAN FORMULATION

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ABSTRACT
The dynamics of bubbles formed during underwater explosions are numerically investigated using an Arbitrary Lagrangian-Eulerian three-dimensional finite-element code and compared with experimental data. Both experimental and numerical results show good qualitative and quantitative agreement and suggest that the excitation Rayleigh-Taylor instability is a major cause of interface instability. Migrations also have been carried out to investigate bubble-bubble interactions. Results show the formation of a water jet as one bubble slaps onto the other, in agreement with recent experimental observations. Finally, the collapse of a bubble near a rigid wall and the formation of high-velocity re-entrant jet onto the wall have been successfully simulated. The well-known vortex ring bubble during the collapse process has been numerically captured.

INTRODUCTION
Vapor and gas bubble dynamics are of great practical interest in prediction and prevention of cavitation erosion of marine propeller turbine blades. The destructive nature of strong underwater explosions near walls is well known. Detailed reviews (e.g., Blake i Gibson, 1987; Prosperetti, 1982) have summarized experimental and numerical results. Experimental studies are too many to list completely; however, most past studies focused on cavitation (all) bubbles. Among the studies that focused on large scale explosions are the studies reported in Cole (1948) for freely-illuminated, deep sea explosion bubbles and the studies of bubble assembly near walls (e.g., Tomita and Shima, 1986). Bubble-bubble interactions have also been studied in the past (e.g., Warren and Rice, 1942). However, in most cases, due to difficulties in acquiring field data, only limited information has been obtained. Recently, experiments were carried out to investigate large-scale bubble oscillations (Menon and Lal, 1996; Lal and Menon, 1996a, b). These experiments were conducted in shallow water due to an interest in understanding the dynamics of bubble-wall interaction in such flows to investigate the feasibility of targeting buried mines for destruction in beaches. The data obtained from these experiments have been used to validate the numerical model discussed in this paper.

Numerical studies in the past range from simple 1-D analytic solutions (e.g., Lauterborn, 1976; Piesset, 1971; Prosperetti, 1982) to more complex 2D/3D studies. Many studies employed the Boundary Element Method (BEM) or its variants (e.g., Chahine and Perdue, 1988; Duncan and Zhang, 1991; Blake et al., 1986; Piesset and Chapman, 1970). This method has some inherent limitations. For example, compressibility in the gas cannot be included and the study of bubble collapse near a surface. BEM can be used only up to the point of jet formation. To model the flow beyond the point of bubble collapse, BEM has been modified by introducing vortex elements (e.g., Zhang and Duncan, 1994; Zhang et al., 1993; Best and Kucera, 1992; Best, 1993). Furthermore, to set up this problem, recourse to experimental observation is required to obtain characteristic parameters. Such an approach is not general and cannot be used when the details of the explosion dynamics is unknown.

There are other assumptions used in past studies that are known to be in error. For example, significant compressibility effects are known to occur in the collapse phase especially in deep sea strong explosions. Bubble shape is also known to quickly deviate from sphericity at bubble maximum, thereby, violating axisymmetric assumptions used in the past (e.g., Szymczak et al., 1993; Zhang and Duncan, 1994) and requiring full 3D treatment. Thus, simple 1-D or 2-D/axisymmetric analysis or incompressible methods cannot completely resolve the bubble and the flow dynamics. Furthermore, such simplified treatments also do not to provide detailed of the flow field inside and outside the bubble and cannot account for the interaction between the vapor and the liquid phases. Conventional numerical treatments (even using full 3D) such as Lagrangian or Eulerian techniques are also not practical, since the expansion and collapse of bubbles create severe fluid motion so that a Lagrangian approach (in which the grid points move with the fluid resulting in severe grid distortion) becomes inappropriate, while an Eulerian approach, adequate resolution in the regions of interest is very difficult to achieve since the bubble's shape changes very rapidly.

A numerical method that includes both compressibility and an ability to capture the entire bubble collapse in complex configuration.
is used in this study. This numerical code (ALE3D) combines
lagrangian and eulerian features and is based on the Arbitrary
Lagrangian-Eulerian (ALE) scheme and is developed at the Lawrence
Livermore Lab. Past applications of this method include the 2D (e.g.,
Tipton, et al., 1992) and the full 3D (Milligan et al., 1995) studies of
bubble collapse. However, it appears that so far, the full capabilities
of the ALE3D have not yet been exploited to simulate and investigate
the dynamics of underwater explosions. This paper reports some recent
results of both single and double bubble explosions both in free field
and in the vicinity of a rigid wall.

2. THE NUMERICAL METHOD

ALE3D (Anderson et al., 1994) is an explicit, 3D finite element
code that simulates the fluid motion and elastic-plastic response on an
structured grid. The grid may consist of arbitrarily connected
eahedral shell and beam elements. The ALE algorithm is
plemented by carrying out a complete Lagrangian calculation
ollowed by an advection step. After each lagrangian step, a new mesh
created using a finite element based equilibration method to relax
the distorted grid. In the eulerian advection step, the fluid variables
ach as mass, density, energy, momentum and pressure are reevaluated
in the new mesh by allowing fluid motion. The details of the
stitutive models are described elsewhere (e.g., Steinberg, 1991)
d, therefore, are not described here for brevity.

The advection step uses a second order, monotonic advection
gorithm. This can create mixed material elements (i.e., liquid and
por). Material interfaces are not explicitly tracked but for the
pose of carrying out mixed element advection, they are inferred
om volume fractions. Separate state variables are kept for each
ponent of a mixed element.

RESULTS AND DISCUSSION

In this section, the results obtained for the various test cases are
mmarized and discussed. These studies serve to identify the
ibilities and limitations of the ALE3D code and to identify areas
for further study.

1. Free Field Single Bubble Oscillation

This simulation employed test conditions similar to the
perimental set-up of Menon and Lai (1996) and is modeled as a
ely oscillating bubble placed in the center of a 1.5 m x 1.5 m x 1.5
ank filled with water. The initial bubble diameter is 6.34 cm and
ial explosion pressure is 9.34 atmospheres. The water pressure
osophere. The ALE mesh treatment is applied to all the
ents in the bubble and in the vicinity of the bubble. But away
m the bubble where the bubble explosion does not cause much grid
rtion, lagrangian mesh treatment is used. The number of elements
d to resolve the bubble and the surrounding water was varied to
irm that the results are grid independent. For a typical 3D
ulation, 64512 elements were used to discretize the domain, but as
ny as 150000 elements were used for carrying out the grid
ependence tests for this test case. Although various cases have been
ulated, only characteristic results are discussed below.

The bubble grows after the explosion due to the high vapor
sure inside the bubble. Because of inertia, this results in an over
ation and the pressure inside the bubble falls below the ambient
ner pressure. As a result, the bubble collapses and reaches a bubble
imum at which time the internal pressure again exceeds the
eral pressure. Thus, a bubble oscillation process is set up and
continues as long as there is sufficient energy available. However,
ergy is continuously lost during the oscillation due to irreversible
ancial work done on water and vapor and due to the onset of
arious instabilities. Detailed analyses of the losses and the instability
achanisms were performed earlier (Menon and Lai, 1996) and results
uggest that during the collapse process the Rayleigh-Taylor (R-T)
stability occurs at the interface. This results in a distortion of the
apor-water interface. This phenomenon has been captured in the
mberical study. For example, Figs. 1a and 1b shows snapshots of
bble at the first maximum and the first minimum. As can be seen,
near the bubble minimum, wave-like distortion appears along the
ipop interface. Figures 2a-b show the corresponding velocity vector
id inside and outside the bubble. Figure 2a shows the outward
otion of the bubble just before the bubble maximum and Fig. 2b
ows the outward motion of the bubble just after the first bubble
imum. The magnitude of the velocity vectors also indicate that the
cceleration of the fluid is minimum at the beginning and end of
pression or expansion phases.

The deviation from sphericity and the formation of waves on the
bble interface are characteristics of Rayleigh-Taylor instability.
To ensure that this interface distortion is not due to acoustic reflections
rom the wall, calculations were carried out by moving the wall further
and by replacing the rectangular domain by a spherical domain.
Results showed that, although there are changes in the bubble
oscillation period, the interface distortion appears near the bubble
imum in all cases. The R-T instability can also be inferred by
alyzing the variation of the radius with time. For example, Fig. 3
ows the region (near bubble minimum) where $d^2R/dt^2 > 0$ (which
is a necessary condition for R-T instability).

Figure 4 compares the pressure history during the first oscillation
in the bubble with experimental data (Menon and Lai, 1996). It can be
en that the computed first period of oscillation (around 15 ms), the
peak pressure (at first bubble maximum) and the maximum radius (at
bubble maximum) agrees well with data. The differences during the
contraction phase may be due to differences between the two setups.
For example, the experimental set-up employed a glass globe (which
 contained the stoichiometric fuel-air mixture) with a metal insert that
 contained the pressure transducer and the spark generator. While these
features were ignored in the numerical model. In addition, the effect
of glass fragments (during the explosion) have not been included in the
numerical model.

Figure 5a shows the time trace of pressure in the tank away from
the bubble and close to a wall. It is very similar to the high frequency
pressure oscillations as recorded by the tank transducer in the
experiments (Menon and Lai, 1996) and is shown in Fig. 5b. The
light differences in the two plots may be attributed to the idealization
of the tank as a cube with walls all around (whereas, for the
experiment, the top surface was a free surface; see below).

As mentioned earlier, simulations were carried out to ensure grid
independence, and to confirm that the presence of walls do not effect
the overall dynamics. It has been determined that the presence of walls
does effect (decrease) the oscillation period; however, the bubble
dynamics are captured relatively accurately. To simulate true free field
explosion requires using outflow boundary conditions on the far wall
or by moving the walls far enough to minimize the impact of the
acoustic reflection. However, at present, the ALE3D code requires
that the far field boundary be modeled as a solid reflecting wall. This
limination of the code can be removed only by modifying the source
code. This is will be investigated in the future.

To extend the applicability of ALE3D to real underwater
explosions is quite trivial. To demonstrate this capability a deep sea
underwater explosion was simulated using pentolite as the explosive.
The time period (not shown) scales as approximately two times the non-dimensional time based on the maximum radius of the bubble, the ambient (water) pressure and the water density \( \left( \frac{R_m}{\sqrt{\frac{P_m}{\rho_w}}} \right) \), as found in the above simulations and earlier studies (e.g. Chahine and Perdue, 1988).

3.2 Bubble-Bubble Interactions

To investigate bubble-bubble interactions, a series of studies were carried out using bubbles of various sizes. A limitation of the current ALE3D code is that it does not allow phase difference between the two explosions to be incorporated into the model. However, by using different bubble sizes (or using different explosive strength) the net effective energy release from each bubble can be varied. The effect of inter-bubble distance on the interaction process was also studied. Due to space limitation only characteristic results are discussed below.

When two identical bubbles (of initial radii 3.17 cm and placed 8 cm apart) are exploded the bubbles expand and then collapse onto each other and a reentrant water jet with a high speed (30 m/s) is formed in both vertical and horizontal directions. Figures 6a-c show snapshots of the bubble-bubble interaction, and the corresponding velocity fields are shown in Figs. 7a-c, respectively. Fig. 8a-c show photographs from the experiments (Lal and Menon, 1996a) for the present case with two bubbles of same size exploding in phase with each other. Although not clearly seen in the experimental Fig. 8c, studies have show the presence of vortex ring bubble. The jet directed towards the adjacent bubble impinges on its counterpart as in a stagnation point flow. As the bubble-bubble process continues, two outer vortex rings are formed with the velocity between the bubble increasing to as high as 50 m/s. There is reasonable agreement between the experimental observations and the present computations.

When same size bubbles were exploded at the same distance as above, but with one bubble containing four time more energy than the other, a similar result was obtained except that in this case, the weaker bubble is sucked into the other bubble with a velocity reaching a maximum of around 85 m/s (not shown). The reentrant water jet is first formed in the weaker bubble during the first oscillation and the vortex ring formed merges into the (still coherent) stronger bubble. The bubble formation in the stronger bubble is delayed until the second oscillation, at which time the second bubble also collapses.

When two bubbles of different size (e.g., of radii 3.17 cm and 17 cm (and thus, with different total explosion energy) are exploded, the results are quite similar to the case discussed above. During the explosion phase, the greater inertia and explosion strength of the larger bubble inhibits the smaller bubble. During the collapse, the pressure drop in-between the bubbles is more than on the other side of the larger bubble. This pressure differential causes the smaller bubble to be engulfed by the larger bubble. The center of motion of the water jet directed towards the bubbles does not immediately adjust to the motion of the bubbles and, thus, the water motion is directed off-center of the bubble. This creates a very high pressure on the side of the smaller bubble, away from the larger bubble. This high pressure and the low pressure in-between the bubbles creates enough momentum to form a jet through the bubbles which penetrates to the other side of the bubble. Final stage of the jet formation is shown in Figs. 9a and 9c.

The velocity vector field shows the formation of the water jet in agreement with experimental studies (Lal and Menon, 1996a). A water bubble was also observed in the experiments when two identical bubbles were exploded out-of-phase, as shown in Fig. 9b. Out-of-phase explosion essentially changes the relative strength of the bubble explosion during interaction and is, therefore, similar to the present case with two unequal bubbles exploding simultaneously and the similarities can be seen in Figs. 9a-c. However, as mentioned earlier, the current ALE3D code cannot simulate phase difference between the adjacent explosions. This feature will be included in the code at a later date.

Finally, Fig. 10 compares the pressure between the two bubbles for the various test cases. All cases have the same period of oscillation. However, the case with increased energy content shows the strongest water jet formation (around 85 m/s) and the largest impact pressure at the first bubble minimum.

3.3 Bubble-Wall Interactions

Bubble collapse near a rigid wall is of significant interest due to a variety of reasons related to its ability to cause serious damage to the structure. This is because when the bubble collapses near a rigid surface, a strong reentrant water jet is formed that is directed towards the wall. The peak impact pressure on the wall due to this water jet can be substantially higher that the explosion pressure especially when the initial explosion energy is very large. Various simulations were performed by varying the explosion strength and distance of the bubble from the rigid plate. However, only characteristic results are discussed here to highlight the pertinent observations.

Two cases are discussed here with bubble placed 5 cm above (buoyancy inhibiting jet formation) and 5 cm below (buoyancy aiding the jet formation) the wall. Figures 11a-d show the velocity field at various stages of the collapse for the first case. Initially, the bubble is almost spherical but begins to distort as it collapses. The physics of the jet formation is quite similar to the bubble-bubble case. Since there is less volume of water between the wall and the bubble during the collapse, the pressure drop is quite large relative to the pressure on other sides of the bubble. This pressure differential further forces the bubble towards the wall. Since steam is lighter, the bubble tends to move further away from the wall (due to buoyancy) for the case where the gravitational force is inhibiting jet formation, while for the second case, the bubble is further accelerated towards the wall (Fig. 12). The water surrounding the bubble is directed off-center relative to the bubble geometric center, thereby, creating a higher pressure on the side of the bubble away from the wall. The combination of these effects causes the water to penetrate the bubble from the high pressure side and to form a high-speed water jet that impacts the rigid surface. As the jet impacts the rigid plate, a ring bubble vortex is formed as shown in the figures. The maximum jet velocity obtained is around 40 m/s. It scales as approximately 11 times the non-dimensional velocity scale based on the ambient (water) pressure and water density \( \left( \frac{P_m}{\rho_w} \right) \) and this scaling is in excellent agreement with earlier results (e.g. Chahine and Perdue, 1988).

The effect of buoyancy in the formation of jet is very evident in fig. 13 where the impact pressure on the wall is plotted versus time. The buoyancy aided case almost doubles the impact pressure than for the buoyancy inhibited case and is as much as two-and-half time that of the peak explosion pressure. For the buoyancy inhibited case (as in the experiments, Lal and Menon, 1996b).

The present study was able to capture the vortex ring bubble as shown above. This vortex ring bubbles after the jet impact has been also observed both in experiments (e.g., Tomita and Shima, 1986; Vogel, et al., 1989) and in numerical studies (Best, 1993; Szymczak, et al., 1993; Zhang and Duncan 1994).
4. CONCLUSIONS

These studies show that the ALE3D code can be used for bubble explosions. The basic code has been validated using shallow water explosion data. In addition to isolated bubbles, bubble-bubble and bubble-wall interaction studies were also performed. It has been shown that all the features observed in past experiments have been captured in these studies. The formation of reentrant waterjet as the bubble collapses near a rigid surface and the formation of ring vortex bubble have been captured in the simulation. These features are in good agreement with experimental data.

Some limitations of the current ALE3D code have also been identified in these studies. For example, the current code is unable to simulate bubble-bubble interaction with a phase difference between the explosions. However, these features can be incorporated by proper modifications to the code. These issues are still under investigation.

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REFERENCES


Figure 1: Freely Oscillating bubble at bubble maximum and bubble minimum.

Figure 2: Vector fields around the maximum and minimum of a freely oscillating bubble.

Figure 3: $\frac{d^2 R}{dt^2}$ versus time plot to identify the regions of R-T stable and unstable regions.

Figure 4: Time trace of pressure in the freely oscillating bubble compared with the experiments.
Figure 4: The acoustic pressure signature in the tank away from the bubble and near the walls. a) Numerical Simulation and experiments (Menon and Lai, 1996).

Figure 6: Time sequence of two bubbles of same size interacting.
   a) At bubble maximum, b) Just before the jet formation and c) Formation of toroidal double ring bubble.

Figure 7: Velocity vectors for the cases discussed in fig. 6.
Figure 8: Snapshots of the bubbles in the expt. (Lal and Menon, 1996) for in-phase explosions of same size. a) Corresponds to bubble maximum, b) During Collapse & c) During rebound.

Figure 9: ALE3D Bubble Shape(a), corresponding expt. (Lal and Menon) snapshot for out of phase explosion(b) and corresponding velocity field(c) at the time of jet formation for the bubbles of different sizes interacting.

Figure 10: Time trace of the pressure in between the bubbles for the different cases of double bubble interactions.
Figure 11: Velocity field around a bubble collapsing near a wall (Buoyancy Inhibiting). a) At bubble maximum, b) & c) Just before the jet formation and d) After the toroidal bubble is formed during rebound.

Figure 12: Velocity field around the bubble collapsing near a rigid wall with buoyancy aiding the collapse.

Figure 13. Impact pressure on the wall for both the buoyancy aiding and inhibiting cases.
Research in Underwater Explosions

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Research in Underwater Explosions

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1. Introduction

Underwater detonation of explosive material converts the unstable material into a more stable gas void at high temperature and pressure. The high pressure of the remnants of an underwater detonation sets forth an expansion-collapse cycle of the resulting underwater explosion bubble which is repeated several times before the bubble goes through interface instabilities and eventually disintegrates into a cloud of smaller bubbles. The interface instability problem is an interesting and complex subject and has recently been addressed based on experimental and analytical methods by Menon and Lal (1998). Various instability mechanisms at play during the bubble oscillation cycles were addressed and it was shown that the Rayleigh-Taylor instability occurs during the bubble collapse and plays a major role in the eventual collapse of the bubble.

The presence of a solid surface in the vicinity of a pulsating bubble manifests itself as an asymmetry in the flow field. A dominant feature in the collapse of a bubble in such a flow is the development of a reentrant water jet. The asymmetry in the flow causes one side of the bubble to accelerate inward more rapidly than the opposite side resulting in a high-speed reentrant jet which pierces the bubble in the direction of its migration and produces an impact pressure much larger than the explosion pressure. This increased pressure on the surface can cause structural damage especially when the explosion energy (and hence the bubble size) is large. Other asymmetries (i.e., gravity or a free surface) can also cause the formation of the reentrant jet. The jets caused by gravity are directed upward and those caused by free surfaces are directed away from them.

This report summarizes the pertinent results obtained under this research program. Most of the details of this effort are given in the Appendices attached to this summary report and therefore, avoided here for brevity. In the following, a brief summary of the experimental and numerical efforts is given.

2. Experimental Studies

A series of experiments were carried out to investigate underwater explosions in shallow water (1 atmosphere ambient pressure) to understand the dynamics of bubble-wall interaction in such flows and to investigate feasibility of targeting and destroying mines buried in beaches. In this configuration (shown in Fig. 1), the free water surface is close enough to the bubble-wall
interaction region to allow it to play a role in modifying the dynamics of the bubble collapse. The free surface provides a constant pressure boundary in close proximity to the wall. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration. Since, both the Bjerknes force and the buoyancy force, the two competitive forces acting on a bubble near a free surface, act in the same direction, the presence of the free surface above the bubble collapse region is likely to increase the net impact pressure on the wall. Another issue that was investigated is the behavior of the impact process when the rigid surface is buried below a layer of sand, as would be the case for buried mines. Some interesting results have been obtained and summarized in the papers in the Appendices.

Underwater explosion experiments near a solid boundary were conducted in a wooden tank of dimension 2 m × 1.5 m × 1.5 m, coated with fiberglass resin from inside. The tank has windows on three sides for optical imaging. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe over a steel plate of dimension 36.83 cm × 60.96 cm × 0.635 cm (shown in Fig. 2). Two different sizes of glass globes were used for present experiments with average radii of 2.54 cm and 3.2 cm. The glass globe has an electric spark ignition system connected to a 3000V DC power supply that ignites the premixed fuel-air stoichiometric mixture contained in the globe. The explosion takes place at a constant volume until the globe bursts. Since the experiments were conducted in a laboratory shallow water setup and using a gaseous explosive mixture, the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep-sea explosions. Recently, Menon and Lal (1998) addressed the dynamics and instability issues of such a bubble in free field and they showed by means of extensive geometric and dynamic similarity analyses that the explosion bubble thus formed is a reasonable subscale approximation of a deep sea underwater explosion bubble. They have presented detailed scaling parameters, energy partitioning and also various interface instability mechanisms. Repeatability and experimental uncertainty have also been addressed and it has been shown in particular that repeated experiments produced error bands for the explosion pressure, maximum radius and time period of 5.88%, 3.7% and 6.06%, respectively.

The pressure inside the bubble during its oscillation was measured by a KISTLER transducer that is mounted inside the plug. Additionally, eight KISTLER pressure transducers were mounted on the plate as shown in Fig. 2 to obtain a surface distribution of the impact pressure field. These dynamic pressure transducers have low and high frequency response of 0.001 Hz and 50 kHz, respectively, and the resonant frequency of 300 kHz. They are, therefore, well suited for the current experiments as the bubble oscillation frequency (time period of approximately 15 ms) lies well within the above mentioned bounds. Signals from these pressure transducers were digitized using National Instrument’s AT-MIO-16X analog-to-digital converter board, and were recorded into a microcomputer. Ten-channel data recording was performed with a sustained sampling rate of 10,000 samples per second per channel.

The distance between the globe and the plate was varied to investigate the effect of solid wall location relative to the explosion. The plate was later covered with sand or clay to simulate explosion over a buried wall. The thickness layer of sand and clay above the instrumented plate was also varied to determine how the porous material above the plate modifies the impact pressure. The water surface was lowered to study shallow water bubble-wall, bubble-sand-wall and bubble-clay-wall interactions. Either direct overhead floodlights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis illuminated the tank. The optical recording of the bubble motion was performed by a CCD enhanced digital video camera at a speed of 1000 frames per second in order to obtain a full screen image.
Underwater explosion bubbles are created near an exposed or buried rigid boundary by detonating a mixture of oxygen and Carbon Monoxide in glass globes submerged in a water tank. A variable depth of either play sand or general purpose purge clay is used to bury a solid steel plate in order to simulate explosion over a buried rigid boundary. Eight pressure transducers mounted on the plate are used to map the pressure distribution on the plate and instrumented tubes and plugs measure pressure inside and outside the bubbles. A cinematography technique is employed to capture entire interaction process. There exists a critical distance above the plate where the reentrant water jet produces the maximum impact pressure on the plate. The jets formed by the explosions above this distance have to pierce the water layer between the bubble and the plate and hence yield lower impact pressures. The growth of bubbles formed by explosions below the critical distance is inhibited by the presence of the plate and hence their maximum sizes are comparatively restrained. The water jet is very focused and symmetrical about the center of impact. The effect of covering the flat plate with sand or clay is in general, to reduce the impact pressure and to smooth its distribution over the plate. However, when an explosion occurs very close to the sand surface loose sand particles are ejected and displaced as the bubble expands. This reduces the effective sand thickness and as a result, an increased impact pressure is achieved. This recovery of impact pressure increases in shallow water cases due to the free surface effect. Explosions were also carried out above clay surface to view the shape of the crater formed. Results show that double craters (i.e., secondary crater within the primary crater is formed for certain initial locations of the explosive above the surface).

3. Numerical Studies

A series of numerical experiments were performed using a 3D finite element code called ALE3D that was obtained from Lawrence Livermore Laboratory. Results of these studies are given in the Appendices attached to this report. Here, we briefly summarize the pertinent results.

3.1 Bubble-Wall Interactions

An unsteady, 3D finite-element compressible code (ALE3D) has been successfully applied to study underwater explosions. Results clearly demonstrate that the ALE3D code can be used for bubble explosions. The basic code has been validated using shallow water explosion data. It has been shown that nearly all the features observed in experiments have been captured in these studies. There is excellent qualitative and reasonable quantitative agreement with the experimental data.

Results show that during collapse of a freely oscillating bubble, the bubble loses spherical symmetry and the bubble interface becomes unstable due to the excitation of Rayleigh-Taylor instability. Stability analysis confirms that this instability can occur and energy partition analysis indicates that there is a reasonable amount of energy missing that could be used to excite this instability. This verifies the earlier experimental demonstration that R-T instability is one of the primary mechanisms in bubble collapse and breakdown. The simulation of the collapse of a bubble near a rigid wall showed that the jet velocity and the impact pressure on the wall are functions of the explosion pressure and the distance of the bubble from the wall. The results indicate that for a given explosion pressure there is an optimal distance of the bubble from the wall for which we obtain maximum impact pressure. This trend and the peak impact pressure are in good agreement with the experimental results. It has been shown that the optimal location is due to two different physical effects as the bubble collapses near the wall. The evolution of the
vortex ring bubble, reported in earlier experimental and numerical studies, is also accurately predicted.

Some limitations of the current ALE3D code have also been identified. However, most of these limitations can be corrected by proper modifications to the code. Current effort is directed towards this goal so that more realistic (i.e., using real explosives) deep and shallow water explosion studies can be carried out. Extension to the code to handle sand surface properties is also being investigated for eventual study of explosions near buried surface.

3.2 Shaped Charge Explosions

The dynamics of bubbles formed during underwater explosions is numerically investigated using an Arbitrary Lagrangian-Eulerian, three-dimensional finite-element code. The collapse of bubbles formed by spherical, cylindrical and shaped charge explosion near a rigid wall has been simulated. It is shown here that the impact signature (the pressure footprint on a rigid surface) is a direct function of the explosive shape and explosion energy distribution. Shape modifications are studied here to understand the correlation between the explosive properties (shape, size and, energy/volume) and the impact process. Analysis of the simulations demonstrate that the final stages of the collapse, including the formation of a vortex ring bubble and a high velocity re-entrant jet, are successfully captured for all the simulated cases. The jet velocity and the impact pressure on the wall are functions of the explosion pressure and the distance from the wall. The results indicate that, for a given explosion energy, there is an optimal distance above (or below) the wall which results in the maximum impact pressure on the wall. For spherical explosions, this trend and the magnitude of the peak impact pressure are in good agreement with the experimental results. Spherical explosions result in the maximum peak impact pressure on the wall when compared to other shaped charges (for the same initial energy density and location) and is due to the highly focused impinging jet formed when the spherical bubble collapses. For other shaped explosions, the peak impact pressure is lower but proper shaping of the initial explosive increases the impact footprint. Simulations indicate that it is possible to correlate the initial explosive shape to the impact pressure and footprint size. These results suggest that by properly shaping the explosive charge it will be possible to increase the impact area, thereby, controlling the effect of explosion bubble collapse near a rigid surface.

3.3 Detonation Cord Explosions

Simulations of underwater detonation cord mesh explosion have been carried out using a three-dimensional arbitrary Lagrangian-Eulerian finite-element code. Earlier, this code was successfully employed to capture both qualitatively and quantitatively the dynamics of underwater explosions near rigid surfaces. In this study, the code was used to study the effect of detonation cord explosions on a stainless steel rod placed vertically within the mesh. This metal rod mimicked the trigger arm of a buried mine. The explosion strength was chosen to be larger than the yield strength of the metal rod. It was shown that when the metal rod is placed directly in the center of the mesh, the explosion bubble collapse causes a very high pressure along the diagonal axes and results in the rod getting squeezed and lengthened. On the other hand, when the rod was placed away from the mesh center, asymmetric forces are generated so that in all cases, fracture of the rod into multiple pieces occurs. These results demonstrate that to ensure repeatable destruction of a buried mine, asymmetric design of detonation cord (by changing energy density and/or geometry) is desired. Further calculations are planned to determine if an optimum design of the detonation mesh exists.
4. Conclusions

This report gives all the recent results obtained under this project. Both numerical and experimental studies were carried out to address the same physical problem. The problem of interest is the dynamics of gas bubble explosions underwater and its subsequent interaction with rigid and porous walls (i.e., rigid wall buried under sand). Of particular emphasis was the investigation of an earlier (previous year) observation that even when the rigid wall is not directly exposed to the explosion, it still feels a strong impact pressure. In terms of direct payoff this should be of considerable interest to NAVY since this maybe a means to target and explode buried mines on beaches without direct human involvement. Our experiments were performed in shallow water configurations so that the effects of both the wall and the free surface (as in natural beachfront) are simulated. Results described in the attached Appendices clearly demonstrate the key results of this study.

Our numerical studies developed the ALE3D code (from Lawrence Livermore Lab) to study these explosions. We have shown that the code is capable of capturing all the necessary features of the bubble-wall interaction and has been validated against the experimental data. This code is different from the codes currently being employed by NSWC (Indian Head) but has the same capabilities (actually more since it can also be used to investigate structural failure and crack propagation).

Publications under this Grant (ordered as in the Appendix)


On the dynamics and instability of bubbles formed during underwater explosions

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Abstract

Dynamics of explosion bubbles formed during underwater detonations are studied experimentally by exploding fuel (hydrogen and/or carbon monoxide–oxygen mixture in a laboratory water tank. Sub-scale explosions are instrumented to provide detailed histories of bubble shape and pressure. Using geometric and dynamic scaling analyses it has been shown that these sub-scale bubbles are reasonable approximations of bubbles formed during deep sea underwater explosions. The explosion bubble undergoes pulsation and loses energy in each oscillation cycle. The observed energy loss, which cannot be fully explained by acoustic losses, is shown here to be partly due to the excitation of instability at the interface between the gaseous bubble and the surrounding water. Various possible mechanisms for the dissipation of bubble energy are addressed. The analysis of the experimental data gives quantitative evidence (confirmed by recent numerical studies) that the Rayleigh–Taylor instability is excited near the bubble minimum. The dynamics of the bubble oscillation observed in these experiments are in good agreement with experimental data obtained from deep sea explosions © 1998 Elsevier Science Inc. All rights reserved.

Keywords: Underwater explosion; Bubble dynamics; Bubble instability; Fluid dynamics; Interface instabilities; Explosion bubble; Rayleigh–Taylor instabilities

1. Introduction

The dynamics of pulsating bubbles of hot gases at high temperature and pressures formed during an underwater explosion of solid explosives are still not very well understood. During an underwater explosion, a significant portion of the explosion energy is carried away by the detonation shock wave and is imparted into the surrounding water. The gas bubble left behind rapidly expands due to high internal pressure generated during the explosion. However, due to the high inertia of water, the bubble overexpands and its internal pressure drops below the pressure of the surrounding water. As a result, the bubble, once reaching the maximum radius, collapses and eventually reaches a bubble minimum. The collapse process is then reversed and the bubble expands again when the internal pressure exceeds the water pressure. Depending upon the remnant energy available, the bubble can undergo multiple oscillations that are highly damped and exhibit a nearly cuspid behavior at the minimum size. The bubble pulsation is not indefinite since the thermodynamic internal energy available for the oscillation continuously decreases during the oscillation process till eventually the bubble loses its coherence and breaks up into a cloud of smaller bubbles that quickly disperse.

Although some sources of the observed energy loss have been identified in the past (for example, the radiation of shock wave and acoustic energy to the surrounding medium, the cooling of the hot bubble, mass loss and compressibility effects that occur near the bubble minimum) the bubble collapse process cannot be fully explained by these mechanisms [1–4]. The deviation of the bubble shape from sphericity [5] and various interface instability mechanisms (for example, the Kelvin–Helmholtz shear instability [6,7], the Rayleigh–Taylor [7,8] and Birkhoff [9,10] instabilities and Landau–Darrieus evaporative instability [11,12]) have all been proposed [13] to account for additional energy loss. However, no clear quantifiable evidence has been obtained so far, and it is still not clear how all these instabilities (if at all) interact and contribute to bubble collapse.
Detailed measurements and imaging of pulsating bubbles formed during deep sea explosions are very difficult due to a variety of obvious reasons [14] and, therefore, there is insufficient data available to analyze the bubble collapse process. Controlled experiments, such as the ones described here, can be used to investigate the physical processes that contribute to the bubble instability. Since the experiments were conducted in a laboratory shallow water setup and using a gaseous explosive mixture (with significantly lower energy release), the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep sea explosions. Therefore, the results have to be justified for their relevance to large scale underwater explosions. Using both geometric and dynamic scaling analyses (e.g., [13]), it is shown that the present experiments reasonably simulate deep sea bubble dynamics. Also, it has been shown earlier [15] that the explosion temperature is quite comparable to deep sea detonation temperature. It will be demonstrated here that although the explosion pressure is much lower when compared to deep sea detonations, the scaling parameters for the subsequent bubble pulsations have a relatively weak dependence on the explosion pressure.

2. Experimental procedure and typical results

Underwater explosion experiments were conducted in a water tank of dimension $2m \times 1.5m \times 1.5m$. The tank, shown in Fig. 1, has windows on three sides for optical imaging. The explosion bubble is generated by centrally igniting a mixture of explosive gases (e.g., Hydrogen and/or Carbon monoxide), oxidizer (Oxygen), and sometimes inert gases (Helium and Argon) in a hand-blown glass globe. The glass globe, which is shown in Fig. 2, weighs about 5–6 g and has an average diameter of 6.34 cm. Thus, the initial explosion source is of the order of the globe diameter. The gaseous explosive mixture is ignited in the glass globe by using an electric spark generated by spark wires placed inside the glass globe and connected to a 3000 volt DC power supply. The explosion takes place as in the constant volume combustion bomb of Flock et al. [16], until the globe breaks. Because of the water inertia, the combustion process is essentially completed at constant volume.

To simulate free-field explosions, the glass globe is supported around 0.65 m below the water free surface by two thin steel wires (of diameter 1 mm) that span the tank width between two windows (Fig. 1) and attached to the walls by means of four suction cups. The method used to support the glass globe allows one to mimic free-field explosions. However, due to buoyancy effects the bubble will (and does) migrate upwards. However, this migration process is noticeable only after the completion of the first oscillation and is similar to that observed in deep sea explosions [1]. Since, as shown below, the bubble interface instability is initiated during the first oscillation, it is assumed that the physics controlling the bubble instability is only weakly affected by buoyancy. The effect of buoyancy may become important during the later stages of the bubble oscillation. As the bubble begins to migrate upwards, some of the bubble energy could be lost to the vertical motion of the water and to the initiation of shear induced instabili-
ities (i.e., Kelvin-Helmholtz) which could contribute to the eventual collapse of the bubble. These issues are discussed further in Section 4.

The pressure response (due to the propagation of acoustic waves) in the water surrounding the bubble is recorded during these experiments. To record the acoustic wave reflections from the walls, a hydrophone (Bruel & Kjaer Model No. 8103), with a frequency range of 0.1 Hz–180 kHz and a working pressure of up to 4 × 10^9 Pa, is installed in the tank close to the wall. This hydrophone is used primarily as an event marker and to record the strength of the wall-reflectenced acoustic waves. The pressure data in the surrounding water ahead of the oscillating bubble is obtained by using KISTLER dynamic piezoelectric pressure transducers as shown in Fig. 1. These transducers are fitted at the end of stainless steel (0.5 in. diameter) tubes which are bent at right angles so that the transducer faces towards the bulb. The range of these transducers is 0–35 bar which is sufficient for the present sub-scale experiments. These transducers have low frequency response of 0.001 Hz, high frequency response of 50 kHz and have the resonant frequency of 300 kHz. The bubble oscillation time period in the present experiments is nominally about 15 ms which yields a frequency lying well within the frequency bounds of the transducers. Analysis of the pressure signals from the transducers located at different positions (as shown in Fig. 1) showed that the pressure response ahead of the bubble is nearly independent of the location of the transducer and, therefore, most of the data presented here are from a single (the same) transducer.

To measure the pressure inside the bubble, the glass globe is equipped with another KISTLER dynamic pressure transducer (range 0–35 bar) coated slightly with RTV cement to protect it against the thermal loads generated upon ignition (see Fig. 2). The signals from the pressure transducers and the hydrophone are recorded using a PRESTON analog-to-digital converter and a HP 1000 A-700 computer at a sampling rate of 20,000 samples/s, per channel giving a temporal resolution of 50 μs. Data are continuously recorded until the bubble completely disintegrates.

To investigate the bubble instability process, direct imaging of the bubble during its pulsation is carried out using a high speed, CCD enhanced video camera (Kodak Ektapro EM Motion Analyzer Model No. 1012). To obtain the optical record, the tank is illuminated by either direct flood lights or an argon-ion laser sheet. Although the camera is capable of recording at a speed of 10,000 frames/s, images were obtained at 6000 and 4000 frames/s so that the full bubble can be viewed during its oscillation. Most of the images analyzed and reported in this study were obtained at 4000 frames/s. At this speed, the typical image size was 239 × 48 pixels and all the images were recorded in a video recorder using two speeds (30 frames/s and 5 frames/s) for image analysis.

Table 1 summarizes the various types of explosive mixtures studied in the present experiments. A variation in the fuel–air mixture changes not only the molecular weight (M) and the combustion product but also changes the explosion pressure (P0), the peak pressure at bubble minimum (P1) and \( \gamma \) (the ratio of specific heats), which depends on both the combustion product and explosion temperature. Since both carbon monoxide and hydrogen have similar heat release on molar basis (284 kJ/mol for CO and 242 kJ/mol for H2), the explosion temperature is nearly the same for different stoichiometric fuel–oxygen mixtures considered here. When an inert gas is mixed with the fuel–oxygen mixture, some of the explosion energy is used to raise the temperature of the inert gas to the explosion temperature. This results in a reduction of the peak temperature achieved during the explosion. However, as shown by Strahle and Liou [15], when the inert gas was added to change the molecular weight (i.e., from 12.98 to 36.33), only a slight variation in explosion temperature (a variation between 3038 and 3372 K) occurred. Therefore, the temperatures achieved in all these experiments are considered comparable (and also comparable to deep sea explosion temperature of 2900 K [5]). Furthermore, since the specific heats of combustion products (water vapor and CO2) have only a weak dependence on temperature [17], \( \gamma \) does not change appreciably in the temperature range of interest. Therefore, all values of \( \gamma \) in Table 1 have been computed at a reference temperature of 3200 K, which suffices for the purpose of qualitative study of the bubble dynamics for different gas mixtures. Values of \( P_0 \), \( P_1 \) and \( P_1/P_0 \) listed in Table 1 are those measured experimentally in this study and \( \phi \) denotes equivalence ratio of the fuel–oxygen mixture, which is defined as \( \phi = (F/O)/(F/O)_{stoichiometric} \). Here, \((F/O)_{stoichiometric}\) indicates fuel to oxygen ratio, and \( \phi = 1 \) indicates a stoichiometric mixture. \( \phi < 1 \) a fuel-lean mixture and \( \phi > 1 \) a fuel-rich mixture.

Analysis of the results summarized in Table 1 shows that the explosion and the first minimum pressures depend on the equivalence ratio of gas mixture. Stoichiometric mixtures without any inert gas produce the strongest explosion pressure \( P_1 \) and, also, have the largest peak pressure at the first bubble minimum \( P_1 \). The pressure \( P_1 \) is of the same order (in some cases larger) than the explosion pressure for these cases. Increasing the inert gas content or decreasing the equivalence ratio weakens the explosion and, hence, decreases the corresponding bubble pressure. Although all these cases were analyzed, only representative results are shown in this paper.

Fig. 3(a) shows the pressure trace recorded by the test plug transducer for two typical explosions. The signal shows the signature of the explosion (the first maximum in the pressure), followed by the second pressure maximum corresponding to the first bubble minimum that occurs approximately 15 ms after the first peak. The subsequent peaks approximately represent the pressure maxima achieved during the second and third bubble pulsations, respectively. These peaks are much lower suggesting a significant energy loss near the first bubble minimum. This loss mechanism is addressed in detail in
### Table 1

Bubble explosion test conditions

<table>
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<th>Group</th>
<th>No.</th>
<th>Gas mixture composition</th>
<th>$M$</th>
<th>$\phi$</th>
<th>$P_b$ (kPa)</th>
<th>$P_0/P_b$</th>
<th>$\gamma$</th>
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</table>

This paper. Fig. 3(b) and (c) show, respectively, the pressure signatures recorded by the transducer in the water outside the bubble and by the hydrophone near the tank wall. The higher frequency signals observed in Fig. 3(b) and (c) are indicative of the acoustic reflections from the walls of the tank. Since the frequency of this pressure fluctuation is higher and the amplitude significantly lower (by at least an order of magnitude when compared to the explosion pressure and the pressure at the first bubble minimum), it is expected that the acoustic reflections from the walls do not significantly contaminate the bubble instability process.

This issue was recently addressed using a full three-dimensional, numerical simulation of the experimental configuration [18]. Very similar high frequency pressure fluctuations at comparable amplitudes were observed in the calculations. The possibility of acoustic interference or excitation of interface instability (discussed below) was also investigated by moving the walls further out and by replacing the rectangular tank walls with a shell. Computed results showed excellent agreement with the experimental data for the bubble pressure, and analysis of the results showed that, although there are some subtle effects of acoustic reflection from the laboratory tank walls (such as a slight decrease in the first oscillation period), the overall dynamics of the bubble oscillation was qualitatively and quantitatively very similar. It was, therefore, concluded that acoustic reflections from the walls are not causing any significant modifications of the bubble oscillation.

Fig. 4 shows the typical variation of the bubble radius (nondimensionalized by the initial bubble size, $R_0$, which is assumed in the present experiments to be the glass globe size since the explosion is completed before the bubble begins to expand) as a function of time (nondimensionalized by a time scale $t_\tau = R_0/\sqrt{(P_0/P_\tau)}$, where, $P_0$ is the explosion pressure and $P_\tau$ is the density of water) for two typical explosions using stoichiometric mixtures of $H_2-O_2$ and $CO-O_2$. Consistent with the pressure signature seen in Fig. 3, Fig. 4 clearly shows how quickly the bubble weakens regardless of the fuel-oxygen mixture used. The magnitude of the maximum bubble radius, the pressure maximum at the first bubble minimum and the period of oscillation vary with the changes in the explosive mixture. However, the qualitative behavior of the rapid energy loss is quite similar and, therefore, Fig. 4 is representative of all the cases studied here.
As shown in Fig. 4, for both $\text{H}_2-\text{O}_2$ and $\text{CO}-\text{O}_2$ stoichiometric mixtures, the second bubble maximum is roughly 0.26 of the maximum achieved during the first oscillation (correcting for the initial bubble size which is unity in the non-dimensional coordinate used in Fig. 4). The decrease in the bubble maximum relative to the second bubble maximum is not that significant for the third pulsation. This suggests that most of the energy loss occurs either prior to or during the second bubble pulsation. The time period for pulsation also decreases with the second pulsation requiring around 33% of the time required to complete the first oscillation. The bubble radius variation observed in the present experiments is remarkably similar to that of deep sea explosions. For example, data from an explosion of 250 g of tetryl at a depth of 91 m [5] showed that the bubble radius maximum reached during the second pulsation was around 0.56 of maximum achieved during the first pulsation, and the time period for the second pulsation was 79% of the time required to complete the first pulsation. Data from TNT explosions [14] at greater depths (e.g., 152 m) also suggested a similar (0.61) reduction of second bubble maximum when compared to the first maximum. However, the deep sea data suggests that the losses are not as severe as observed in the laboratory. There could be many reasons for this discrepancy. For example, in the laboratory setup, the bulk holder (which contains the spark igniter and the pressure transducer) could be playing a role in damping the bubble oscillation, resulting in the observed reduction in the second bubble oscillation. In addition, much larger energy release occurs during TNT and pentolite explosions [1,3,4] which could result in relatively large amounts of energy left after the first oscillation. In general, however, this (qualitative) comparison suggests that the rapid decrease in the bubble size during multiple pulsations captured in the current experiments is reasonably similar to that observed in deep sea explosions.
To investigate bubble instability, the bubble images were analyzed. Typical bubble images near bubble minimum extracted from the video are shown in Fig. 5 for the stoichiometric $\text{H}_2-\text{O}_2$ explosion (as a typical example). To analyze these images, the video frames were digitized and the bubble interface was extracted by enhancing the contrast. To enhance the bubble edge, these images were purposely blurred. Then, the contour of the bubble was traced using a commercially available image processing software and converted into a binary data file. The digitized images (corresponding to Fig. 5(a) and (b)) are shown in Fig. 5(c) and (d), respectively. The bubble contours shown in Fig. 5(c) and (d) clearly suggest that the bubble interface is corrugated and that both small and large wavelength corrugations exist along the bubble interface.

There are various possibilities for the observed interface corrugation. One possibility is that the glass fragments (formed from the bulb during the initial explosion) are causing the observed corrugation. To determine this, a series of experiments were carried out by coating the glass with black paint and then visualizing the explosion using back lighting. Under this condition, the glass fragments became clearly identified. Fig. 6(a) and (b) show, respectively, typical images near the first bubble maximum and minimum. It appears that in all the experiments, the glass fragments are typically long thin fragments (approximately 2-3 mm in width and 20 mm in length) that move outwards with the bubble during the expansion phase. Thus, it is feasible that the corrugation seen on the interface near the bubble maximum is due to the presence of the glass fragments. However, it appears that the bubble collapse is much faster than the inward motion of the fragments and the glass fragments are lagging behind. Thus, it is likely that the large wavelength interface corrugations seen near the first bubble minimum are not directly due to the presence of the glass fragments in the surrounding water.

Another possibility for the interface corrugation near the bubble minimum is the excitation of interface instabilities. There are various forms of possible instabilities and these are discussed in more detail in Section 4. To
2.1. Experimental uncertainties

In order to determine the experimental uncertainties, several experiments were conducted under same experimental conditions for stoichiometric CO–O₂ explosion mixture and, initial radius, \( R_0 \), explosion pressure, \( P_0 \), maximum bubble radius, \( R_{max} \), and time period of oscillation, \( T \), were measured. Then, the quantities \( P_0/(4\pi R_0^3/3) \), \( R_{max}/R_0 \) and \( T/\tau \), were tabulated and their maximum, minimum and mean values were used to find error bands. The error bands for the three quantities mentioned above were found to be 5.88%, 3.7% and 6.06%, respectively.

3. Scaling analysis

In the present experiments, the explosion pressure is of the order of only 10 bar, while in a deep sea explosion it is of the order of 100 kbar [5,14]. In view of this discrepancy, the applicability of the present data for deep sea explosions must be examined. In this section, it will be shown that nearly all the geometric and the dynamic parameters required to ensure proper scaling are preserved between the current sub-scale experiments and deep sea explosion.

The noncompressive radial motion is the simplest approximation to the true motion of the gas bubble, where it is assumed that the motion of the surrounding water is entirely radial and there are no migration and buoyancy effects. This motion is expressed mathematically [5] as:

\[
\frac{d^2 r}{dt^2} + \frac{3}{2} \left( \frac{dr}{dt} \right)^2 = \frac{P_r - P_i}{\rho_i},
\]

where \( r \) is the instantaneous bubble radius, \( P_r \) is the pressure inside the bubble (assumed uniform throughout the bubble), \( P_i \) is the pressure in the water at the explosion depth, and \( \rho_i \) is the density of water. Eq. (1) has been extensively studied, for example, by Herring [19] and later by Trilling [20] who included acoustic energy losses and modified Eq. (1) for bubble motion with spherical symmetry to obtain the following equation (often called the Trilling–Herring equation):

\[
\left( 1 - 2C \frac{dz}{dr} \right) \frac{d^2 z}{dr^2} + \frac{3}{2} \left( 1 - \frac{4}{3} \frac{dz}{dr} \right) \left( \frac{dz}{dr} \right)^2 = \frac{P^* - \tilde{P}}{z} + C \frac{dP^*}{dr}.
\]

Here \( z \) is the normalized bubble radius \((z = r/R_0)\) and \( C \) is the acoustic loss factor. The quantities \( P^* \), \( \tilde{P} \), and \( C \) are respectively, \( P_r/P_0 \), \( P_i/P_0 \), and \( \sqrt{P_0/P_i}/c_i \), where, \( c_i \) is the speed of sound in water. Eqs. (1) and (2) have been investigated extensively in the past [13]. Various other forms of this equation have also been studied, for example, the Rayleigh–Plesset equation that has been extended to include surface tension effects, compressibility of the liquid and mass transfer at the interface [21–24]. In
the present study, we will limit ourselves to the forms given in Eqs. (1) and (2) since the focus of this study is not on the numerical investigation of the bubble instability problem.

The Herring–Trilling equation, Eq. (2), can be solved to obtain the temporal variation of the bubble radius. However, it is clear from the image data (Figs. 5 and 6) that the one-dimensional assumption is clearly violated (due to the loss of sphericity) as the bubble collapses and nears the bubble minimum. Therefore, it is likely that the solution of the one-dimensional model (even with various factors such as mass transfer, compressibility, etc. included) will not agree with the experimental data especially near the first bubble minimum and for subsequent pulsations. This can be demonstrated by solving Eq. (2) numerically, starting with \( \zeta = 1 \) and \( \zeta / \tau = 0 \) and \( \tau = 0 \) using a perfect gas law for adiabatic expansion, i.e., \( P^* = \frac{z^3}{z} \).

The comparison of the computed variation of bubble radius with time is shown in Fig. 7 for both H\(_2\)-O\(_2\) and CO-O\(_2\) stoichiometric mixtures. Only the first pulsation is shown here since the model (2) includes only acoustic losses and does not agree with the experimentally observed rapid decay in bubble radius with subsequent pulsations. Clearly, there is very good agreement between the measured and numerically predicted bubble radius for the first expansion phase. A similar observation has been made earlier [15]. As the bubble collapses and approaches the bubble minimum, the computed radius begins to deviate from the experimental data. Especially near the bubble minimum, the computed radius is larger than the experimentally observed value suggesting that additional losses are occurring in the experiments as discussed in Section 3.2.

The first two terms in Eq. (2) represent the change in the rate of the water kinetic energy, and the two terms on the right-hand-side are related to the mechanical work rate of the gas bubble on the water and the energy loss due to acoustic radiation, respectively. Eq. (1), when integrated with respect to time (using appropriate initial conditions) yields

\[
\frac{3}{2} \left( \frac{4\pi}{3} \frac{1}{r^2} \right) \frac{dr}{dt} + \frac{4\pi}{3} P r^3 + E(r) = \gamma,
\]

where \( E(r) \) is the internal energy and is defined later in Eq. (5). \( \gamma \) is a constant of integration and represents the total energy associated with the radial flow of water. The internal energy is relatively unimportant over much of the expansion. Therefore, setting \( dr/dt = 0 \) in Eq. (3) gives an estimate for the energy \( \gamma \) in terms of \( R_{\text{max}} \) at sufficiently expanded stage of the motion [5, pp. 274–275]:

\[
\gamma = \frac{4\pi}{3} P \frac{r^3}{R_{\text{max}}}.
\]

As shown below, \( \gamma \) is related to the bubble oscillation period and modified forms of \( \gamma \) are used in Section 3.2 to study the energy partitioning issues.

### 3.1. Geometric and dynamic similarity analysis

The above expressions can be used to estimate parameters necessary to carry out scaling analysis of the present data. For solid explosives, such as TNT [5, p. 274], there is a very weak dependence of \( \gamma \) on \( P_0 \); namely, \( \gamma \approx 6 E(r)/P_0^{1/5} \). Furthermore, since

\[
E(r) = \frac{P_0 Y(r)}{\gamma - 1} = \frac{4\pi P_0 r^3}{3(\gamma - 1)},
\]

\( E(r) \) and \( \gamma \) can be eliminated from Eqs. (4) and (5) to yield an expression for \( R_{\text{max}}/r \):

\[
\left( \frac{R_{\text{max}}}{r} \right)^3 = \frac{6 P_0^{1/5}}{(\gamma - 1) P_i}.
\]

Since, \( P_i \approx n P_0 \) [where \( n = O(1) \)] when \( r = R_{\text{min}} \), it can be seen that \( R_{\text{max}}/R_{\text{min}} \) has an extremely weak dependence on \( P_0 \), i.e., \( R_{\text{max}}/R_{\text{min}} \propto (P_0)^{1/5} \). Comparing the explosion pressures of gas explosion (approximately 10 bar) and deep sea explosion (approximately 100 kbar), it would seem that the bubble amplitude ratio, \( R_{\text{max}}/R_{\text{min}} \) for gas explosion would be roughly 12 times smaller than that for deep sea explosion. However, since \( P_i \propto (d + 10.33) \), where \( d \) is the explosion depth in meters (e.g., 91 m [5]), and since, in the present study, the ambient pressure, \( P_i \) is much lower than that for a deep sea explosion, the bubble amplitude ratio for shallow water tests becomes comparable to that for deep sea explosions. The period of bubble oscillation, \( T \), can be shown to be proportional to \( Y^{1/2} \) and inversely proportional to \( (P_i)^{1/4} \), and, therefore, using Eq. (6) can be expressed as \( T \propto \sqrt{P_i/P_{\text{max}}} \).

To be useful for practical scaling studies, the bubble motion should preserve both geometric and dynamic similarities [13]. Geometric similarity implies invariance of \( R_{\text{max}}/d \) and \( R_{\text{max}}/R_{\text{min}} \) and dynamic similarity essentially implies invariance of the Froude number, \( Fr = U^2/gd \), which in turn implies an invariance of \( T^2/R_{\text{max}} \). Estimates for these parameters for the first bubble oscillation are given below for both the present experiments and the past deep sea studies.

For the present experiments of gas explosion, \( R_{\text{max}} \approx 3.9 \), \( R_{\text{min}} \approx 0.7 \), \( \gamma \approx 3.15 \text{ cm} \), \( T \approx 0.01 \text{ s} \), and
$d = 0.6477$ m. This gives $R_{\text{max}}/(d + 10.33) \approx 8.607 \times 10^{-3}$, $R_{\text{max}}/R_{\text{min}} \approx 3$, and $T^2/R_{\text{max}} \approx 2.381 \times 10^{-5}$ s$^2$/cm. In a deep sea explosion of 249.5 grams of tetryl charge fired at a depth of 91.44 m below the water surface [5, p. 271], $R_{\text{max}} = 45.11$ cm, $R_{\text{min}} \approx 12.7$ cm, $R_0 = 3.5$ cm, $T \approx 0.028$ s, and $d = 91.44$ m. This gives $R_{\text{max}}/(d + 10.33) \approx 4.432 \times 10^{-3}$, $R_{\text{max}}/R_{\text{min}} \approx 3.55$, and $T^2/R_{\text{max}} \approx 1.738 \times 10^{-4}$ s$^2$/cm. Data from TNT explosions at a depth of 152 m [14] suggests similar results: $R_{\text{max}}/(d + 10.33) \approx 2.55 \times 10^{-3}$, $R_{\text{max}}/R_{\text{min}} = 2.255$, and $T^2/R_{\text{max}} \approx 1.596 \times 10^{-5}$ s$^2$/cm. Scaling analysis of the amplitude of the bubble oscillation can also be carried out by defining an equilibrium radius $R_e$ which is defined as the radius of the bubble when the pressure inside it equals the hydrostatic pressure at the explosion depth [3]. The present results suggest that $R_{\text{max}}/R_e = 1.6212$ and $R_{\text{max}}/R_0 = 0.853$, whereas the experimental data [5] suggest: $R_{\text{max}}/R_e = 1.48$ and $R_{\text{max}}/R_0 = 1.03$. Here, the subscripts 1 and 2 denote, respectively, the first and second bubble maxima. These comparisons clearly suggest that the present experiments preserve reasonably close geometric and dynamic similarities for all the parameters except for $R_{\text{max}}/d$.

3.2. Energy partition

As noted earlier, the explosion energy can be redistributed among the various modes (e.g., shock wave energy, potential, kinetic and dissipated energies, etc.) initiated by the explosion. Previous studies [3,4,14] have attempted to use the experimentally obtained data from deep sea studies to evaluate how the total energy from explosion is partitioned between these modes. In the present study, particular interest lies in determining how much of the energy is available for exciting the bubble instability (to be discussed in Section 4). Vokurka [4] suggested that the energy balance for an expanding system (i.e., as the shock wave propagates outwards and the bubble begins to expand after the explosion) can be written as:

$$\Delta E_{\text{r}} = \Delta E_{\text{pe}} + \Delta E_{\text{de}} + E_k,$$  

(7)

where $\Delta E_{\text{pe}}$ and $\Delta E_{\text{r}}$ are, respectively, the change in potential and internal energy of the gas bubble (from its initial state at time $t = 0$), $\Delta E_{\text{de}}$ is the energy dissipated and $E_k$ is the liquid kinetic energy. Estimates for each of these terms can be obtained from the experimental data. The total ( nondimensional) energy available for the explosion is determined from the relation $E_0 = QW/E_0$, where $Q$ is the detonation energy per kg of explosive, $W$ is the weight in kg of the explosive and $E_0 = 4\pi R_0^2 P_0/3$ is the energy of the initial gas volume. For a stoichiometric hydrogen-oxygen mixture, $Q = 242$ kJ/mol of hydrogen and using representative values for the current experiments, one obtains: $E_0 = 66.8735$. This is total amount of explosion energy which is to be distributed into various modes defined in Eq. (7) as the bubble expands. The change in internal energy ( nondimensionalized using $E_0$) as the bubble expands and reaches bubble maximum can be written as [4]:

$$\Delta E_{\text{pe}} = E_k - E_k = \left( \frac{P_0}{P_i} \right) \left[ 1 - \left( \frac{r}{R_0} \right)^{-3\gamma - 1} \right] / (\gamma - 1),$$

(8)

where the subscript 1 indicates the first bubble oscillation and the superscript e indicates that the energy estimates is obtained during the expansion phase. The change in the potential energy (again nondimensionalized by $E_0$) is:

$$\Delta E_{\text{pe}} = \left[ \left( \frac{r}{R_0} \right)^3 - 1 \right].$$

(9)

Here, $r > R_0$ is the instantaneous bubble radius as the bubble expands from its initial diameter $R_0$ to its first maximum bubble radius $R_{\text{max}}$. Further, if one assumes that $E_k \approx 0$ near the first bubble maximum [4], an estimate for the energy dissipated can be obtained as: $E_{\text{de}} = \Delta E_{\text{pe}} - \Delta E_{\text{pe}}$. Estimates for all these quantities are given in Table 2. As can be seen from Table 2, approximately 26% of the energy is dissipated as the bubble expands and reaches its first maximum. Previous studies [1,3] suggest that this dissipated energy is entirely carried away by the shock wave and the remaining energy (74%) at the first bubble maximum is the total energy available for the bubble collapse and subsequent oscilla-

<table>
<thead>
<tr>
<th>$r$</th>
<th>$E_k$</th>
<th>Energy partition</th>
<th>Total energy available for next pulsation (% of $E_0$)</th>
<th>Total energy lost (% of $E_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r = R_0$</td>
<td>$E_k$</td>
<td>66.8735</td>
<td>66.8735 (100%)</td>
<td>0 (0%)</td>
</tr>
<tr>
<td>$r = R_{\text{max}}$</td>
<td>$E_k$</td>
<td>34.4235</td>
<td>49.3259 (-73.76%)</td>
<td>17.5476 (-26.24%)</td>
</tr>
<tr>
<td>$r = R_{\text{max}}$</td>
<td>$E_{\text{pe}}$</td>
<td>39.4041</td>
<td>39.0911 (-58.46%)</td>
<td>27.7824 (-41.54%)</td>
</tr>
<tr>
<td>$r = R_{\text{max}}$</td>
<td>$E_{\text{de}}$</td>
<td>-0.3130</td>
<td>-0.3130</td>
<td>-0.3130 (-0.48%)</td>
</tr>
<tr>
<td>$r = R_{\text{max}}$</td>
<td>$E_{\text{de}}$</td>
<td>37.1503</td>
<td>38.4688 (-57.52%)</td>
<td>28.4047 (-42.48%)</td>
</tr>
<tr>
<td>$\Delta E_{\text{pe}}$</td>
<td>1.3185</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
tions. The current estimate of 26% for the dissipated energy (which may or may not be totally associated with the shock wave) is lower than the value computed by Vokurka [4] who estimated for a TNT explosion [14] that approximately 50% of the total energy was dissipated as the bubble reached the first maximum. However, note that $E_0$ (and hence, $P_0$ and $R_{\text{max}}$) is much larger for the deep sea TNT explosion and, as a result, the difference between Eqs. (8) and (9) will be substantially larger than for the present case.

A similar analysis can be carried out for the contraction phase (i.e., as the bubble collapses from the first bubble maximum and reaches its first minimum) and then for the second expansion phase as the bubble rebounds to its second maximum. For the contraction phase, the energy balance can be written as in Eq. (7) except that now, the energy available for the various modes is limited to the amount left after 26% of the explosion energy has been dissipated. The energy balance at this stage is quite crucial since the observed large decrease in the second bubble maximum suggests that a significant amount of energy is dissipated either during the first bubble contraction or during the second bubble expansion. An estimate of the energy dissipated can be inferred by estimating the changes in the internal and potential energy as the bubble reaches, respectively, the first minimum and the second maximum by assuming that at both the first minimum and the second maximum, $E_k \approx 0$. For example, during the contraction phase, the change in potential energy can be written as [4]:

$$\Delta E_{\text{pc}} = \left( \frac{R_{\text{max}}}{R_0} \right)^3 \left[ 1 - \left( \frac{r}{R_{\text{max}}} \right)^3 \right]. \quad (10)$$

where, $r ( < R_{\text{max}} )$ is the instantaneous bubble radius as the bubble contracts from the first maximum bubble radius $R_{\text{max}}$ and reaches its first minimum. The superscript c indicates the contraction phase. The change in internal energy can also be expressed in terms of the bubble properties as

$$\Delta E_{\text{ie}} = \frac{1}{\gamma - 1} \left( \frac{P_{\text{max}}}{P_r} \right) \left( \frac{R_{\text{max}}}{R_0} \right)^3 \left[ 1 - \left( \frac{r}{R_{\text{max}}} \right)^{3(\gamma - 1)} \right]. \quad (11)$$

Using $\gamma = 1.173$ and the experimentally measured pressure at the first bubble maximum, $P_{\text{max}} = 8.0826$ kPa, in Eqs. (10) and (11) one can obtain $\Delta E_{\text{pc}}$ and $\Delta E_{\text{ie}}$ at the first bubble minimum i.e., when $r = R_{\text{min}}$. Table 2 gives estimates for these energies at the first bubble minimum as well as the amount of energy dissipated, $\Delta E_{\text{de}}$, during the first contraction of the bubble and the internal energy of the bubble at the first minimum, $E_{\text{ie}}$. To obtain the energy partition at the second bubble maximum, the energy left after $\Delta E_{\text{de}}$ is first subtracted from the available energy and then partitioned using Eqs. (8) and (9), except for the subscript 2 (indicating the second pulsation).

The results show very interesting trends. It appears that around 15% of the total energy ($E_0 = 66.8735$) is dissipated between the first bubble maximum and the first bubble minimum, and a very negligible part (less than 0.1%) of the total energy is dissipated between the first bubble minimum and the second bubble maximum. In terms of the actual energy available after the first expansion, $E_{\text{ie}} + \Delta E_{\text{pc}}$ (i.e., after the 26% energy dissipated near the first bubble maximum is removed), the energy dissipated by the first minimum is around 21% of the available energy. Thus, nearly 42% of the total available energy is dissipated in first oscillation cycle. Between the first bubble minimum and the second bubble maximum, the amount of energy dissipated is negligible and the bubble radius data (Fig. 4) suggests that beyond the second oscillation, the energy loss is similar to the losses occurring during the second pulsation. Thus, this energy partition study shows that a significant amount of the explosion energy is dissipated during the first bubble pulsation. Clearly, some of this dissipated energy (during the first expansion phase) is carried away by the shock wave; however, there still appears to be a significant amount of energy that cannot be accounted for by the internal and potential energies associated with the bubble after the first pulsation. A similar conclusion has been obtained in earlier studies [1,4,22,25] where it was suggested that this dissipation may be due to various phenomena such as (i) turbulence induced in the water, (ii) mass loss from the gaseous bubble, (iii) Taylor instability at the interface, and (iv) gas cooling and steam condensation near the interface. However, none of the above studies were able to clearly identify the mechanism(s) behind the observed energy loss. As demonstrated in Section 4, the present study suggests that the energy loss is likely to be related to the excitation and amplification of interface instabilities.

4. Bubble interface instability

Analysis of the bubble images (e.g., Fig. 5(c) and (d)) showed that significant corrugation of the bubble interface occurs near bubble minimum. Past studies [13,15] have suggested that this corrugation is the result of the bubble interface undergoing some sort of instability. Although various sources of instability (both hydrodynamic and evaporative) have been proposed it is still not clear how these instabilities contribute to the bubble interface collapse. However, if these instabilities do occur then it would explain the (so far) unexplained loss of energy during the first bubble pulsation. Therefore, an attempt was made in this study to identify the sources of the bubble interface instability and to determine, using theoretical stability analysis, if these instabilities are possible for the test conditions.

Two types of hydrodynamic instability mechanisms have been proposed: Rayleigh–Taylor (RT) and Kelvin–Helmholtz (KH) instability. The RT instability occurs when there are two fluids of different densities adjacent to each other and the lighter fluid is accelerated towards the denser fluid while the KH instability re-
quires the presence of a shear motion along the circumference of the bubble. The evaporative instability, often called the Landau–Darrieus (LD) instability [11], is a much more complex process and is due to the mass and heat transfer occurring across the bubble interface.

4.1. Hydrodynamic instabilities

Both or either of the RT and KH instability could play a role in destabilizing the bubble interface leading to the bubble collapse. Here, these two instabilities are addressed to identify their importance.

4.1.1. Rayleigh–Taylor instability

As shown by Birkhoff [9], for the case of underwater bubble, collapsing bubbles are unstable even though the denser liquid (water) is being accelerated towards the lighter vapor (explosion product), with surface tension having negligible effect on instability. He further showed that the general instability criteria allow two types of instability: the classical RT instability, which occurs at large wavelengths (small wave numbers) and has an exponential growth rate and the Birkhoff instability, which occurs at small wavelengths (large wave numbers) and has an algebraic growth rate. For small perturbations in bubble radius, the disturbed interface $r(\phi, t)$ in spherical coordinates can be expressed in terms of Legendre polynomials by:

$$r(\phi, t) = R(t) + \sum_{n=1}^{\infty} b_n(t) P_n(\cos \phi).$$

(12)

where $R(t)$ is the mean bubble radius, $\phi$ is the bubble polar variable and $n$ is the order of Legendre polynomial with the corresponding amplitude $b_n$. The differential perturbation equation can be obtained, in the absence of gravity, for the condition of constant internal pressure as [10]

$$\frac{d^2b_n}{dt^2} + \frac{3}{2} \frac{d}{dt} \frac{d}{dt} b_n - 4b_n = 0.$$  

(13)

where $\lambda$ is given by [26]:

$$\lambda = \left[ \frac{n(n-1) - (n+1)(n+2)(\rho_2/\rho_1)}{5} + (n+1)\rho_1/\rho_2 \right].$$

(14)

Here $\rho_1$ is the density of water, $\rho_2$ is the density of the gas in the bubble, $\sigma$ is the surface tension and $z$ is the dimensionless radius, $R(t)$. The general stability criteria for the stability of the Eq. (13) was derived by Birkhoff [10] and shown to be:

$$A < 0 \quad \text{and} \quad 64 \frac{d^2A}{dt^2} + R \frac{dA}{dt} < 0.$$  

(15)

The first criterion in Eq. (15) reduces to the familiar RT stability criterion, $d^2R/dt^2 < 0$, for $\rho_1 \gg \rho_2$ and negligible surface tension effects. The second criterion is the Birkhoff stability criterion and entails that $R^2A$ should be a decreasing function of time for stability. Fig. 8 shows the variation of $A$ for different values of $n$ for the H₂–O₂ explosion case of Fig. 7. Here a value of 70 dynes/cm for the surface tension of water, $\sigma$, has been assumed. Except for $n = 1$ (which corresponds to translation of the bubble center rather than deformation of the spherical shape), it can easily be observed that the Birkhoff stability criterion is violated as the bubble approaches the minimum radius. These stability criteria have been discussed in [27] where the stability of the growth phase has been analyzed and the preferentially amplified wavelength has been shown. The expansion and collapse of a large two-phase bubble of hot water in a tank of cold water has been shown in [28], where the growing interface has been shown to undergo RT instabilities of high wavenumber of 10.

The stability of Eq. (13) can be studied using Liapunov's theorems for nonlinear systems [29] by constructing a positive-definite Liapunov functional. Consider the second order differential equation $q'' + f(x)q' + g(x) = 0$, where $x' = dx/dt$ and in the state variable notation where $x_1 = x$ and $x_2 = x'$, the energy functional can be expressed as $V(x_1, x_2) = x_2^2/2 + U(x_1)$. If $g(x)$ is continuously differentiable, $g(0) = 0$ and $x(g(x)) > 0$ for all non-zero $x$, then $U(x_1)$ can be selected as $U(x_1) = \int_0^1 g(s)ds$. According to Liapunov's theorem, a sufficient condition for the stability of the system of above mentioned second order differential equation in a region of state space is $V > 0$ and $W \equiv dV/dt < 0$. Considering $x = b_n$, one obtains $f(x) = (3\sigma/4\varepsilon^2)/z^2$ and $g(x) = -4x$ from Eq. (13). The Liapunov's energy functional becomes $V = x_2^2/2 + A x_1^2/2$, which will be positive for all negative values of $A$. Therefore, by considering the range of $\tau$ defined by the points $a$ and $b$ in Fig. 8, one obtains $V > 0$. Since $W = -3(\sigma/\varepsilon^2)x_1^2/2$, a sufficient condition for the stability of the system defined by Eq. (13) becomes $dz/d\tau > 0$. Fig. 9 shows $dz/d\tau$. It can easily be inferred that the bubble growth is stable and the Liapunov stability criterion is violated during the bubble collapse phase.

Another method to study the stability of a second order differential equation with time-varying coefficients, like Eq. (13), is by means of the (so called) phase plane method [30]. In this method, the system trajectory is...
plotted in the state space. One can employ the delta method and Eq. (13) can be rewritten as \( \frac{dz}{d\tau} = -\frac{(x_3 + \delta)}{x_2} \), where \( \delta = 3(dz/d\tau)x_2/z - (1 + A)x_3 \) can be assumed constant over an infinitesimal interval \( \Delta \tau \). The integrated form of this equation is \( x_2^2 + (x_3 + \delta)^2 = r^2 \), where \( r \) is the constant of integration. One can start with an initial point \( P(x_{i0}, x_{z0}) \) at \( \tau = 0 \) and calculate \( \delta \) and \( r \). Next, an infinitesimal arc of radius \( r \) is drawn through \( P \) with the center at \((-\delta, 0)\). The time required to move from the initial point to the next point in the phase plot is given by

\[
\Delta \tau = \sin^{-1} \left( \frac{x_{i0} + \Delta x_3 + \delta}{r} \right) - \sin^{-1} \left( \frac{x_{z0} + \delta}{r} \right)
\]

\[
= \cos^{-1} \left( \frac{x_{z0} - \Delta x_3}{r} \right) - \cos^{-1} \left( \frac{x_{i0}}{r} \right), \tag{16}
\]

which can be rearranged to solve for \( \Delta x_1 \) and \( \Delta x_3 \) in terms of \( \Delta \tau \). Thus, the coordinates of the next point in the state space are determined by \( \Delta \tau \) from the initial point \( P \) by using the delta method and a forward marching scheme can be set forth by updating the initial point and incrementing time by \( \Delta \tau \) in each step. By making \( \Delta \tau \) sufficiently small, this method can produce a set of values of \( x_1 \) and \( x_3 \) without actually solving Eq. (13). Since, this set of data is all one needs to investigate the stability or instability of Eq. (13), a phase plot is actually not needed. One can start with any arbitrarily small nonzero initial values of \( x_1 \) and \( x_2 \), and study their variations as time progresses. It is interesting to note that for \( n > 1 \) the qualitative variations of \( x_1 \) and \( x_2 \) with \( \tau \) is quite independent of the choice of their initial values. Furthermore, the values of \( x_1 \) and \( x_3 \) grow rapidly without limit as the bubble minimum is reached and they keep growing even when the second pulsation starts indicating that the bubble minimum is unstable.

To examine the second criterion of Birkhoff's instability, the quantity \( R^A \) (again in terms of the nondimensional variable \( z \)) estimated from the experimental data is shown in Fig. 10 for the \( H_2O_2 \) mixture case. Interestingly enough, this quantity decreases continuously till bubble radius maximum and, thereafter it increases. Thus, based on the criteria established by Birkhoff, the conditions near bubble minimum can give rise to catastrophic instability. This is consistent with the observation seen earlier in Fig. 7 and in experiments. [5]) that the rate of change of bubble radius is virtually discontinuous near bubble minima. To correlate the various regions of potential instability, the bubble radius time history along with the different regions for potential instability are shown in Fig. 11. Three regions are identified in this figure. In region A, \( dR/d\tau < 0, d^2R/d\tau^2 < 0 \), in region B, \( dR/d\tau < 0 \) or \( dr/d\tau > 0, d^2R/d\tau^2 > 0 \) and, in region C, \( dR/d\tau > 0, d^2R/d\tau^2 < 0 \). Clearly, the second criterion of Birkhoff suggests that the region B is unstable to perturbation.

The above analysis suggests that the bubble is unstable to perturbation near the bubble minimum and is susceptible to both RT and Birkhoff type of interface instability. To determine which of these instabilities is dominant, it is necessary to determine the most unstable
wavelength of the instability. It was noted earlier [13], that within the range of the maximum wavelength (which is determined by the size of the bubble) and the minimum wavelength (which depends on the effects of the surface tension), there exists a preferentially amplified most unstable wavelength. To determine this, the power spectral density of the bubble diameter near bubble radius minimum (obtained by carrying out FFT analysis of the bubble diameter, as described in Section 2) was analyzed and is shown in Fig. 12 for the H$_2$-O$_2$ mixture as a function of c/λ. Here, c is the bubble circumference and λ is the wavelength. Two curves are shown in the figure corresponding to 1 ms and 0.5 ms just prior to bubble minimum. It can be seen that there is a peak in power spectral density (which is actually the square root of the sum of the squares of mode amplitude coefficients, and should occur at integral fractions of bubble circumference because a periodic trace is being analyzed) for a value of the wavelength, which though large, is finite. Infinite wavelength corresponds to the mean bubble radius, and is, consequently, accompanied by a large peak (which is removed from these plots to facilitate comparison of the amplification of the other smaller wavelengths).

Fig. 12 clearly shows that the large wavelength instability is most dominant and the power spectral density of the largest wavelength increases as the bubble minimum is approached. Such a large-wavelength perturbation is characteristic of the RT type of interface instability which grows with an exponential growth rate. The general criteria of Birkhoff are satisfied suggesting that both RT and Birkhoff instabilities can be (and do get) excited near bubble minimum. However, since the effect of surface tension is always stabilizing and its effect increases rapidly with increasing n [26], it is plausible that, although Birkhoff-type of instability occurs near bubble minimum, it may be stabilized by the surface tension effect. The fact that the high wavenumber instability is not observed in the bubble diameter spectra suggests that even though the Birkhoff instability may be excited first near bubble minimum, the stabilizing effect of surface tension is suppressing the small wavelength disturbances, while, due to its exponential growth rate, the large wavelength RT instability becomes more dominant and, thereby, controls the growth of the bubble interface instability.

These results were confirmed using a full three-dimensional numerical simulation of the bubble explosion using an advanced finite-element code ALE3D [18,31]. Results showed that as the bubble collapses, a large wavelength variation appears on the bubble interface. A typical result is shown in Fig. 13. This wave-like interface corrugation seen in the computations occurred in all test cases, including simulations with the tank walls moved outwards and changed from rectangular to spherical geometry. These computations were carried out without including the glass bulb and other experimental artifacts, such as, the pressure transducer and the spark. Thus, it appears that the formation of the interface corrugation is a physical property of the bubble dynamics.

4.1.2. Kelvin–Helmholtz instability

Yet another fundamental instability mechanism is the KH instability, which occurs when there is a shearing motion at the interface. A dispersion relation for gravity waves with a shear velocity $U$ tangential to the interface in the upper fluid [6,8] can be given as:

$$\Omega_{KH} = -ik \frac{U p_2}{(\rho_1 + p_2)} + \left[ k^2 U^2 - \frac{\rho_1 p_2}{(\rho_1 + p_2)^2} - kg \frac{\rho_1 - p_2}{(\rho_1 + p_2)} \right]^{1/4}. \tag{17}$$

![Fig. 12. Power spectral density analysis of the bubble diameters for the H$_2$-O$_2$ explosion case. The dotted and the solid curves correspond, respectively, to the bubble data 0.5 ms and 1 ms just prior to collapse. Results show that the large wavelength RT instability is dominant near the first bubble minimum.](image)

![Fig. 13. Numerically predicted (using ALE3D) bubble shape near the first bubble minimum clearly showing the presence of large wavelength RT instability.](image)
where $\Omega_{KH}$ is the growth rate, $k$ is the wave number and $g$ is the local acceleration (note that for the present case, the local acceleration is given by $R_0/t_i^2$). The stability criterion can be shown to be [6]

\[(\rho_1 - \rho_2)g > kU^2 \rho_2,\]  \hspace{1cm} (18)

which, for the particular case of water–gas interface ($\rho_1 \gg \rho_2$) reduces to $\rho_1 g > kU^2 \rho_2$. Since, $\rho_1 \gg \rho_2$ and $g$ can also be very large near bubble minimum, the quantity $\rho_1 g / \rho_2$ is usually very large and the condition given by Eq. (18) is not difficult to satisfy suggesting that KH instability will not become relevant unless the tangential shear velocity $U$ becomes very significant. Such shear motion would occur in the initially symmetric situation only if some interface corrugation has already occurred.

Fig. 14 shows a plot of the distance of the bubble center from the reference (unexploded) bubble location as a function of time. It can be seen that between the explosion and the first bubble maximum, there is negligible upward migration of the bubble (this justifies the earlier neglect of buoyancy effect for the study of RT instability). However, as time progresses, the bubble migrates upwards at an almost linear rate. Fig. 14 can be used to estimate the migration velocity which is determined to be approximately, 104 cm/s. The quantity $\rho_1 g / \rho_2$ assumes a value of approximately $2 \times 10^8$ m/s, for example, for stoichiometric H$_2$–O$_2$ mixture near bubble minimum. A surface corrugation of wavenumber as large as 150 m$^{-1}$ ($c / \lambda \approx 30$) would also require a tangential shear velocity of magnitude larger than 1000 m/s to violate the stability criterion given by Eq. (18). Therefore, it is concluded that KH instability does not play a major role in the initiation of the bubble instability process.

4.2. Evaporative instability

When mass transfer is induced at the interface in the form of evaporation or condensation, the Landau–Darrieus evaporative instability can also contribute to the interface distortion. Mass transfer across the gas–water interface can occur through several mechanisms: evaporation, condensation, dissolution of product gases into water, explosive boiling of superheated liquids, and direct entrainment of gases into the liquid water [13]. In the present case, it is possible that due to presence of hot combustion products in the bubble, the surrounding water layer could be made to vaporize. However, the actual evaluation of the evaporation rate is not possible without detailed heat and mass transfer calculations since both the evaporative kinetics and the diffusion processes enter into the heat and mass transfer problem in a complex manner [12]. In order to determine the mass flux, $j$, the energy equation in spherical coordinates in presence of spherical symmetry,

\[\frac{\partial T}{\partial t} + \frac{R^2}{r^2} \frac{\partial T}{\partial r} = D \left( \frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} \right),\]  \hspace{1cm} (19)

has to be solved in the liquid surrounding the bubble, subject to the boundary conditions. Here, $T$ is the temperature, $r$ is the radial coordinate measured from the bubble center, and $D$ is the thermal diffusivity of water. The boundary condition at the bubble interface is given by:

\[4\pi R^2 k \rho \left( \frac{\partial T}{\partial r} \right)_{r = R} = -\frac{d}{dt} \left( \frac{4}{\rho} \right),\]  \hspace{1cm} (20)

where $k$ is the thermal conductivity of water, $L$ is the latent heat of vaporization, and $\rho$ is the density of vapor. This boundary condition simply states that the heat required for evaporation of the liquid is supplied by the hot bubble by means of conduction through the interface. To solve Eq. (19), a transformation to the Lagrange coordinates can be implemented by defining $h = (r^2 - R^2(t))/3$, so that the interface always lies at $h = 0$. Two finite difference equations are obtained for Eqs. (19) and (20) in terms of the space step, $\Delta r$ and time step, $\Delta t$. The computation starts with an initial temperature distribution in water. A tridiagonal and diagonally dominant matrix equation is obtained and solved for the unknown temperatures at the grid points at later time $\Delta r$ [32]. Thus a forward marching scheme in time can be set to determine the temperature distribution in water with bubble motion. The temperature of water at the interface determines the mass flux across the interface at any time.

The above equations were numerically solved for the experimental test conditions. Results showed that the interface temperature during the bubble expansion is around 344 K which is lower than the temperature needed to evaporate the water surrounding the bubble. Therefore, it is concluded that LD instability is not likely to occur near the bubble maximum.

In summary, it appears that both Birkhoff and RT instabilities can contribute to the interface instability near the bubble minimum. Near the bubble minimum, the interface becomes highly corrugated and the flow becomes locally turbulent. A portion of the bubble energy could be dissipated into heat in the local turbulent flow as the

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Fig. 14. Upward migration of the bubble center from its original location as a function of time. The vertical motion due to buoyancy is negligible only for the first bubble pulsation. Using a linear fit to this data, a migration velocity of around 104 cm/s is estimated for the present case.
bubble reaches its minimum. Of the hydrodynamic instabilities, the B-instability is likely to be excited first. However, the surface tension effect tends to stabilize the small wavelength (high wavenumber) B-instability, whereas, due to its exponential growth, the large wavelength (low wavenumber) RT instability becomes dominant. The excitation of RT instability extracts (or dissipates) significant amounts of total energy (as estimated in Section 3). This may explain the observed rapid decrease in the bubble size and pulsation amplitude after the first oscillation. As the bubble continues to pulsate (albeit with a significantly reduced amplitude), the effect of buoyancy causes an upward migration of the bubble. This can result in an effective shear velocity (however, data suggests that this velocity is not large enough to contribute sufficiently to cause the excitation of the KH instability).

5. Practical significance/usefulness

The dynamics of underwater explosions are of considerable interest due to the well known effects of the bubbles formed during the explosion. Many of the past and present studies are motivated by the need to characterize the mechanism by which these bubbles interact with underwater surfaces, such as, submarines, etc. The erosive and destructive nature of cavitation bubbles (usually these are very small micro-bubbles) on turbine blades and propellers are well known. However, when bubbles are formed due to large explosions near a surface, more destructive interaction occurs, since, the collapse process results in the formation of a water jet that impacts the surface with a peak pressure much larger (by an order of magnitude than the original explosion pressure [4.5]. The dynamics of such large-scale bubble-surface interaction is not well understood. The present study forms the first phase of an ongoing investigation on the effects of bubble dynamics. Understanding the basic mechanism of bubble oscillation is the first step towards investigation of more complex fluid-structure interactions. For example, bubble-bubble interactions were recently used [33] to enhance the remnant energy in one bubble (at the expense of the other) so that the stronger bubble is less susceptible to instability and can maintain its coherence for a longer period. This has implications for focusing underwater bubble explosions.

6. Conclusions

This paper discusses results obtained in an experimental investigation of gaseous bubbles formed during underwater explosions. Experiments were carried out in a sub-scale laboratory test facility using gaseous fuel-oxygen mixtures as the explosion source. The typical bubble images and the pressure signatures (both outside and inside the bubble) were recorded during these experiments for a range of test conditions. Using both geometric and dynamic scaling analyses it has been demonstrated that the characteristic nondimensional parameters for the present experiments are reasonably similar to those measured in deep sea explosions. Thus, the present results have practical implications for understanding the characteristics of high-explosive deep sea explosions (which are very difficult to quantify).

Partition analysis of the total energy released during a typical explosion showed that nearly 26% of the total explosion energy is dissipated (or lost) during the first bubble expansion phase and another 15% of the total available energy is dissipated during the first contraction phase. As a result, the bubble size and the oscillation amplitude is decreased significantly after the first pulsation. This rapid decrease in bubble size and amplitude is similar to observations of deep sea explosions. Past studies attributed this observed decrease partly to the energy dissipated by the initial shock wave and partly to some unaccounted mechanism(s). The analysis of the present data suggests that the unaccounted-for energy goes into interface instability.

Various sources of the interface instability have been elucidated in the present study using the experimental data and stability analysis. Results suggest that neither the evaporative LD instability nor the KH instability is likely to play an observable role in the interface instability. However, near the bubble minimum, the hydrodynamic RT and Birkhoff instabilities are excited. It has been shown here that although the Birkhoff instability is excited first, surface tension effects damp the high wavenumber, small wavelength Birkhoff instability. However, the RT instability (with its exponential growth rate) is not damped and eventually dominates the interface instability. This study confirms earlier conjectures [1.3,13] that hydrodynamic interface instabilities play a major role in the energy loss process and the rapid decrease in bubble size and oscillation dynamics.

7. Recommendations

The current experiments were limited to shallow water, gaseous explosions. Although scaling analysis demonstrates that the present results are meaningful for real deep sea explosions, this needs to be confirmed by carrying out similar controlled explosions under higher pressure. Some such studies have been conducted in the past but the data acquired under these conditions are very limited and difficult to interpret. More detailed measurements are needed for realistic explosives (e.g., pentolite).

Nomenclature

<table>
<thead>
<tr>
<th>English letters</th>
<th>Description</th>
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<tbody>
<tr>
<td>$b_n$</td>
<td>amplitude of Legendre polynomial</td>
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<tr>
<td>$c_s$</td>
<td>speed of sound in water, m/s</td>
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</table>
\[ C = \text{acoustic loss factor} \left( = \sqrt{P_0/\rho_0/c_l} \right), \text{nondimensional} \]
\[ d = \text{explosion depth, m} \]
\[ D = \text{thermal diffusivity of water, m}^2/\text{s} \]
\[ E(r) = \text{internal energy of the bubble, J} \]
\[ k = \text{wavenumber, m}^{-1} \]
\[ L = \text{latent heat of vaporization, J/kg} \]
\[ M = \text{molecular weight, gm/mole} \]
\[ P_i = \text{pressure inside bubble, KPa} \]
\[ P^* = \text{normalized pressure} \left( = P_i/P_0 \right), \text{nondimensional} \]
\[ \bar{P} = \text{normalized pressure} \left( = P_i/P_0 \right), \text{nondimensional} \]
\[ r = \text{instantaneous bubble radius, cm} \]
\[ t = \text{time, s} \]
\[ T = \text{bubble oscillation period, s} \]
\[ Y = \text{total energy associated with the radial flow of water, J} \]
\[ z = \text{normalized bubble radius, nondimensional} \]

**Greek Symbols**

\[ \gamma = \text{ratio of specific heats, dimensionless} \]
\[ \lambda = \text{wavelength, m} \]
\[ \sigma = \text{surface tension of water, dynes/cm} \]
\[ \tau = \text{time} \left( = t/\tau_i \right), \text{nondimensional} \]
\[ \tau_i = \text{time scale} \left( = R_0/\sqrt{P_0/\rho_0} \right), \text{s} \]
\[ \phi = \text{bubble polar variable, rad} \]

**Superscripts**

\[ \text{c} = \text{contraction phase} \]
\[ \text{e} = \text{expansion phase} \]

**Subscripts**

\[ 0 = \text{initial state} \]
\[ 1 = \text{corresponding to the first oscillation} \]
\[ 2 = \text{corresponding to the second oscillation} \]
\[ \text{de} = \text{dissipated energy} \]
\[ \text{e} = \text{equilibrium state} \]
\[ \text{ie} = \text{internal energy} \]
\[ k = \text{kinetic energy} \]
\[ l = \text{ambient state at the explosion depth} \]
\[ \text{max} = \text{maximum state} \]
\[ \text{min} = \text{minimum state} \]
\[ \text{pe} = \text{potential energy} \]

**Acknowledgements**

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**References**

Simulations of Underwater Explosion Bubble Dynamics

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Abstract

The dynamics of bubbles formed during underwater explosions is numerically investigated using an Arbitrary Lagrangian-Eulerian, three-dimensional finite-element code. Comparison of isolated bubble oscillation results with experimental data show good qualitative and quantitative agreement. Analysis using both stability considerations and energy balance estimates suggest that the excitation of Rayleigh-Taylor (R-T) instability is a major cause of interface instability during bubble collapse. The collapse of an explosion bubble near a rigid wall has also been simulated. The entire collapse process, including the formation of a vortex ring bubble and a high velocity re-entrant jet, are successfully captured in the simulations. The jet velocity and the impact pressure on the wall are functions of the explosion pressure and the distance of the bubble from the wall. The results indicate that, for a given explosion energy, there is an optimal distance of the bubble from the wall for which the impact pressure on the wall is maximum. This trend and the magnitude of the peak impact pressure are in good agreement with the experimental results. The evolution of the vortex ring bubble, reported in earlier experimental and numerical studies, is also accurately predicted. Available scaling laws for the time period and the peak velocities generated during bubble collapse are also reviewed in light of the current results. The present results are shown to obey many of the geometric and dynamic scaling laws when compared to deep sea explosions.

1. INTRODUCTION

Vapor and gas bubble dynamics are of great practical interest in the prediction and prevention of cavitation erosion of marine propeller and turbine blades. The destructive nature of strong explosions near submerged rigid surfaces is also well known. Detailed reviews (e.g., Blake and Gibson, 1987; Prosperetti, 1982) have summarized past experimental and numerical results. Experimental studies are too many to list completely; however, most past studies focused on cavitation (small) bubbles. Among the studies that focused on large scale explosions are the studies reported in Cole (1948) for freely oscillating, deep sea explosion bubbles and the studies of bubble collapse near walls (e.g., Tomita and Shima, 1986). Bubble-bubble interactions have also been studied in the past (e.g., Warren and Rice, 1964). However, in most cases, due to difficulties in acquiring detailed data, only limited information has been obtained. Recently, experiments were carried out to investigate relatively large-scale bubble explosions (Menon and Lal, 1998a; Lal and Menon, 1996). These experiments were conducted in shallow water (1 atmosphere ambient pressure) due to an interest in understanding the dynamics of bubble oscillation and bubble-wall interaction in such flows (including the effect of a free water surface) and to investigate the feasibility of targeting mines buried in beaches. The data obtained from these experiments have been used to validate the numerical model discussed in this paper.

Numerical studies in the past range from simple 1-D analytic solutions (e.g., Lauterborn, 1976; Plesset, 1971; Prosperetti, 1982) to more complex 2D/3D studies. Many studies employed the Boundary Element Method (BEM) or its variants (e.g., Chahine and Perdue, 1988; Duncan and Zhang, 1991; Blake et al., 1986; Plesset and Chapman, 1971). BEM is computationally very efficient since only the flow on the boundary surface is computed which allows the reduction of the dimensionality of the problem by one. Past results have demonstrated that this approach can capture many features of the bubble oscillation and collapse process. However, this method has some inherent limitations. For example, BEM does not allow for variations of density and pressure inside the bubble even though studies suggest that considerable variations can occur at bubble minimum. In addition, the BEM fails at the point of jet formation, as the simple domain becomes double connected.
The flow evolution beyond the point of bubble collapse has been modeled using BEM by explicitly introducing vortex elements (e.g., Zhang and Duncan, 1994; Zhang et al., 1993; Best, 1993). This approach requires understanding where to introduce such vortex elements. Another assumption employed in BEM is that the ambient water is incompressible. This assumption need not be true in strong underwater explosions, especially near jet formation.

There are other assumptions used in past studies that are known to be questionable. For example, bubble shape is known to quickly deviate from sphericity at bubble maximum (radius), thereby, violating axisymmetric assumptions used in many past studies (e.g., Szymczak et al., 1993; Zhang and Duncan, 1994) and requiring full 3D treatment. Thus, simple 1-D, 2-D/axisymmetric or incompressible methods cannot completely resolve the bubble and the associated flow dynamics. Furthermore, simplified treatments cannot account for the interaction between the vapor and the liquid phases. To investigate the physics of such interaction, the details of the flow field both inside and outside the bubble is needed. Conventional numerical treatments (even using full 3D) such as Lagrangian or Eulerian techniques are also not practical, since the expansion and collapse of bubbles create severe fluid motion so that a Lagrangian approach (in which the grid points move with the fluid resulting in severe grid distortion) becomes inappropriate, while in an Eulerian approach, adequate resolution in the regions of interest is very difficult to achieve since the bubble's shape changes very rapidly.

A numerical method that includes both compressibility and an ability to capture the entire bubble collapse in complex configuration is used in this study. This numerical code combines lagrangian and eulerian features and is based on the Arbitrary Lagrangian-Eulerian (ALE) scheme developed at the Lawrence Livermore Laboratory. This code has been typically employed in the past to investigate structural fracture, and only limited studies of underwater explosion dynamics have been reported. Past attempts include 2D (e.g., Tipton et al., 1992) and 3D (e.g., Couch et al., 1996) studies of single bubble collapse. However, these studies provided only qualitative information and did not investigate the details of the collapse process. Furthermore, the ability of this code to capture accurately large
explosion bubble collapse has not yet been established. Validation under these conditions is necessary for the next phase of research as noted below.

In this paper, quantitative comparisons with experimental data is carried out to establish the ability of this code. In addition, the ALE3D code is also extended to investigate both single and double bubble explosions in free field and bubble collapse in the vicinity of a rigid wall. The motivation behind these studies is three fold. First, there are not that many 3D unsteady, compressible codes currently available to study such complex flow problems. For example, an Eulerian-Lagrangian (restricted) code called DYSMAS is currently used by the U. S. Navy to study underwater explosions. DYSMAS is made up of three different codes: an Eulerian solver, a Lagrangian solver and a coupler and is quite different from the ALE formulation. The ability and accuracy of this code to simulate complex bubble collapse and shock propagation is sometime very difficult (if not impossible) to verify since very little data is available for problems of interest. One of the objectives of the present study (using ALE3D) is to validate a code that can be used to investigate flow configurations that were studied using DYSMAS thereby proving an independent verification of the predicted results. The validation of the ALE3D code is discussed in this paper. Its application to complex problems of more practical interest will be reported in the future.

The second objective of this study is to investigate the dynamics of the bubble collapse. It was determined experimentally recently (Menon and Lal, 1998a) that during the bubble collapse, Rayleigh-Taylor interface instability is excited as the bubble reaches its minimum radius. Energy balance and stability analysis of the experimental data suggested that this instability could play a role in the bubble collapse and in its eventual breakdown. Earlier studies (e.g., Shephard, 1988) had suggested that hydrodynamic instability could be important in bubble collapse but until recently there had not been any clear evidence of this mechanism in both experimental and numerical data. The recent experiments only provide limited data due to the inherent difficulty in carrying out measurements. The present numerical study (under nearly identical conditions) provides an independent capability to address this fundamentally important issue. Since, the present numerical code is a
fully compressible solver it can capture interface instability due to density variation and therefore, can be used to address this issue. The results discussed in this paper clearly demonstrate that Rayleigh-Taylor interface instability is excited during the bubble collapse and that a portion of the bubble energy is being used to excite this instability. These results are discussed in some detail in this paper.

Finally, the third objective is to validate a simulation tool that can be utilized to investigate large (i.e., strong) underwater explosions near complex surfaces and the subsequent bubble-surface interactions. A numerical solver that can simulate simultaneously fluid dynamic and material surface interaction is needed. In the present study, ALE3D validation is carried out for bubble collapse near a rigid surface. However, the eventual goal is to study underwater explosion dynamics near surfaces buried beneath a sand surface. This type of study will require development and implementation of material models that mimic sand surface properties and is a focus of current research.

2. THE NUMERICAL METHOD

ALE3D (Anderson et al. 1994) is an explicit, 3D finite element code that simulates the fluid motion and elastic-plastic response on an unstructured grid. The grid may consist of arbitrarily connected hexahedral shell and beam elements. The ALE algorithm is implemented by carrying out a complete lagrangian calculation followed by an advection step. After each lagrangian step, a new mesh is created using a finite element based equipotential method to relax the distorted grid. In the Eulerian advection step, the fluid variables such as mass, density, energy, momentum and pressure are reevaluated on the new mesh by allowing fluid motion. The details of the constitutive models are described elsewhere (e.g., Steinberg, 1991) and, therefore, are not described here for brevity.

The advection step uses methods similar to those developed for 2D ALE code, CALE (Tipton, 1990), and the 3D Eulerian code, JOY (Couch et. al., 1983). For pure zones, a second order, monotonic advection algorithm is used (Van Leer, 1977). This advection
step can create mixed material elements (i.e., liquid and vapor). Material interfaces are not explicitly tracked but for the purpose of carrying out mixed element advection, they are inferred from volume fractions. Separate state variables are kept for each component of a mixed element. The well known compressible Euler equations are solved for the Eulerian advection step and therefore, are not shown here for brevity. Only the equations governing the Lagrangian step are summarized below. Further details of this code are given in the above mentioned references. The Lagrangian equations for conservation of mass, momentum and energy are given, respectively as:

$$\frac{d\rho}{dt} = -\rho \mathbf{\nabla} \cdot \mathbf{U}$$  \hspace{1cm} (1)

$$\rho \frac{d\mathbf{U}}{dt} = -\mathbf{\nabla} p + \mathbf{F}_0$$  \hspace{1cm} (2)

$$\frac{d\epsilon}{dt} = -p \frac{d\nu}{dt} + \dot{\epsilon}_0$$  \hspace{1cm} (3)

where $\rho$ is the mass density, $\nu = 1/\rho$ is the specific volume, $\mathbf{U}$ is the Lagrangian velocity, $\epsilon$ is the specific energy, $p$ is the pressure, $\mathbf{F}_0$ is the force from the artificial viscosity and $\dot{\epsilon}_0$ is the heating rate from the artificial viscosity. The artificial viscosity is part of the dissipative algorithm in the code since there is no natural viscous dissipation included in this formulation.

In ALE3D the proper choice of the equations of state for the various materials as required. Only a brief summary is given here primarily to highlight some of the features of the code. The explosion bubble is assumed to be of noncondensible steam and its equation of state is represented as: $p = (\gamma - 1)(1 + \mu)E$ where, $\gamma$ is the ratio of specific heats, the relative volume is $\mu = \rho/\rho_0 - 1$ and $E$ is the internal energy per unit volume.
The surrounding water is modeled using a Gruneisen form given as:

$$p = \rho_0 \left( \frac{1 + \left( 1 - \frac{\gamma_0}{2} \right) \mu - \frac{a}{2\mu^2}}{1 - (S_1 - 1)\mu - S_2\frac{\mu^2}{\mu + 1} - S_3\frac{\mu^3}{(\mu + 1)^2}} \right) + (\gamma_0 + a\mu)E$$

For expanded material, the above expression is replaced by $p = \rho_0 C^2 \mu + (\gamma_0 + a\mu)E$. Here, $C$ is the intercept of the shock velocity-particle velocity curve (the Hugoniot curve), $S_1$, $S_2$ and $S_3$ are the coefficients of the slope of the shock velocity-particle velocity curve, $\gamma_0$ is the Gruneisen gamma and "$a$" is the first order volume correction to $\gamma$. Our present interest is in underwater explosions in shallow water. Comparison of the pressure predicted by the above equation of state using available coefficients in the literature with the data obtained from NIST (1988) in the appropriate temperature and pressure range of interest showed significant discrepancies. In order to address this problem, water regime data was used to obtain new coefficients. It was determined that with these modified coefficients, the equation of state very closely agreed with the experimental results.

A limitation of the current ALE3D input structure is that it requires that the equation of state be in an analytical form (such as Eq. 5). The behavior of a material like water which has a discontinuous transition from steam to water can not be fully represented by an analytical form. Therefore, modifications were carried out to the coefficients to achieve a best fit to the experimental data. A more general approach would involve reading the data directly from tables rather than fitting it to an equation. This capability is currently being investigated for incorporation into the code. High explosive materials are also required to obey a specific form (many forms are included in ALE3D) of the state equation. Modifications are currently being studied to allow investigation of explosive materials and porous surfaces (e.g., sand) for which there are no analytical equation of state.
Another issue is that ALE3D is an inviscid code and surface tension effects are not included in this model. These limitations of the code are acceptable for the present study since the Reynolds numbers (Re) encountered in the flow based on the bubble diameter is the order of 100,000. Thus, neglecting viscous terms in the momentum and energy equations in such high Re flows should not significantly alter the results, as noted earlier (Wilkerson, 1989). Furthermore, the study of the hydrodynamic Rayleigh-Taylor instability can be carried using an inviscid model since this instability is due to density gradients in the flow. Neglecting surface tension is also considered acceptable since it was noted earlier (Plesset, 1954) that surface tension has a very limited stabilizing effect on the bubble interface and is only critical when there is very little pressure difference across the bubble interface (which is not the case in explosive bubbles). Other studies (e.g., Vokurka, 1985) also showed that surface tension effects are important for only very small bubble.

3. RESULTS AND DISCUSSION

In this section, the results obtained for the various cases are summarized and discussed. Wherever possible, comparison with experimental data is carried out to validate the simulated results. Analysis is also carried out to understand the fluid motion generated both inside and outside the bubble during collapse and to understand the dynamics of the instability occurring during collapse.

3.1 Bubble Oscillation Dynamics

Free field oscillating bubble collapse was simulated using test conditions similar to the experimental set-up of Menon and Lal (1998a) to enable direct comparison and to investigate the afore-mentioned interface instability seen in the experiments. The experiments were conducted in a water tank of dimension 2m x 1.5m x 1.5m. The underwater explosion bubble were generated in the experiments by centrally igniting a mixture of explosive gases (e.g., Hydrogen and/or Carbon Monoxide), oxidizer (Oxygen) in a hand-blown glass globe. The glass globe weighed about 5-6 grams, and had an average diameter of 6.34 cm. Thus, the initial explosion source is of the order of the globe diameter. The gaseous explo-
sive mixture was ignited by using an electrical spark generated by spark wires placed inside the glass globe and connected to a 3000 volts DC power supply. Further details are given elsewhere (Menon and Lal, 1998a).

In the present study, the computational domain, the bubble initial explosion energy, the bubble location and ambient conditions were all chosen identical to the values in the experiments. Thus, the freely oscillating bubble is modeled in the center of a 1.5 m x 1.5 m x 1.5 m tank filled with water (as in the experiments). The initial bubble diameter modeled is 6.34 cm, the initial explosion pressure is 9.34 atmospheres (again, as in the experiments) and the water pressure is 1 atmosphere. The only differences between the numerical treatment and the earlier experiments are that the effect of the glass bulb and the sting used to hold the electric spark assembly are not modeled. This omission is particularly important since it removes any possibility that these features of the experimental setup could play a role in exciting interface instability.

The ALE mesh treatment (i.e., mesh regridding) is applied to all the elements in the bubble and in the vicinity of the bubble. But away from the bubble where the bubble explosion does not cause much grid distortion, Lagrangian mesh treatment is used. The number of elements used to resolve the bubble and the surrounding water was varied to confirm that the results are grid independent. For a typical 3D simulation, 72,576 elements were used to discretize the domain, but as many as 220,000 elements were used for grid independence tests. Most of the results on the bubble dynamics discussed here (unless otherwise noted) are on the finest grid resolution. In addition to these studies, the effect of the computational domain (extent and shape) was also studied to ensure that the results do not depend upon these parameters.

Figures 1a and 1b show, respectively, the global grid and a close-up of the grid around the bubble. Most of the grid resolution is employed around the bubble and ALE region while the far field is resolved using a relatively coarse grid. Although various cases have been simulated, only characteristic results are discussed below to highlight the results.
Figures 2a and 2b show respectively, the pressure (normalized by the initial liquid pressure \( p_1 \) which is 1 atmosphere) and the bubble radius (normalized by the bubble maximum radius, \( R_{\text{max}} \)) history during the first oscillation under various test conditions. Here, time is normalized by \( \tau_1 = \frac{R_{\text{max}}}{\sqrt{\frac{p_1}{\rho_1}}} \) which is the characteristic time scale for the bubble to reach its maximum radius. Also, \( p_1 \) is the ambient water pressure and \( \rho_1 \) is the water density. In Fig. 2a, the pressure in the bubble is compared with experimental data (Menon and Lal 1998a). It can be seen that the computed period of oscillation (around 15 ms), the peak pressure and the maximum radius (Fig. 2b) agrees well with data. It can also be seen that the shape and the extent of the computational domain do not play a major role in dictating bubble dynamics. This can be observed from Fig. 2a since the time period and peak pressure changed only by around 10% (increase) and 5% (decrease), respectively, when the walls are moved away by a factor of seven. Further increase in the wall locations did not significantly change the predicted pressure profile. Changing the outer boundary shape (from rectangular to spherical) also did not change the oscillation period and the bubble dynamics (discussed below). This suggests that the geometry of the outer domain does not significantly affect the local physics of the bubble oscillation and that “nearly” free-field explosion condition can be simulated when the walls are moved further away. However, to obtain “true” free field conditions, outflow boundary conditions are required. This feature is currently being investigated for implementation.

The bubble radius history (Fig. 2b) also shows similar agreement with data. During the contraction phase there are some differences between the calculations and the experiments. Note that, the experimental set-up employed a glass globe (which contained the stoichiometric fuel-air mixture) with a metal insert that contained the pressure transducer and the spark generator (Menon and Lal, 1998a) while these features were ignored in the numerical model. In addition, the effect of glass fragments have not been included in the numerical model. Thus, some discrepancies are expected.

In spite of these differences it is interesting to observe that the numerical and the experimental results are in relatively close agreement. This suggests that the experimental
artifacts (identified above) are not significantly modifying the dynamics of the bubble oscillation process. Therefore, the present numerical study serves to provide an independent validation of the results described earlier in Menon and Lal (1998a).

Figure 3a shows the time trace of the non-dimensional pressure in the tank close to a wall. It is very similar to the high frequency pressure oscillations as recorded by the tank transducer in the experiments (Menon and Lal, 1998a) and shown in Fig. 3b. The differences in the frequency content (the high frequency fluctuations are missing from the numerical data) may be attributed in part to coarse grid used in the far field (which would effectively damp high frequency waves). This is demonstrated in Fig. 3a using pressure data obtain on two different grid resolutions.

3.2 Bubble Instability during Collapse

The basic physics of bubble oscillation has been known for some time. The bubble grows after the explosion due to the high pressure inside the bubble. Because of inertia, this results in an over expansion and the pressure inside the bubble falls below the ambient (water) pressure. As a result, the bubble collapses and reaches a bubble minimum at which time the internal pressure again exceeds the external pressure. Thus, an oscillation process is set up and continues as long as there is sufficient energy available. However, energy is continuously lost during the oscillation. As a result, the maximum bubble size decreases during the subsequent oscillations and the bubble typically breaks up into smaller bubbles after 2-3 oscillations. The source of this energy loss leading to bubble collapse has been an issue of investigation for some time. Past studies (e.g., Arons et al., 1948; Pritchett, 1971; Plesset, 1971; Vokurka, 1987; Shepherd, 1988) have attempted to identify these sources. The propagation of the initial shock wave, acoustic losses, compressibility and heat transfer effects, mechanical work done on water and vapor, etc. have all been used to explain the energy loss. However, even when all these effects were included the total observed energy loss still could not be fully explained.
Other sources of energy loss and bubble breakdown have been proposed but without any detailed experimental or numerical proof. For example, onset of various interface instabilities have been suggested (Shepherd, 1988). Recent experimental studies focused on the instability mechanisms (Menon and Lal, 1998a) and the analysis suggested that during the collapse process, the Rayleigh-Taylor (R-T) instability occurs at the bubble interface. Analytically, R-T instability has been shown to occur for the case of underwater collapsing bubbles by Birkhoff (1956) even though the denser fluid (water) is being accelerated towards the lighter fluid (gaseous explosion product), with surface tension having negligible effect on instability. He further showed that the general instability criteria allows two types of instability: the classical R-T instability, which occurs at large wavelengths (small wave numbers) with an exponential growth rate, and the Birkhoff instability, which occurs at small wavelengths with an algebraic growth rate. Analysis of the experimental data (Menon and Lal, 1998a) showed that both instabilities are excited near bubble minimum but the Birkhoff instability is quickly overtaken by the exponentially growing R-T instability so that R-T instability eventually dominates.

Understanding the mechanism of bubble interface instability is important since this would partly explain the energy loss during oscillation and is a possible mechanism for the bubble breakdown. This phenomenon has never been numerical captured (at least to the present authors' knowledge). Thus, the present numerical results appear to be the first such demonstration in open literature. Figure 4a shows the computed bubble surface (only the rectangular computational domain case is shown since the spherical case gave identical result) at the maximum bubble radius and Figs. 4b and 4c show respectively, the bubble shape at its first minimum radius for the rectangular and spherical outer domains. Both these simulations were conducted using 220,000 cells (simulations using coarser grids also showed very similar structure and therefore, are not shown). For comparison, Fig. 4d shows a typical snapshot (at the bubble minimum) from the experiments. Note that Figs. 4b and 4c have been magnified (relative to Fig. 4a) to facilitate visualization. As can be seen, near the bubble minimum large-wavelength wave-like distortion appears along the bubble interface in remarkable similarity with the distortion seen the experimental picture. Note that, there are sufficient number of grid points in the azimuthal direction to reason-
ably resolve the bubble interface and it was confirmed that the interface distortion occurs independent of the grid resolution. It can also be seen that changing the computational domain either by increasing it or by changing the shape (see more discussions below) did not change the form or character of the wave-like distortion on the bubble surface. Thus, this interface distortion feature appears to be a physical phenomenon.

Figures 5a and 5b show, respectively, the velocity vector field at the bubble maximum in the vicinity of the bubble for the rectangular and spherical domains and Figs. 6a and 6b show respectively, the velocity vector field just after the bubble minimum for the rectangular and spherical domains. Figures 5a and 5b shows that the fluid (both the gas inside the bubble and the liquid outside) are undergoing an outward motion of the bubble. Changing the shape of the computational domain does not change the local fluid motion significantly. In Figs. 6a and 6b, the bubble is just starting to expand after reaching the minimum radius and the fluid has reversed direction and is in the outflow direction. It can be seen that near the bubble minimum the fluid motion has lost spherical symmetry. These figures show that both the bubble shape and the associated fluid motion in the vicinity of the bubble are relatively insensitive to the shape of the computational domain. Of course, the oscillation period and the bubble maximum radius are modified slightly when the outer domain is extended, as shown earlier in Figs. 2a and 2b, but the local fluid dynamics appears to be relatively insensitive.

As noted above, Figs. 6a and 6b show that fluid motion inside the bubble is no longer symmetric indicating a deviation from sphericity. This is consistent with observations made earlier (Cole, 1948). The deviation from sphericity and the formation of waves on the bubble interface are characteristics of Rayleigh-Taylor instability.

The R-T instability can also be inferred by analyzing the variation of the radius with time. As shown by Birkhoff (1956), for the case of underwater bubble, collapsing bubbles are unstable even though the denser liquid (water) is accelerated towards the lighter fluid (explosion product). He showed using a perturbation analysis of the one-dimensional Trilling equation (Trilling, 1952) that the general stability criteria is given by
\( \frac{d^2 R}{dt^2} < 0 \), where \( R(t) \) is the mean bubble radius. This criteria can be evaluated from the current numerical (and the earlier experimental) data. Figure 7a shows this quantity near bubble minimum. Clearly, near the bubble minimum the R-T stability criteria is violated and confirms that R-T instability is occurring near bubble minimum.

Another approach to analyze the bubble surface instability is to use Liapunov's theorem for nonlinear systems (Menon and Lal, 1998a; Kolk and Lerman, 1992). The sufficient condition for stability using this analysis for the bubble expansion and collapse process reduces to \( \frac{dz}{dt} > 0 \), where \( z \) is the normalized bubble radius \( z = \frac{r}{R_0} \). Figure 7b shows the variation of this quantity as a function of normalized time. It can be seen that the bubble growth phase is stable. However, the Liapunov stability criterion is violated during the bubble collapse phase. Thus, it appears that the Rayleigh-Taylor instability does occur as the bubble nears its minimum radius.

The analysis of the earlier experimental data indicated that both Birkhoff and Rayleigh-Taylor instabilities are likely to be excited near the bubble minimum. The Birkhoff stability criteria is part of the general stability criteria (see Menon and Lal, 1998a). The experimental data suggested that both types of instabilities are excited but that the exponentially growing R-T instability quickly dominates the interface distortion. The present analysis confirms that the general stability criteria is violated near the bubble minimum and the visualization of the bubble shape shows that the interface instability appears as a large wavelength distortion which is characteristics of R-T instability. No sign of Birkhoff instability (which is short wavelength, high frequency instability) is observed in the calculations. This may be due to lack of sufficient grid resolution to resolve the high frequency, short wavelength distortion even with the high resolution employed. Nevertheless, the results clearly confirm the earlier observation that R-T instability does occur during the bubble collapse phase.
3.3 Scaling Analysis

To be useful for practical applications, the bubble motion should preserve both geometric and dynamic similarities (Cole 1948). Geometric similarity implies invariance of \( R_{\text{max}}/d \) and \( R_{\text{max}}/R_{\text{min}} \) and dynamic similarity essentially implies invariance of the Froude number, \( Fr=U^2/gL \), which in turn implies an invariance of \( T^2/R_{\text{max}} \). Here, \( R_{\text{min}}, d \) and \( T \) are, respectively, the bubble minimum radius, water depth and time period of oscillation. Estimates for these parameters for the first bubble oscillation are given below for the present study and compared to the past deep sea studies.

For the present studies of gas explosion, \( R_{\text{max}} = 3R_0, R_{\text{min}} = R_0, R_0 = 3.15 \, \text{cm} \) (initial bubble size), \( T = 0.015 \, \text{sec} \), and \( d = 0.6477 \, \text{m} \). This gives \( R_{\text{max}}/(d+10.33) = 8.607\times10^{-3}, R_{\text{max}}/R_{\text{min}} = 3, \) and \( T^2/R_{\text{max}} = 2.381\times10^{-5} \, \text{s}^2/\text{cm} \). In a deep sea explosion of 249.5 grams of tetryl charge fired at a depth of 91.44 m below the water surface [Cole 1949, page 271], \( R_{\text{max}} = 45.11 \, \text{cm}, R_{\text{min}} = 12.7 \, \text{cm}, R_0 = 3.5 \, \text{cm}, T = 0.028 \, \text{sec}, \) and \( d = 91.44 \, \text{m} \). This gives \( R_{\text{max}}/(d+10.33) = 4.432\times10^{-3}, R_{\text{max}}/R_{\text{min}} = 3.55, \) and \( T^2/R_{\text{max}} = 1.738\times10^{-5} \, \text{s}^2/\text{cm} \).

Data from TNT explosions at a depth of 152 m (Arons et al. 1948) suggests similar results: \( R_{\text{max}}/(d+10.33) = 2.55\times10^{-3}, R_{\text{max}}/R_{\text{min}} = 2.255, \) and \( T^2/R_{\text{max}} = 1.596\times10^{-5} \, \text{s}^2/\text{cm} \). These comparisons suggest that the present study preserves reasonably close geometric and dynamic similarities with the past deep sea explosion data for all the parameters except for \( R_{\text{max}}/d \) (which is clearly impossible since current studies are in shallow water). This good agreement is primarily due to the fact that in the present study, \( R_{\text{min}} = R_0 \) whereas in deep sea studies they are significantly different.

3.4 Energy Loss Estimate

The explosion energy is redistributed among the various modes (e.g., shock wave energy, potential, kinetic and dissipated energies, etc.) initiated by the explosion. Earlier (Menon and Lal, 1998a) used the experimentally obtained data to evaluate how the total energy
from explosion is partitioned between these modes. The energy balance for an expanding system can be written as (e.g., Vokurka, 1987): \( \Delta E_i = \Delta E_p + \Delta E_d + E_k \), where, \( \Delta E_i \) and \( \Delta E_p \) are respectively, the change in internal and potential energy of the gas bubble (from its initial state at time \( t=0 \)), \( \Delta E_d \) is the energy dissipated and \( E_k \) is the liquid kinetic energy. Estimates for each of these terms can be obtained (or approximated) from the simulation.

For the experiments, the total (nondimensional) energy was determined from the relation \( \bar{E}_0 = QW/E_0 \), where \( Q \) is the detonation energy per kg of explosive, \( W \) is the weight in kg of the explosive and \( E_0 = 4\pi R_0^3 p_1/3 \) is the energy of the initial gas volume. For a stoichiometric hydrogen-oxygen mixture, \( Q = 242 \) KJ/mole of hydrogen and using representative values it was determined that \( \bar{E}_0 = 66.9 \). In the present numerical study, the explosion pressure was matched to the experimental pressure which results in an energy estimate of \( \bar{E}_0 = 56.5 \). The discrepancies is likely to be due to the idealized estimate used in the experiments where it was assumed that the entire explosion energy is converted to pressure. No correction for incomplete combustion, energy to break up the glass bulb, and energy lost to the sting were included in the experimental estimate. Thus, the present numerical estimate is considered reasonable. \( \bar{E}_0 \) is total amount of explosion energy which is to be distributed into various modes as the bubble expands. The change in internal energy (nondimensionalized using \( E_0 \)) as the bubble expands and reaches bubble maximum can be written as (Vokurka, 1987):

\[
\Delta E_i^e = \left( \frac{P_0}{P_1} \right) \left( 1 - \left( \frac{r}{R_0} \right)^3 (\gamma - 1) \right)^{\gamma/(\gamma - 1)}
\]

(5)

where the subscript 0 indicates the first bubble oscillation and the superscript \( e \) indicates the expansion phase. The change in potential energy (again nondimensionalized by \( E_0 \)) is:

\[
\Delta E_p^e = \left( \left( \frac{r}{R_0} \right)^3 - 1 \right)
\]

(6)
Here, \( r > R_0 \) is the instantaneous bubble radius as the bubble expands from its initial diameter \( R_0 \) to its first maximum bubble radius \( R_{\text{max}} \). Further, since \( E_k = 0 \) near the bubble maximum (Vokurka, 1987), an estimate for the energy dissipated based on the above relations suggests that approximately 14% of the energy is dissipated as the bubble expands and reaches its first maximum. This estimate is lower than the experimental result of 26% (Menon and Lal, 1998a). However, as mentioned earlier, this discrepancy may be due to uncertainties in the experimental estimate.

A similar analysis can be carried out for the contraction phase (i.e., as the bubble collapses from the bubble maximum and reaches its minimum radius). For the contraction phase the energy available is limited to the amount left after the above noted fraction of the explosion energy has been dissipated. However, the current numerical study is based on an inviscid code and thus, the energy lost from the bubble during the expansion phase is not fully dissipated. Some fraction is dissipated due to numerical and artificial dissipation in the numerical scheme. This fraction can be determined by determining the total energy in the computational domain during the calculation. Estimate suggests that approximately 3-4% of the energy is dissipated due to artificial dissipation.

The change in potential energy during the contraction phase can be written as (Vokurka, 1987):

\[
\Delta E^c_p = \left( \frac{R_{\text{max}}}{R_0} \right) \left[ 1 - \left( \frac{r}{R_{\text{max}}} \right)^3 \right]
\]

(7)

where, the superscript "c" indicates the contraction phase. The change in internal energy can also be expressed in terms of the bubble properties as:

\[
\Delta E^c_i = \left( \frac{1}{\gamma - 1} \right) \left( \frac{p_{\text{max}}}{p_1} \right) \left( \frac{R_{\text{max}}}{R_0} \right)^3 \left[ \left( \frac{r}{R_{\text{max}}} \right)^{-3(\gamma - 1)} - 1 \right]
\]

(8)

Using the computed data in (7) and (8) it appears that around 15% of the total energy is dissipated between the first bubble maximum and minimum. This amount is in very
good agreement with the experimental estimate of 15%. Since it is estimated here that around 3-4% is numerically dissipated, it leaves around 11-12% unresolved. A possible avenue for this energy loss is in the excitation and amplification of the interface instabilities which lead to the eventual bubble collapse.

In summary, the above study confirms the earlier experimental observation that Rayleigh-Taylor instability occurs near the bubble minimum. Stability analysis confirms that this instability can occur and energy partition analysis indicates that there is a reasonable amount of energy missing that could be used to excite this instability.

3.5 Bubble-Wall Interactions

Bubble collapse near a rigid wall is of significant interest due to its ability to cause serious damage to the structure (cavitation damage of underwater propellers is a well known example). This is because when the bubble collapses near a rigid surface, a strong reentrant water jet is formed that is directed towards the wall. The peak impact pressure on the wall due to this water jet can be substantially higher than the explosion pressure. The dynamics of this collapse process has been under investigation for some time; however, experimental capability to record the effects of the interaction process is limited due to the difficulty in accessing the bubble collapse region. Past numerical studies have been able to capture the collapse process but, as noted before, such calculations resorted to obtaining information from experiments to ensure that the simulation's initial conditions agreed with experimental data. Furthermore, earlier studies did not fully resolve the fluid motion both outside and inside the collapsing bubble. Thus, the earlier studies failed to fully explain the dynamics of the impact process and the effect of initial bubble location on the measured impact pressure on the wall.

In the present study, the impact process is investigated as a function of explosion strength, gravity orientation (i.e., wall above and below the bubble), and the bubble location relative to the wall. Conditions were chosen to mimic the experimental conditions as much as possible. For a typical simulation, a total of 25,000 elements are used to resolve
the bubble and the wall region and another 35,000 elements are used for the rest of the
domain. Again, grid independence studies using over 200,000 elements were conducted to
ensure that the results were grid independent. Characteristic results that highlight some
interesting physics of the problem are discussed here.

Three cases are analyzed here to highlight the various physical phenomena associated
with the bubble collapse near a wall and its dependence on the proximity of the wall.
These cases correspond to an initial bubble location of 5 cm (case 1), 6.34 cm (case 2) and
4 cm (case 3) above the wall (gravity inhibiting case, i.e., bubble above the wall as in the
experiments, Menon and Lal, 1998b), respectively. Figures 8a-c show the velocity field at
various stages of the collapse for case 1. At the bubble maximum, the bubble is almost
spherical but begins to distort as it collapses. Since there is less volume of water between
the wall and the bubble during the collapse, the pressure drop is quite large relative to the
pressure drop on other sides of the bubble. This pressure differential further forces the
bubble towards the wall. This migration of the bubble causes the water surrounding the
bubble to be directed away relative to the bubble geometric center and thereby, creating a
higher pressure on the upper side of the bubble away from the wall resulting in the well
known Bjerknes force (Blake et al., 1986). The iterative combination of these effects (as
the bubble moves closer to the wall) cause the water to penetrate the bubble from the high
pressure side and to form a high-speed water jet that impacts the rigid surface. As this jet
impacts the rigid plate, a toroidal vortex ring bubble is formed, as shown in Fig. 8c. This
ring bubble qualitatively compares well with those observed in both experiments (e.g.,
Tomita and Shima, 1986; Vogel et al., 1989) and in numerical studies (Best, 1993; Szymczak
et al., 1993; Zhang and Duncan, 1994).

Figures 9a and 9b show, respectively, the velocity fields for cases 2 and 3 at a time just
before the jet formation. Comparison with Fig. 8b indicates that when the bubble is far-
thest from the wall (case 2) it has time to collapse to a smaller volume and thus, generates
higher velocities while in case 3, the bubble cannot collapse as much as the other two
cases and thus, lower velocities are observed. The implication of these differences on the
impact pressure generated on the wall is discussed below.
Figure 10 compares the time traces of peak impact pressure on the rigid wall for the three cases. The non-dimensional time periods (scaled with the time scale, $\tau_l$), are 2.021, 2.017 and 2.048, respectively, for the three cases. The peak velocities are observed slightly before the jet impacts the wall. These velocities scaled with non-dimensional velocity scale based on the ambient (water) pressure and water density ($\sqrt{\rho_l} \sqrt{p_1}$) are 6.9, 7 and 6.4, respectively. These values compare well with earlier results (e.g., Chahine and Perdue, 1988; Blake et al., 1986). It can be observed that even though case 2 generated the maximum velocity, it does not correspond to the maximum peak impact pressure. This is because the jet loses some of its kinetic energy as it penetrates through the water layer between the toroidal bubble and the wall. In case 1, the lower and the upper bubble surfaces are closer to (in fact just touching) the wall before impingement and thus, there is no water layer for the jet to penetrate. This results in higher impact pressures on the wall. On the other hand, for case 3, the bubble is so close to the wall that it does not have the freedom to fully expand before collapsing. As a result, the overall jet speed achieved in this case is lower than case 1 and as a result, the impact pressure is lower than for case 1.

These results suggest that due to two different physical reasons, there is an optimum distance above the wall for which a peak impact pressure is achieved. This has been confirmed by direct comparison with experimental data. Figure 11 shows the peak impact pressure (normalized by the initial explosion pressure) variation with normalized distance of the bubble center above the wall. Two sets of numerical results for gravity inhibiting case (with wall below the bubble) and gravity aided case (wall above the bubble) are compared to the experimental data (gravity inhibiting case). Both these cases show similar behavior except that there is a slight shift to higher impact pressure for the gravity aided case. This agrees with physical intuition that buoyancy aids the migration of the bubble towards the wall and thus, enhances the impact process. It is likely that the effect of buoyancy will be more apparent for deep sea large-scale explosions. This is an issue currently under study.
It is clearly seen in Fig. 11 that the present numerical study has captured both the trend and the magnitudes of the peak impact pressures seen in the experiments for a range of initial bubble locations. The experimental data shows a slightly lower maximum peak impact pressure at a farther distance when compared to the numerical data. However, this discrepancy can be attributed primarily to the limitations of the experimental set up (Menon and Lal, 1998a; 1998b). As noted earlier, the experiments used a metal sting (which contained a pressure transducer) to hold the glass bulb that contained the explosive mixture. The presence of this sting and the glass fragments very close to the wall (not included in the numerical study) are likely to effect the measurements.

In spite of this discrepancy, it is encouraging to note that there is considerable agreement between the numerical and experimental results. This provides confidence in the capability of the simulation model and provides a research tool that can now be utilized for detailed studies of more realistic explosions both in shallow water and deep sea situations. Current studies (to be reported in the near future) are focused on bubble dynamics due to realistic high explosive shaped (e.g., cylindrical detonation cord) charge explosions near rigid surfaces.

Finally, Fig. 12 shows the variation of impact pressure along the wall for case 1 just after the jet formation. This figure shows that the jet is very narrow and is not spreading at all. This is reflected in the observed high impact pressure at the jet centerline. There is a rapid decay of pressure away from the impingement point which is very typical of stagnation point flows. The need to adequately resolve this highly coherent but narrow jet structure is one of the reason that makes it very difficult to simulate the bubble collapse process accurately using conventional schemes without causing prohibitive increase in computational cost. The ability of ALE3D to capture such a flow field without requiring enormous increase clearly demonstrates its capability.
4. CONCLUSIONS

In this paper, an unsteady, 3D finite-element compressible code has been successfully applied to study underwater explosions. Results clearly demonstrate that the ALE3D code can be used for bubble explosions. The basic code has been validated using shallow water explosion data. It has been shown that nearly all the features observed in experiments have been captured in these studies. There is excellent qualitative and reasonable quantitative agreement with the experimental data.

Results show that during collapse of a freely oscillating bubble, the bubble loses spherical symmetry and the bubble interface becomes unstable due to the excitation of Rayleigh-Taylor instability. Stability analysis confirms that this instability can occur and energy partition analysis indicates that there is a reasonable amount of energy missing that could be used to excite this instability. This verifies the earlier experimental demonstration that R-T instability is one of the primary mechanisms in bubble collapse and breakdown. The simulation of the collapse of a bubble near a rigid wall showed that the jet velocity and the impact pressure on the wall are functions of the explosion pressure and the distance of the bubble from the wall. The results indicate that for a given explosion pressure there is an optimal distance of the bubble from the wall which gives the maximum impact pressure. This trend and the peak impact pressure are in good agreement with the experimental results. It has been shown that the optimal location is due to two different physical effects as the bubble collapses near the wall. The evolution of the vortex ring bubble, reported in earlier experimental and numerical studies, is also accurately predicted.

Some limitations of the current ALE3D code have also been identified. However, most of these limitations can be corrected by proper modifications to the code. Current effort is directed towards this goal so that more realistic (i.e., using real explosives) deep and shallow water explosion studies can be carried out. Extension to the code to handle sand surface properties is also being investigated for eventual study of explosions near buried surface.
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Figure 12. Variation of the pressure on the wall away from the jet stagnation point. Plot shows that the impinging jet is narrow and highly focussed. This results in a high impact pressure.
Experimental Studies of Underwater Explosions near Exposed and Buried Rigid Surfaces

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ABSTRACT

Underwater explosion bubbles are created near an exposed or buried rigid boundary by detonating a mixture of oxygen and Carbon Monoxide in glass globes submerged in a water tank. A variable depth of either play sand or general purpose purge clay is used to bury a solid steel plate in order to simulate explosion over a buried rigid boundary. Eight pressure transducers mounted on the plate are used to map the pressure distribution on the plate and instrumented tubes and plugs measure pressure inside and outside the bubbles. A cinematographic technique is employed to capture entire interaction process. There exists a critical distance above the plate where the reentrant water jet produces the maximum impact pressure on the plate. The water jet is very focused and symmetrical about the center of impact. The effect of covering the flat plate with sand or clay is in general, to reduce the impact pressure and smoothen its distribution over the plate. However, when an explosion occurs very close to the sand surface loose sand particles are ejected and displaced as the bubble expands. The reduces the effective sand thickness and as a result, an increased impact pressure is achieved. This recovery of impact pressure increases in shallow water cases due to the free surface effect. Explosions were also carried out above clay surface to view the shape of the crater formed. Results show that double craters (i.e., secondary crater within the primary crater is formed for certain initial locations of the explosive above the surface.)
1. INTRODUCTION

Underwater detonation of explosive material converts the unstable material into a more stable gas void at high temperature and pressure. The high pressure of the remnants of an underwater detonation sets forth an expansion-collapse cycle of the resulting underwater explosion bubble which is repeated several times before the bubble goes through interface instabilities and eventually disintegrates into a cloud of smaller bubbles. The interface instability problem is an interesting and complex subject and has recently been addressed based on experimental and analytical methods by Menon and Lal (1998). Various instability mechanisms at play during the bubble oscillation cycles were addressed and it was shown that the Rayleigh-Taylor instability occurs during the bubble collapse and plays a major role in the eventual collapse of the bubble.

The presence of a solid surface in the vicinity of a pulsating bubble manifests itself as an asymmetry in the flow field. A dominant feature in the collapse of a bubble in such a flow is the development of a reentrant water jet. The asymmetry in the flow causes one side of the bubble to accelerate inward more rapidly than the opposite side resulting in a high-speed reentrant jet which pierces the bubble in the direction of its migration and produces an impact pressure much larger than the explosion pressure. This increased pressure on the surface can cause structural damage especially when the explosion energy (and hence the bubble size) is large. Other asymmetries (i.e., gravity or a free surface) can also cause the formation of the reentrant jet. The jets caused by gravity are directed upward and those caused by free surfaces are directed away from them.

Much of the research activities in the area of underwater bubble dynamics have been focused on the behavior of cavitation bubbles. Vapor and gas bubble dynamics are of great
practical interest in prediction and prevention of cavitation erosion of marine propeller and turbine blades. The fact that the collapse of these tiny microbubbles near a solid boundary is also characterized by the formation of reentrant jets leading to formation of damage pits on the solid wall has led to the huge amount of research activity in the field. Researchers have resorted to experimental (e.g., laser induced bubbles) and various numerical (e.g., boundary-integral method) techniques to model and predict the behavior of cavitation bubbles. On the other hand, large bubbles are created by larger underwater explosion. These large bubble due to their tremendous destructive capabilities upon collapse near a rigid boundary find practical applications in underwater weaponry. However, detailed measurements and imaging of pulsating bubbles formed during deep sea explosions are very difficult due to a variety of obvious reasons and therefore, there is insufficient data available to analyze the dynamics of interaction of such large bubbles with a solid boundary. Controlled experiments as described in this paper can be very helpful in providing some insight into the problem.

Recently, a series of experiments were carried out to investigate underwater explosions in shallow water (1 atmosphere ambient pressure) to understand the dynamics of bubble-wall interaction in such flows and to investigate feasibility of targeting and destroying mines buried in beaches. In this configuration (shown in Fig. 1), the free water surface is close enough to the bubble-wall interaction region to allow it to play a role in modifying the dynamics of the bubble collapse. The free surface provides a constant pressure boundary in close proximity to the wall. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration (e.g., Birkhoff, 1957; Blake and Gibson, 1981; Chahine, 1977; Chahine, 1982; Wilkerson, 1989). Since, both the Bjerknes force and the buoyancy force, the two competitive forces acting on a bubble near a free surface, act in the same direction, the presence of the free surface above the bubble collapse region is likely to increase the net impact pressure on the wall. Another issue that was investigated is the behavior of the
impact process when the rigid surface is buried below a layer of sand as would be the case for buried mines. Some interesting results have been obtained and summarized in this paper.

This paper reports the results of the experiments carried out in a laboratory water tank to study the interaction of a pulsating bubble created by underwater explosion of flammable gas contained in a glass globe with a nearby solid wall. The wall was later covered with sand or clay to simulate explosion over a buried wall. The location of the globe with respect to the wall was varied for a parametric study. The water surface was lowered to study shallow water interactions.

2. EXPERIMENTAL PROCEDURES

Underwater explosion experiments near a solid boundary were conducted in a wooden tank of dimension 2 m × 1.5 m × 1.5 m, coated with fiberglass resin from inside. The tank has windows on three sides for optical imaging. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe over a steel plate of dimension 36.83 cm × 60.96 cm × 0.635 cm (shown in Fig. 2). Two different sizes of glass globes were used for present experiments with average radii of 2.54 cm and 3.2 cm. The glass globe has an electric spark ignition system connected to a 3000V DC power supply that ignites the premixed fuel-air stoichiometric mixture contained in the globe. The explosion takes place at a constant volume until the globe bursts. Since the experiments were conducted in a laboratory shallow water setup and using a gaseous explosive mixture, the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep sea explosions. Recently, Menon and Lal (1998) addressed the dynamics and instability issues of such a bubble in free field and they showed by means of extensive geometric and dynamic similarity analyses that the explosion
bubble thus formed is a reasonable subscale approximation of a deep sea underwater explosion bubble. They have presented detailed scaling parameters, energy partitioning and also various interface instability mechanisms. Repeatability and experimental uncertainty have also been addressed and it has been shown in particular that repeated experiments produced error bands for the explosion pressure, maximum radius and time period of 5.88%, 3.7% and 6.06%, respectively.

The pressure inside the bubble during its oscillation was measured by a KISTLER transducer which is mounted inside the plug. Additionally, eight KISTLER pressure transducers were mounted on the plate as shown in Fig. 2 to obtain a surface distribution of the impact pressure field. These dynamic pressure transducers have low and high frequency response of 0.001 Hz and 50 kHz, respectively, and the resonant frequency of 300 kHz. They are, therefore, well suited for the current experiments as the bubble oscillation frequency (time period of approximately 15 ms) lies well within the above mentioned bounds. Signals from these pressure transducers were digitized using National Instrument's AT-MIO-16X analog-to-digital converter board, and were recorded into a microcomputer. Ten channel data recording was performed with a sustained sampling rate of 10,000 samples per second per channel.

The distance between the globe and the plate was varied to investigate the effect of solid wall location relative to the explosion. The plate was later covered with sand or clay to simulate explosion over a buried wall. The thickness layer of sand and clay above the instrumented plate was also varied to determine how the impact pressure is affected by the porous material above the plate. The water surface was lowered to study shallow water bubble-wall, bubble-sand-wall and bubble-clay-wall interactions. The tank was illuminated by either direct overhead flood lights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis. The optical recording of the bubble motion was performed by a CCD enhanced digital video camera at a speed of 1000 frames per second in order to obtain a full screen image.
3. VISUALIZATION OF THE BUBBLE-WALL INTERACTION

The collapse process near the wall (with and without the presence of the sand layer) was dynamically similar. Figure 3 shows a typical collapse process of a bubble near a rigid plate. The bubble expands subsequent to the explosion, however, the extend of the expansion (for a free field explosion, these bubbles were found to expand up to three times the initial diameter) depends on the relative position of the free surface and the rigid plate. As the bulb is brought closer to the plate, up to a certain distance an increase in the impact pressure is recorded by all transducers. If the distance between the bulb and the plate is further reduced beyond the certain distance, a reduction in the impact pressure is noted. The optimum distance is determined to be \( d/R_o = 2 \); where \( d \) is the distance between the bulb and plate, and \( R_o \) is the initial globe radius. Such an optimum distance was also observed in recent numerical studies (Menon and Pannala, 1998) where it was shown that this is due to effect of two different physics. When the bubble is too close to the wall it cannot fully expand to its maximum diameter and thus, does not generate sufficient jet speed during collapse. On the other hand, when the bubble is too far away, there is a water layer between the collapsing bubble and the wall that the reentrant water jet has to penetrate. This also reduces the peak impact pressure on the wall. When the bubble is at the optimum location, the bubble expands to its maximum and just touches the wall before undergoing the collapse process.

The pressure recorded by the transducer no. 8 located at the center of the plate and directly underneath the globe was found to be the maximum, \( P_{imp}/P_o = 4.19 \) at the optimum distance, where \( P_{imp} \) is the impact pressure on the plate at the center and \( P_o \) is the explosion pressure inside the bubble. Figure 4 shows the impact pressure (normalized by the explosion pressure) variation with distance from the plate. It also shows the pressure traces recorded by two off-center transducers (see Fig. 2 for locations). Although the impact pressure is noted to be
highest for $d/R_o = 2$, the actual magnitude of the pressure is much lower than the value at the central location. This implies that the impinging water jet is highly focused and does not spread at all. Figure 5 shows the distribution of the impact pressure on the plate and confirms earlier conjecture.

For a bubble collapse at the optimum distance, the time period of oscillation is found to be 19 ms, while it is around 15 ms in free field configuration. Therefore, as the bubble is brought close to the surface, an increase in the time period of oscillation is observed. If the bubble is brought further close to the surface beyond the optimum distance, the time period reduces slightly to 18 ms.

Experiments were also conducted to simulate underwater explosion over a buried surface as near a beach by covering the plate with varying depths of sand on the top. The typical parameters are identified in Fig. 1. The bubble collapse process optically obtained for this case is shown in Fig. 6. The bubble is once again attracted towards the plate and a reentrant jet is formed in the bubble in the direction of its migration. It can be seen from this figure that the bulb is almost touching the sand. The effect of covering the plate with sand is to reduce the impact pressure on the plate. Figure 7 shows the impact pressure at the center of the plate for various sand depths. When the plate is covered with the sand while maintaining the same distance between the plate and the bulb, a reduction in the impact pressure at the center of the plate is observed. However, when the sand depth is further increased so as to bring the bulb closer to the sand, a partial recovery of the impact pressure occurs as shown in Fig. 7. Here, the distance of the bubble from the sand surface becomes more important. An explosion close to the sand surface creates a crater in the sand. The sand particles are ejected or displaced from the center and are deposited at the rim of the crater in the form of ejecta. This process is quite similar to crater formation even when there is no water present as on planetary surfaces. The key result of the ejecta formation is that the effective sand thickness at the center decreases and this
in combination of the water jet can be attributed to slight recovery in the impact pressure. As the
bubble is moved away from the plate, crater formation is not observed and the impact pressure is
solely due to the water jet impact.

In order to simulate the explosion near a beach and to investigate the effect of the
proximity of the free surface, the water depth, \(d_w\), was lowered. Since water free surface is
known to repel the bubble, the free surface should aid in the impact process. This was confirmed
in the experiments. Figure 8 shows the impact pressure on the plate for cases with sand covering
as a function of varying water depth. A lower water depth increases the impact pressure and
decreases the time period of oscillation. The time period of oscillation of the bubble shown in
Fig. 6 is about 16 ms. When the water depth is decreased for this experiment such that the bulb
center is only about 10 cm below the free surface, the period of oscillation further reduces to 14
ms.

The water jet impinges on the flat plate almost symmetrically. This fact can be seen in
Fig. 9 which shows the impact pressure distribution over the plate covered with 5.08 cm of sand.
Notice also that the distribution of impact pressure is not as focused as it used to be for no sand
cases (compare the pressure drop between center and off-center transducers from Fig. 4).
Therefore, the effect of burying the flat plate under sand is to smoothen the impact pressure
distribution. Figure 10 shows the impact pressure recorded at the plate center for varying
distance of explosion over the sand and that of explosion depth. For shallow water cases, two
peaks are observed with varying distance of explosion from the plate. Examination of the
recorded images suggests that the second peak is due to similar dynamics as in no sand cases.
That is, the peak corresponds to the case when the bubble touches the sand surface at its
maximum diameter. The first peak, however, is due to the crating phenomena as discussed
erlier as a case of explosion close to sand surface. This crating phenomena can be visually seen
in next two figures. Figure 11 shows an explosion close to sand surface and does indeed exhibit
the crating phenomenon (the formation of the crater lip can be seen). Figure 12 corresponds to an explosion away from the sand surface and the bubble completes one oscillation cycles before it hits the plate. No crating is observed to be taking place. As the water depth is increased, the peaks in impact pressure vanish (Fig. 10). This signifies the fact that the water free surface actually helps in crating.

The dynamics of bubble collapse over a clay layer is very much identical to those corresponding to the sand. The explosion leaves behind a distinct crater in the clay and the shape and size of the crater depends on the proximity of the explosion and the mechanical properties of the clay. An explosion close to the clay surface makes a crater in which the existence of two craters can be identified. The outer (wider but shallow) crater can be attributed to the bubble expansion and the deeper inner crater is due to the impact of water jet. Figure 13 shows a typical digitally regenerated crater formed in a 7.6 cm thick clay layer by an explosion occurring 2.54 cm above the clay surface. As the explosion is moved away from the crater surface, the outer crater becomes non-existent and the inner crater becomes predominant. The issues concerning crating phenomena is still under investigation and the results will be reported in the future.

4. ANALYSIS OF THE RESULTS

The fact that the interaction of two bubbles oscillating in phase with each other is physically equivalent to that of a single bubble near a solid boundary was exploited by Lal and Menon (1998) to demonstrate how the water jets formed in the two bubbles violently collapse on to each other. The dominant feature of the asymmetric collapse of a bubble near a solid boundary is the possibility of a liquid/solid impact (the impact of the water jet on the solid surface) with the generation of a “water hammer” pressure given by (Field, 1993)
\[ P_{\text{sh}} = V \rho_i C_i \rho_s C_s / (\rho_i C_i + \rho_s C_s) = V \rho_i C_i \]  

(1)

where \( V \) is the jet impact velocity and \( \rho_1, \rho_s \) and \( C_1, C_s \) are the densities and shock wave velocities of water and solid respectively. The speed of the reentrant jet in a bubble collapsing in a quiescent liquid near a solid wall at the time it impacts the opposite surface of the bubble is given by (Brennen, 1995)

\[ U_j = \xi (\Delta p / \rho_i)^k \]  

(2)

where \( \xi \) is a constant and \( \Delta p = p_{\text{oo}} - p_m \). Here, \( p_{\text{oo}} \) is the hydrostatic pressure at the explosion depth and \( p_m \) is the pressure inside the bubble at its maximum radius. The value of the constant \( \xi \) depends on the size of the bubble and its distance from the wall and has been empirically determined for the cavitation bubbles.

The situation where the bubble is initially located at a considerable distance away from the solid wall is also very interesting. The solid wall in such a case may never experience water hammer pressure because of the reason that the reentrant jet may never reach the solid wall. The reentrant jet may still be formed depending on the strength of Bjerknes force as compared to that of the buoyancy force. The reentrant jet upon hitting the opposite wall of the bubble produces a pressure pulse in the liquid of magnitude \( \rho CV/2 \), which can subsequently interact with the solid wall.

The penetration of the original bubble by the reentrant jet leads to the formation of two toroidal vortex bubbles. For the bubbles starting their oscillation at a moderate distance from the wall, the expansion and collapse is nearly spherical and the reentrant jet penetrates the liquid between the opposite wall of the bubble and the solid boundary while the bubble has already started its second oscillation cycle (i.e., rebounding). When the bubble is initially very close to
the solid wall, the bubble may never attain a spherical shape starting from the very beginning of the expansion phase. For instance, when the glass bulb is initially touching the solid wall, the ensuing bubble elongates in the direction perpendicular to the wall and there would be no amount of liquid trapped between the bubble and the wall. In fact, the graph of the peak pressure produced on the solid wall with respect to the initial distance of the globe from the wall shows that there is an optimum distance from the wall where the peak pressure on the plate is the maximum. A bubble which is farther from the wall collapses to a smaller size and can concentrate its energy over a smaller volume. Earlier numerical studies (Menon and Pannala, 1998; Pannala and Menon, 1997) have presented a detailed convincing argument for the existence of such an optimum distance. Besides, a jet that strikes the wall directly is more capable of damage in spite of its lower jet speed than the one with a higher jet velocity but has to first pierce the liquid volume between the bubble and the wall.

Experimental data indicate that the bubble becomes prolate quite early in the collapse phase, reaching about (12%) prolate just prior to the start of jet formation. Jet formation depends on the curvature of the bubble-liquid interface (Lauterborn and Bolle, 1975). The presence of a nearby solid boundary reduces the motion of the bubble wall closer to it. As the bubble first becomes elongated in the direction normal to the wall, the point of the bubble farthest from the wall will have the maximum curvature and therefore would be the prime candidate for the origin of jet formation. For the pressure difference, Δp, of 1 atm, Plesset and Chapman (1971) found the jet velocities for two cases of bubble location with respect to the solid wall to be around 130 and 170 m/sec, respectively. Perdue (1988) observed the maximum jet velocity to reach a value given by 11.1(Δp/ρ)^1/2.

Since the underwater explosion bubbles in the present study are created by igniting a mixture of Carbon Monoxide and Oxygen, it may be assumed that the bubble contains the non-condensable contents. If this non-condensable gas is considered as ideal and if it is assumed that negligible
heat is exchanged with the surrounding fluid on the time scale of the bubble oscillation, then the pressure, $P_b$, exerted by the bubble contents can be expressed as

$$P_b = P_0 \left( \frac{R_0}{R} \right)^{3\gamma}$$  \hspace{1cm} (3)

where the subscript 0 denotes the initial condition and $\gamma$ is the ratio of specific heats (Menon and Lal, 1998). Furthermore, the buoyancy parameter can be defined as

$$\delta = \left( \frac{\rho g R_m}{P_m} \right)^{\lambda}$$  \hspace{1cm} (4)

where $R_m$ is the maximum bubble radius. Physically $\delta$ signifies the ratio of the bubble half-life to the time it would take a bubble of radius $R_m$ to rise the order of one radius from rest due to buoyancy forces. If the bubble is close to a horizontal rigid boundary and is located a distance $\zeta_0$ from it, then $\lambda = \zeta_0/R_m$ characterizes the Bjerknes attraction of the boundary.

The Kelvin Impulse of a bubble, $I$, is defined such that its time derivative equals the sum of all external force acting on the bubble including Bjerknes and buoyancy forces also. The Kelvin impulse physically corresponds to the impulsive force applied to the bubble surface to generate the observed flow, changing in response to the action of external forces. For motion over a rigid boundary, the Kelvin Impulse is found to be given by (Blake, 1988)

$$I = \left( \frac{2\sqrt{6}\pi}{9\lambda^2} \right) \left[ 2\lambda^2 \delta^2 B(\frac{1}{4}, \frac{1}{2}) - B(\frac{1}{4}, \frac{1}{2}) \right]$$  \hspace{1cm} (5)

where $B$ is the beta function and the Impulse is scaled with respect to $(R_m)^3(\rho p_{\infty})^{1/2}$.

The null impulse state corresponds to a situation where the competing Bjerknes and buoyancy forces are approximately equal in their action. This state is given by
\[ \lambda \delta = \left( \frac{B(\frac{1}{2}, \frac{1}{2})}{B(\frac{1}{4}, \frac{1}{4})} \right)^{1/2} = 0.442 \] (6)

No jetting is observed at this state and the bubble does not migrate in any direction.

All these properties will be verified for the flat plate case in the near future.

5. CONCLUSION

There exists a critical distance above the plate where the reentrant water jet produces the maximum impact pressure. The jets formed by the explosions above this distance have to pierce the water layer between the bubble and the plate and hence yield lower impact pressures. The growth of bubbles formed by explosions below the critical distance is inhibited by the presence of the plate and hence their maximum sizes are comparatively restrained. The water jet, however, is very focused and symmetrical about the center of impact. The effect of covering the flat plate with sand or clay is to reduce the impact pressure and smoothen its distribution over the plate. However, Loose sand particles are displaced during crater formation due to an explosion close to the sand surface. The formation of crater reduces effective sand thickness and a partial recovery in impact pressure is achieved. The water free surface enhances the crater formation and the pressure recovery is indeed substantial. Explosions close to the clay surface display formation of double craters.

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REFERENCES


0.635 cm thick steel plate
of dimension 36.83 cm × 60.96 cm.

Fig. 1. Experimental setup for simulating explosion near an exposed or buried object.
For exposed object, d_s = 0.

Fig. 2. Layout of pressure transducers on the plate
Fig. 3. Time sequence of bubble collapse near an exposed plate for $d/R_0 = 2$. 
Fig. 3. (Continued)
Fig. 4. Pressure time trace recorded by (a) the plug transducer; and (b) the plate transducers for the exposed plate case with d/Ro = 2.
Fig. 4. (Continued)
Fig. 5. Normalized impact pressure measured by (a) transducer no. 8; and (b) transducer no. 6 and 7 (away from the center) as a function of distance of the initial bulb from the plate.
Fig. 6. Bubble collapse near a buried object. $d/R_o = 2.4$, $d_o/R_o = 1.1$ and $d_w/R_o = 24$. 
Fig. 7. Normalized impact pressure at the center as a function of sand layer thickness.
Fig. 8. Normalized impact pressure at the center as a function of the location of free water surface above the bubble.
Pressure distribution around the plate center
(2" sand, 33" water, 0" above sand)
Pressure variation at center versus the distance between the bulb and the solid surface.
2" sand
13" water
0" above
11' sand
33" units
2.6" above
Dynamics of Interaction of Two Underwater Explosion Bubbles

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ABSTRACT

Underwater explosion bubbles are created by detonating a mixture of oxygen and Carbon Monoxide or Hydrogen in glass globes submerged in a water tank. A cinematographic technique is employed to capture entire interaction process in both horizontal and vertical configurations. Instrumented tubes and plugs measure pressure inside and outside the bubbles. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or they may violently repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. When a coalesced bubble is formed by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble in the direction of its migration. Experiments were also conducted to qualitatively and quantitatively study the interaction of a free surface with the explosion bubble(s).

Introduction

Much of the research activities in the area of underwater bubble dynamics has been focused on the behavior of cavitation bubbles. Cavitation bubble dynamics play very important role in underwater acoustics and in predicting and preventing propeller and turbine blade damage. These bubbles, however, seldom occur singly. Actual cavitation fields contain several thousands of
oscillating and translating microbubbles. Study of the behavior of a cloud of bubbles thus becomes inevitable and experimental (e.g., Chahine and Sirian, 1985; Tomita et al., 1984), numerical (e.g., Chahine and Liu, 1985; Chahine, 1991; Chahine and Duraiswami, 1992; Wang and Brennen, 1994) and analytical (e.g., Van Wijngaarden, 1972) techniques have all been developed. The simplest model that has been studied by the researchers is the interaction between two bubbles. Theoretical and numerical studies of the interaction of two spherical or nonspherical bubbles of same or different sizes (e.g., Fujikawa et al., 1985; Fujikawa and Takahira, 1986; Fujikawa and Takahira, 1988; Morioka, 1974; Serebryakov, 1992; Shima, 1971; Takahira, 1988) have also been carried out. Interesting experimental observations of the interaction of a gas bubble with a pressure wave (Shima and Tomita, 1988) or with a vapor bubble (Smith and Mesler, 1972) have also been made. However, most of these observations are for microbubbles and find applications in cavitation, erosion and related topics.

Large bubbles, such as those created by underwater explosion, owing to their tremendous inherent destructive capabilities upon collapse near a rigid boundary, find practical applications in underwater weaponry. Detailed measurements and imaging of pulsating bubbles formed during deep sea explosions are very difficult due to a variety of obvious reasons (e.g., Arons et al., 1948) and therefore, there is insufficient data available to analyze the dynamics of interaction of bubbles. Controlled experiments described here, are required to investigate the physical processes that contribute to the interaction of bubbles. These experiments were conducted in a laboratory shallow water setup using a gaseous explosive mixture. The observations reported here are of practical significance for buried mines detection in shallow water beaches, where the interaction of an explosion bubble with a solid boundary and water free surface is anticipated.

The interaction of two underwater explosion bubbles is a very interesting and complex phenomenon, because of the fact that one bubble is influenced by the time-delayed pressure or shock wave radiated from the adjacent bubble. Radial motion of the bubble may be greatly excited
or subdued due to the interaction depending on their temporal and spatial separations. Though some experimental work has been done on the interaction of gas bubbles with two adjacent underwater explosion bubbles, and it has been shown that strong and complicated interactions ensue, it appears that no detailed results on the interaction of two underwater explosion bubbles have been published in the public domain literature.

This paper reports the results of the experiments carried out in a laboratory water tank to study the interaction between two adjacent bubbles created by underwater explosion of flammable gas contained in glass globes. The globes were placed side-by-side either in a horizontal or a vertical plane. The distance between the two globes and their sizes were both varied. This paper also discusses the interaction of an explosion bubble with the water free surface.

**Experimental Procedures**

Underwater explosion experiments were conducted in a wooden tank of dimension $2 \text{ m} \times 1.5 \text{ m} \times 1.5 \text{ m}$, coated with fiberglass resin from inside. The tank, as shown in Fig. 1, has three windows on three sides for optical imaging. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe. Three different sizes of glass globes were used for present experiments with average radii of 2.9 cm, 3.2 cm, and 3.8 cm. The glass globe, as shown in Fig. 2, has an electric spark ignition system connected to a 3000V DC power supply. The explosion takes place at a constant volume until the globe bursts. Since the experiments were conducted in a laboratory shallow water setup and using a gaseous explosive mixture, the bubbles are relatively smaller (although much larger than cavitation bubbles) than those observed in deep sea explosions. Recently, Menon and Lal (1997) addressed the dynamics and instability issues of such a bubble in free field and they showed by means of extensive geometric and dynamic similarity analyses that the explosion bubble thus formed is a reasonable subscale approximation of a deep sea underwater explosion bubble. They have presented detailed scaling parameters, energy partitioning and also
various interface instability mechanisms. Repeatability and experimental uncertainty have also been addressed and it has been shown in particular that repeated experiments produced error bands for the explosion pressure, maximum radius and time period of 5.88%, 3.7% and 6.06%, respectively. A parallel numerical study using an Arbitrary Lagrangian-Eulerian (ALE3D) code was also carried to investigate bubble dynamics (Menon and Pannala, 1997). Comparison of the computed results with the experimental data for a single and double bubble oscillation showed excellent agreement.

The pressure response in the water around the bubbles were recorded during the experiments by means of 4 KISTLER dynamic piezoelectric pressure transducers fitted at the ends of 4 stainless steel (1.27 cm diameter) tubes bent at right angle, as shown in Fig. 1. A hydrophone is also mounted in the tank and is used for measuring acoustic pressure. Pressure inside the bubble during its oscillation is measured by another KISTLER transducer which is mounted inside the plug, as shown in Fig. 2. These dynamic pressure transducers have low and high frequency response of 0.001 Hz and 50 kHz, respectively, and the resonant frequency of 300 kHz. They are therefore well suited for the current experiments as the bubble oscillation frequency (time period of approximately 15 ms) lies well within the above mentioned bounds. Signals from these six pressure transducers and the hydrophone were digitized using National Instrument's AT-MIO-16X analog-to-digital converter board, and were recorded into a microcomputer. Eight channel data recording was performed with a sustained sampling rate of 10,000 samples per second per channel.

The tank was illuminated by either direct overhead flood lights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis. The optical recording of the bubble motion was performed by a CCD enhanced digital video camera with a maximum speed of 6000 frames per second. Many of the experiments, however, were performed at a lower speed of 1000 frames per second in order to obtain a full screen image.
The two glass globes were supported inside the tank by means of a modified sting which made the pressure transducers mounted inside the two globes to face each other. This facilitated direct measurement of the fluctuation in the pressure inside one bubble due to its interaction with the other. A means for altering the distance between the two bubbles was provided by the six holes drilled in the supporting copper pipe (1.6 cm diameter) of Fig. 1. Experiments were conducted in primarily two configurations; a horizontal configuration, when the supporting pipe was horizontal, and a vertical configuration, when it was vertical. The former configuration prohibited the use of laser light sheet and only the flood lights were used for imaging, while the latter allowed the use of laser light sheet.

Experiments were also conducted to study the interaction of water free surface with the explosion bubble. The motivation for these experiments has obvious reasons. The free surface provides a constant pressure boundary in close proximity of the oscillating bubble. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration (see e.g., Birkhoff, 1957; Blake and Gibson, 1981; Chahine, 1977; Chahine, 1982; Wilkerson, 1989). The observed lack of experimental data for interaction of a bubble with the water free surface (Wilkerson, 1989) motivated the current experiments.

Results and Discussion

The interaction process is highly dependent on the time delay between the two explosions. This time delay is related to a variety of hardware and bubble response characteristics, namely, the gas volume (globe size), the equivalence ratio of the fuel-oxygen mixture, and the gap between the two spark wires, etc. The actual delay (temporal separation) between two explosions is therefore measured from the recorded video images as the time elapsed between the instant when the individual globe bursts. It was therefore deemed necessary to conduct several experiments to
collect statistical information about the range of bubble behavior with respect to the delay between two explosions.

The entire spectrum of delay can be classified into two broad regions: in phase oscillation and out of phase oscillation (see e.g., Morioka, 1974; Shima, 1971; Smith and Mesler, 1972). In most of the past analytical, numerical or even experimental work on the interaction of two cavitation bubbles, interest has been focused primarily on the contraction phase of the bubble oscillation. This yields an in phase oscillation of identical bubbles as they both start collapsing at the same time following a sudden change in the ambient pressure. In phase oscillation is obtained when there is strictly no delay between the explosions. This is the simplest scenario and has been analyzed comprehensively.

Another interesting scenario is a 180° out of phase oscillation and it can be best understood in the interaction of two identical explosion bubbles as a case where one bubble starts its oscillation cycle when the other has already reached its maximum radius. In fact, these are the two scenarios predicted by the analytical theories (Morioka, 1974). Morioka's (1974) theoretical analysis of natural frequencies of two pulsating bubbles predicts the existence of two natural frequencies corresponding to in phase and 180° out of phase oscillations, respectively. Of course, in an experimental setup one can have any amount of delay between zero to 180°, or even beyond 180°. Two bubbles oscillating in phase behave in a nearly identical manner as a single bubble near a rigid boundary and therefore, are of considerable practical interest since it has been shown that the collapse of a bubble near a wall can cause significant damage. The bubbles have an increasing repulsive effect as the delay between two explosions increases, up to the point when they oscillate 180° out of phase (Smith and Mesler, 1972).

Figure 3 shows an example for two underwater explosion bubbles oscillating in phase. The initial volume of the right glass globe is 94 ml and that of the left glass globe is 90.5 ml. They are
in a horizontal configuration and are initially separated by a distance \( d \), where \( d/R_0 = 2.32 \). Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. There is virtually no delay between two explosions (determined from the image data) and since the two globes have almost same volume, they burst also at the same time (time, \( t = 0 \) ms). Since the initial spatial separation between the globes was intentionally kept to a very small value so that a violent interaction can ensue, the bubbles soon come in contact with each other. They deviate more and more from sphericity as they expand with time, collapse violently on to each other and merge together to form a single coalesced bubble.

The plane surface where the two bubbles come in contact may be considered as a rigid boundary in an equivalent single bubble analogy. Figure 4 shows the relative position of this surface with respect to the initial globes' centers. Notice that this surface is almost perpendicular to the line joining the initial globes' centers and is located almost midway. The time period of oscillation of the bubbles shown in Fig. 3 is about 21 ms, while that of an identical bubble in free-field is less than 15 ms. Therefore, for two identical bubbles oscillating in phase, an increase in the bubble period is observed. A similar observation was made by Chahine (1991).

The pressure traces measured around the bubbles show that the bubble behavior is symmetrical. Figure 5 shows the pressure trace measured by the transducer mounted in the right plug. The first peak corresponds to the explosion pressure and a pressure fluctuation of about 700 kPa exhibited near bubble minimum (second peak) demonstrates the severity of collapse of the jets formed in two bubbles on to each other. Figure 6 shows the pressure signature recorded by a transducer mounted in a tube underneath the right globe (see Fig. 1). The pressure drops exponentially as one moves away from the bubble (Cole, 1948). A pressure drop of 70% (from 1000 kPa to 300 kPa) over a distance of 14 cm, and that of 80% (from 1000 kPa to 200 kPa; see Fig. 6) over a distance of 34 cm from the bubble center have been recorded by tank transducers.
The coalesced bubble quickly breaks into cloud of smaller bubbles which migrate upward due to buoyancy effect. The bubble contour is traced and 360 bubble radii are obtained at equal azimuth locations. These data are then Fourier analyzed and the results are shown in Fig. 7, which shows the power spectral density of the coalesced bubble's interface at three instants: just after it is formed and 1 and 5 ms after it collapses. Here \( c \) is the bubble circumference and \( \lambda \) is the wavelength. This technique of obtaining the power spectral density of bubble interface to quantify the interface corrugation has been described in detail by Menon and Lal (1997). Moreover, by using painted globes, they have clearly shown that the glass fragments which are typically long and thin, considerably lag the bubble motion during collapse phase and therefore do not contribute to the interface corrugation. It can be inferred from Fig. 7 that the coalesced bubble starts exhibiting pronounced and distributed peaks in power spectral density soon after collapse. A peak is actually the square root of the sum of the squares of mode amplitude coefficients and occurs at integral fractions of bubble circumference because a periodic trace is being analyzed. The coalesced bubble is therefore very unstable.

A single coalesced bubble does not form only in an in phase oscillation. Another situation where the formation of a single coalesced bubble has been observed repeatedly and most surprisingly, is associated with a nonzero time delay and a very short inter-bubble distance. This case is shown in Fig. 8. The initial volume of the right glass globe is 125 ml and that of the left glass globe is 127 ml. They are almost touching each other such that the initial separation distance between them, \( d \), is given by \( d/R_o = 2.1 \). Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. The right globe explodes first (at time \( t = 0 \) ms) and tries to encompass the left globe as the bubble grows. When the left globe explodes (at time \( t = 8 \) ms), the shock wave emitted by this bubble travels through the right bubble as is evident by its protruding pieces. The right bubble, however, maintains its coherence and sphericity. It seems that the energy of the left bubble is substantially transferred to the right bubble, and it does not even get a chance to expand to its maximum radius. A force field is generated such that when the right bubble
starts to collapse, the left bubble just merges into its predecessor to form a single coalesced bubble, which continues the oscillation cycle as a single explosion bubble. Since no jets are formed and the coalesced bubble is formed by merging the energies of two bubbles, it has more residual energy than that of the previous example and does not disintegrate into smaller bubbles so quickly. In fact, it is even more stable than a single explosion bubble in free field and can, therefore, be used for focusing bubble energy for enhancement of its destructive capabilities.

The available energy for successive pulsations of the coalesced bubble can be calculated using Vokurka's (1986 and 1987) energy balance analysis (e.g., Menon and Lal, 1997). The various formulae for this analysis are given in the cited references and are, therefore, avoided here for the sake of brevity. The energy, \( E_0 = 4\pi R_0^2 P_1/3 \), where \( R_0 \) is the initial globe radius and \( P_1 \) is the ambient hydrostatic pressure at the explosion depth, is used to nondimensionalize energy and the heat release of stoichiometric carbon monoxide is taken to be 284 KJ/mole (Strahle and Liou, 1994). The total nondimensional energy available for oscillation of the right bubble at time \( t = 0 \) can be given by \( \bar{E}_0 = 78.439 \) for an explosion depth of 0.65 m. The nondimensional internal and the potential energies of the right bubble at its maximum radius for an expansion ratio of \( R_{\text{max}} / R_0 = 2.738 \), are estimated to be 38.043 and 19.528, respectively. The energy dissipated into the surrounding water by a shock wave thus equals 20.868\( E_0 \). Therefore, the energy available for successive oscillation of the right bubble at its maximum radius equals 57.571\( E_0 \). A similar analysis for the left bubble for a smaller explosion pressure and expansion ratio of only 1.095 yields the value of the available energy at its maximum radius to be 11.916\( E_0 \). The coalesced bubble should apparently have available energy of 69.487\( E_0 \) for its successive oscillation, which is roughly 20.7% more than what a single explosion bubble should have in a free field condition. The coalesced bubble is, in fact, observed to oscillate for an extended period of time.
It is interesting to note that the explosion pressure for the left bubble (200 kPa) is only about 20% of what it would have been in a free field case. Thus, an expanding bubble inhibits the formation of another explosion bubble in its close proximity by reducing its explosion pressure. This may be the reason why the left bubble does not have sufficient energy to expand to its maximum radius. But, it certainly aids its predecessor to form a coalesced bubble with a greater energy. This time, the right plug transducer lies inside the coalesced bubble as it collapses. The collapse pressure recorded by this transducer is very high (2500 kPa). Except for the plug transducers, the pressure traces recorded by the right and left transducers are once again almost identical, indicating a symmetrical bubble behavior.

Similar dynamic behavior is exhibited by the interaction of bubbles formed by glass globes of initially different sizes. It is not possible, however, to obtain an in phase oscillation because of a simple reason that the two bubbles have different time periods of oscillation. On the other hand, the formation of a coalesced bubble by mergence of two bubbles, have also been exhibited by the bubbles of different sizes when the smaller bubble has been absorbed into the larger bubble. This kind of bubble dynamics is not feasible for large inter-bubble distance.

When the initial inter-globe distance is sufficiently large, the bubbles start repelling each other for a nonzero time delay. The repulsion force increases with the delay between the explosions, up to the point when they are oscillating $180^\circ$ out of phase. In this case, when the predecessor bubble collapses, the successor bubble reaches its maximum radius. At this point the pressure field is abruptly reversed and this causes the formation of a strong reentrant water jet in the successor bubble. An example of out of phase interaction is shown in Fig 9. The initial volume of the right glass globe is 94 ml and that of the left glass globe is 92.5 ml. Both of them are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. They are in a horizontal configuration and are initially separated by a distance $d$, where $d/R_0 = 4$. Figure 9 shows the initial globes' locations. The right globe explodes first (at time $t = 0$ ms) and reaches its maximum radius when
the left globe explodes (at around $t = 7.5$ ms). A strong reentrant water jet is formed in the left bubble as it collapses. Figure 10 shows how this reentrant jet travels with time. Here, $x$ denotes the location of the jet tip relative to the inertial frame (the tank) and $x = 0$ corresponds to the instant when the jet tip becomes visible for the first time. The formation of a strong reentrant water jet has also been observed for two bubbles of different sizes.

As the phase delay between two explosions increases beyond $180^\circ$, the repulsion force as well as the water jet velocity decrease in magnitude. If the phase delay between two explosions increases beyond $360^\circ$ (i.e., if one bubble has already completed one oscillation cycle when the other bubble forms), the resulting interaction is very weak. In this case, though the predecessor bubble manages to create a depression in successor bubble at its maximum radius, formation of a jet is not observed.

The horizontal configuration is very important from a practical standpoint, as it can set a catastrophic bending vibration to a nearby rigid body if tuned properly. The vertical configuration is also equally important, as it can dramatically enhance the impact pressure of a single bubble when collapsed near a rigid body. It is speculated that if two bubbles are placed close to each other along an axis perpendicular to a nearby rigid body, and if these two bubbles are tuned to oscillate $180^\circ$ out of phase with each other, a water jet will be formed directed towards the rigid surface with a velocity which will be higher than that formed by the collapse of a single bubble under similar circumstances. When the interaction of two bubbles is studied in vertical configuration, the orientation of the gravitational force changes. The gravitational effects are known to be controlled by the size of the bubble. If the gravitational effects are dominant, an isolated bubble migrates in an upward direction. But, in the presence of a competing effect, such as another pulsating bubble above or below it, the dynamics of the bubble are controlled by its spatial and temporal separations from it. Therefore, two in phase bubbles separated by a short vertical distance attract each other. On the other hand, two out of phase bubbles separated by a short vertical distance repel each other.
such that the top bubble migrates upward and the bottom one migrates downward and the jets
formed in them pierces them in the directions of their migration. There is an upper bound to the
distance between two identical out of phase bubbles in vertical configuration beyond which their
interaction would cease to exist and its value can be easily estimated as described below.

Experiments were carried out to study the interaction of a free surface with an explosion bubble.
A simple sting mount was used to support the glass globe and the depth of water in the tank was
decreased in a step of 2.54 cm. At larger depths, the gravitational effects are dominant and the
bubble migrates upward. Reduced water depths provide competing effects and the bubble starts to
migrate downward. It was found that the bubble migration velocity smoothly changes its direction
as well as its magnitude. Figure 11 shows the transition of bubble migration velocity with water
depth. As two in phase bubbles are equivalent to a single bubble near a rigid boundary, similarly
two out of phase bubbles are equivalent a single bubble near a free surface. The transition point of
Fig. 11 can thus determine the maximum inter-bubble distance for which two identical out of phase
bubbles in vertical configuration far away from water free surface would start interacting with each
other. The maximum center-to-center distance between two identical interacting bubbles (of
approximate volume of 230 ml) oscillating 180° out-of-phase with each other was found to be
6.8 R₀, R₀ being the initial radius.

Conclusions

Underwater explosion experiments have been conducted in a water tank using flammable gases
in glass globes to study the dynamics of interaction of two explosion bubbles in both horizontal
and vertical configurations. The former configuration can excite a nearby submerged structure in
bending vibration mode, while the latter can easily be tailored for the directionality and
enhancement of the impact pressure resulting from the collapse of an underwater explosion bubble
near a solid boundary. Depending on the delay between two explosions and inter-bubble distance,
the bubbles may either attract each other to form a single coalesced bubble, or they may violently
repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. If a coalesced bubble is formed by mergence of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble. Water free surface repels the bubble and the transition point of bubble migration velocity determines the maximum inter-bubble distance required for the initiation of interaction between two out-of-phase pulsating bubbles.

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References


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Figure 11. Bubble migration velocity for the interaction of a single bubble with water free surface.
Figure 1
Glass globe
Test Plug  3000V DC
RTV coating
Pressure transducer

Pressure transducer protected by RTV coating
Gas introduction
Spark wires
Top view of the test plug

Figure 2
\[ d/R_o = 2.32 \]

\[ d_1/R_o = 1.1; \quad d_2/R_o = 1.22 \]

Figure 4
Figure 5
Figure 11
Simulations of Underwater Mine Destruction Using Detonation Cord Explosives

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Abstract

Simulations of underwater detonation cord mesh explosion have been carried out using a three-dimensional arbitrary Lagrangian-Eulerian finite-element code. Earlier, this code was successfully employed to capture both qualitatively and quantitatively the dynamics of underwater explosions near rigid surfaces. In the present study, this code was used to study the effect of detonation cord explosions on a stainless steel rod placed vertically within the mesh. This metal rod mimicked the trigger arm of a buried mine. The explosion strength was chosen to be larger than the yield strength of the metal rod. It was shown that when the metal rod is placed directly in the center of the mesh, the explosion bubble collapse causes a very high pressures along the diagonal axes and results in the rod getting squeezed and lengthened. On the other hand, when the rod was placed away from the mesh center, asymmetric forces are generated that results in the fracture of the rod into multiple pieces. These results demonstrate that to ensure repeatable destruction of a buried mine, asymmetric design of detonation cord (by changing energy density and/or geometry) is desired. Further calculations are planned to determine if an optimum design of the detonation mesh exists.
Introduction

Earlier studies (Menon and Pannala, 1998; Menon and Yang, 1998) using a three-dimensional Arbitrary Lagrangian-Eulerian (ALE3D) code have established that this code can capture with very good quantitative and qualitative accuracy the dynamics of explosion bubble collapse and the resulting impact pressure generated on the wall. The earlier studies were designed to evaluate the capability of the code and also to determine how the code could be utilized to develop explosive shape designs to achieve particular impact objectives.

A configuration of key interest in the present study is the behavior of explosion bubble collapse for more realistic devices such as a detonation cord net. This type of net is typically deployed on the surf zone of beaches and detonated to destroy buried mines. Due to the complexity of the explosion dynamics, field experiments do not provide detailed visualization of the explosion process and the consequent effect on the buried mine. Recent experiments in a shallow water test facility (Menon et al., 1998) of explosions near exposed and buried (under sand) surfaces have demonstrated that the bubble collapse does result in high impact pressure on the surface even when the surface is buried below a layer of sand. High speed imaging and pressure measurements were carried out to quantify the explosion dynamics and the behavior of the collapsing bubble for a range of parameters. It was determined that the peak impact pressure depends upon the explosion energy, the distance of initial charge above the surface and the sand thickness. Interestingly, it was also determined that the behavior of the sand under the impact force can also play a role in determining the impact pressure on the buried surface. A scaling relation has been obtained to correlate all the data. An attempt to model this complex multiphase behavior is currently underway.
In the present study, we focus on the behavior of detonation cord explosions and its impact of rigid bodies within its neighborhood. The availability of the ALE3D code at this stage provides us with an opportunity to investigate these types of problems. We limit our studies to the case of a single square mesh of detonation cord (made up of four cylindrical charges of same energy) that is located on the ground. Simulations of the explosion of this mesh was first carried out and then, a series of calculations were carried out with a metal (stainless steel) cylinder rod positioned in the center (Case D1), off-center (Case D2) and in the corner (Case D3). Figure 1 shows schematically the various cases studied. The rationale for the choice of this configuration is that buried mines have trigger arms that protrude, in some cases, above the ground. The goal of the detonation cord is to cause a pressure differential force on the arm which in turn would result in the movement of the arm resulting in triggering the detonation. However, field experiments in the past have produced mixed results in terms of the effect of explosion on the trigger arm (it appears the shape of the arm changes the dynamics of the interaction). The present study, albeit highly simplified at present hopefully will provide a preliminary understanding of the explosion dynamics. Subsequently, more realistic scenarios can be studied using this approach.

Results and Discussions

A key feature of the realistic strong underwater explosion is the formation and propagation of a strong shock wave upon the ignition and explosion. This detonation wave propagates at very high speed and can play a major role in the impact process. Earlier studies were carried out to mimic explosions that were carried out in our shallow water test facility. As such, the explosion energy was relatively low. The ALE3D capability has no limitations regarding the type of energy and the shape of the explosive charge. To demonstrate this, Fig. 2 shows the propagation of a detonation shock wave
formed when a high energy device is exploded in free field (only a quarter quadrant is shown). The impingement of such a strong shock wave on a surface can result in significant structural damage. Here, we will not attempt to simulate a real detonation cord energy (which is not available in open literature). Rather, we simulate a representative case using parameters similar to our earlier studies (Menon and Yang, 1998). Table 1 summarizes the test conditions and the properties of the detonation cord used.

Here, we discuss on the cases of the detonation net with the metal cylinder in the middle. Figures 3a-e show respectively, a time sequence of the velocity vector field in the x-y plane at a location just above the ground for case D1 (metal cylinder in the center of the net). The bubble shape is shown in these figures as a solid line. In Fig. 3a, the explosion bubble has reached its maximum and the flow is still outwards in all direction. In Fig. 3b, the bubble begins to collapse and results in a reversal of the fluid motion. However, it can be noted that although the collapse is symmetric, the inward motion of the fluid primarily occurs along the diagonal and the reflection of this incoming fluid from the cylinder results in an outward fluid motion in the x- and y- perpendicular directions. Due to the collapse along the diagonal axes, peak pressures are expected along these directions as will shown below. The key point to note from these figures is that the entire collapse process is symmetric. The implication of this symmetry on the structural forces on the metal cylinder is discussed below.

Figure 4a-c show respectively, three snapshots of the pressure contours in the same x-y plane and at the same location above the ground. As can be seen in Fig. 4c, the diagonal collapse of the impinging jet motion results in high local pressure along the diagonal axes. The pressure is relatively lower in the principal x- and y-axis directions. Thus, one can assumed that the metal cylinder is undergoing compression along the diagonal. If the metal yield strength is unable to withstand this force then it is likely to
undergo failure mode. In the present case, the metal rod is a stainless steel rod with an yield strength of 340 N-m. The initial explosion energy for the detonation cord around 503 N-m. Thus, it is likely that this could result in structural failure of the metal rod as observed in these calculations.

Further effects of the explosion and the explosion bubble collapse on the metal cylinder can be determined by visualization of the velocity vector field and the pressure field in the y-z plane. Figures 5a-e show respectively, a time sequence of the velocity field and Figs. 6a-e show the corresponding pressure contours in the y-z plane at x = xc (which is a plane through the center of the metal cylinder). The metal cylinder is shown in these figures as a solid lined object. The velocity vector field shows the initial outward motion (Fig. 5a) and then the beginning of the collapse (Fig. 5b). The flow impinges on the metal rod and then rebounds and flows away from the rod (Figs. 5d, e).

The pressure contours in the y-z plane shows the impact of the bubble collapse much more clearly. Initially (Figs. 6a,b) region of high pressure is seen on the sides of the rod and on the top region. However, overall the pressure contours are symmetric. It can be seen that as the bubble collapses (Figs. 6c-e) the high pressure region surrounds the bubble and the metal cylinder actually gets squeezed and becomes thinner and taller than its original size. The fluid-structure interaction simulated using ALE3D clearly demonstrate the ability of such simulations to understand this type of complex problem.

In summary, when the metal cylinder is place in the center of a detonation cord mesh then the explosion results in a symmetric flow pattern with the collapse occurring primarily in the diagonal directions. However, due to symmetry the stresses (mostly compressive) on the metal cylinder occur also symmetrically. As a result, the cylinder gets squeezed and elongates under the explosion force.
To understand the collapse dynamics more clearly, calculations were carried out with the metal cylinder located off-center along the x-axis (Case D2) and located at the corner (Case D3). It was expected that these positioning will make the collapse pattern non-symmetric and should result in unbalanced forces on the metal cylinder. This was observed in these calculations. In fact, due to the non-symmetric forces on the metal rod, the forces on the rod were large enough to fracture it. Some representative results for these cases are discussed below.

Figures 7a and 7b show respectively, the pressure contours in the x-y plane near the ground for the off-center case (case D2) and the corner case (Case D3). Since the metal rod was shifted in the positive x-direction the pressure rise is much higher on the left side of the rod. This can be observed by comparing to the earlier case of the rod located in the center. The collapse is still along the diagonal axis in both cases. This results in an asymmetric force on the metal rod.

The velocity vector fields do not show anything significantly different (other than some asymmetry in the flow motion) from the images shown in Figs. 3 and therefore, are not shown. Briefly, the collapse process still occurs along the diagonal with the fluid motion stronger in the left side (since the metal rod was placed off-center to the right and in the top right corner). We discuss below primarily the pressure contours to show the impact of the explosion on the metal rod.

Figures 8a-e shows respectively, the pressure contours in the y-z plane through the metal rod for the off-center case (the corresponding pressure contours for the corner case are not shown since the result was quite similar). The metal rod is shown as a light blue line in these figures and is explicitly marked in these figures. It can be seen that
immediately after the explosion, high pressure is observed on the rod near the ground and half-way above the ground (Figs 8a). This is due to the initial explosion shock impingement. This force appears to be sufficient to cause a fracture in the metal rod. Subsequently, as the bubble collapse and the pressure builds up asymmetrically the rod breaks up as seen in the subsequent figures. This can be clearly observed in Fig. 8c-e where the light blue line (representing the rod) appears as two distinct sections.

A similar result is observed in the case of the rod placed on the corner. It would appear from these simulations that when the metal rod is off-center in any way the forces on the rod become asymmetrical and causes the rod to fracture (if its strength is low enough which was true for the present case). On the other hand, when the rod was in the center of the mesh, due to symmetry the forces just squeezes the rod and lengthens it.

From design standpoint this suggests that to achieve a non-symmetrical force on the metal rod we need to device a detonation code mesh that is NOT symmetric in energy density and/or in geometrical design. Some characteristic configurations are planned for future studies to determine if an optimum design can be developed that will always result in the forces on the metal rod to be asymmetrical. From the standpoint of causing a structural change of the trigger arm of a buried mine, this approach may allow for the detonation cord explosive to trip the arm all the time. We hope to investigate these features in the near future.
Conclusions

Simulations of underwater detonation cord mesh explosion have been carried out using a three-dimensional arbitrary Lagrangian-Eulerian finite-element code. Earlier, this code was successfully employed to capture both qualitatively and quantitatively the dynamics of underwater explosions near rigid surfaces. In the present study, this code was used to study the effect of detonation cord explosions on a stainless steel rod placed vertically within the mesh. This metal rod mimicked the trigger arm of a buried mine. The explosion strength was chosen to be larger than the yield strength of the metal rod. It was shown that when the metal rod is placed directly in the center of the mesh, the explosion bubble collapse causes a very high pressures along the diagonal axes and results in the rod getting squeezed and lengthened. On the other hand, when the rod was placed away from the mesh center, asymmetric forces are generated that in all cases results in the fracture of the rod into multiple pieces. These results demonstrate that to ensure repeatable destruction of a buried mine, asymmetric design of detonation cord (by changing energy density and/or geometry) is desired. Further calculations are planned to determine if an optimum design of the detonation mesh exists.

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Figure 1. Schematic of the various detonation code test cases.

Figure 2. Propagation of a detonation shock wave due to an underwater spherical explosion in free field. Only a quadrant of the full 3D is shown for simplicity.

Figure 3. Time sequence of the velocity vector field in the x-y plane near the ground for a detonation cord mesh explosion with the metal rod in the center of the mesh.

Figure 4. Time sequence of the pressure contours in the x-y plane near the ground for a detonation cord mesh explosion with the metal rod in the center of the mesh.

Figure 5. Time sequence of the velocity vector field in the y-z plane at $x = x_c$ which is a plane through the center of the metal rod. The bubble surface and the metal rod are shown as solid lines. For clarity, the metal rod location has been highlighted as a thickened solid line.

Figure 6. Time sequence of the pressure contours in the y-z plane at $x = x_c$ which is a plane through the center of the metal rod. The metal rod is shown as a highlighted line in the center of the image. Due to symmetric forces on the rod, the rod gets squeezed and lengthened as seen in the last two images.

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Figure 8. Time sequence of the pressure contours in the y-z plane at $x = xc$ (through the center of the rod) for the off-center case D2. The location of the rod is highlighted. Note that as the asymmetric pressure builds up the rod fractures into multiple pieces.
\[ l = 0.14 \, \text{m} \]
\[ w = 0.14 \, \text{m} \]
\[ d_c = 0.015 \, \text{m} \]
\[ h_c = 0.07 \, \text{m} \]

**Total Energy of detonation cord** = \( 503 \, \text{N-m} \)

**Yield Strength of Cylinder** = \( 340 \, \text{N-m} \)

*Figure 1*: Locations for cases D1, D2, D3 are shown in Figure.
a) $t = 4.5$ ms

b) $t = 9$ ms
Figure 3 (cont.)

$e) t = 18\, ms$
Figure 4
Figure 4 (cont.)

(b) $t = 12 \text{ ms}$
Figure 5 (cont.)

d) $t = 15 \text{ m}$

e) $t = 18 \text{ m}$
Figure 6 (cont.)
Figure 7

a) Off-Center, t = 15 ms
Figure 7 (cont.)
a) $t=4.5\,ms$

b) $t=9\,ms$

c) $t=12\,ms$
Figure 8 (cont.)
SIMULATIONS OF UNDERWATER EXPLOSION BUBBLE DYNAMICS USING AN ARBITRARY LAGRANGIAN-EULERIAN FORMULATION

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ABSTRACT

The dynamics of bubbles formed during underwater explosions are numerically investigated using an Arbitrary Lagrangian-Eulerian three-dimensional finite-element code and compared with experimental data. Both experimental and numerical results show good qualitative and quantitative agreement and suggest that the excitation of Rayleigh-Taylor instability is a major cause of interface instability. Simulations have also been carried out to investigate bubble-bubble interactions. Results show the formation of a water jet as one bubble collapses into the other, in agreement with recent experimental observation. Finally, the collapse of a bubble near a rigid wall and the formation of high velocity re-entrant jet onto the wall have been successfully simulated. The well known vortex ring bubble during the collapse process has been numerically captured.

1. INTRODUCTION

Vapor and gas bubble dynamics are of great practical interest in the prediction and prevention of cavitation erosion of marine propeller and turbine blades. The destructive nature of strong underwater explosions near walls is well known. Detailed reviews (e.g., Blake and Gibson, 1987; Prosperetti, 1982) have summarized past experimental and numerical results. Experimental studies are too many to list completely; however, most past studies focused on cavitation (small) bubbles. Among the studies that focused on large scale explosions are the studies reported in Cole (1948) for freely oscillating, deep sea explosion bubbles and the studies of bubble collapse near walls (e.g., Tomita and Shima, 1986). Bubble-bubble interactions have also been studied in the past (e.g., Warren and Rice, 1964). However, in most cases, due to difficulties in acquiring detailed data, only limited information has been obtained. Recently, experiments were carried out to investigate large-scale bubble explosions (Menon and Lal, 1996; Lal and Menon, 1996a, b). These experiments were conducted in shallow water due to an interest in understanding the dynamics of bubble-wall interaction in such flows and to investigate the feasibility of targeting buried mines for destruction in beaches. The data obtained from these experiments have been used to validate the numerical model discussed in this paper.

Numerical studies in the past range from simple 1-D analytic solutions (e.g., Lauterborn, 1976; Plesset, 1971; Prosperetti, 1982) to more complex 2D/3D studies. Many studies employed the Boundary Element Method (BEM) or its variants (e.g., Chahine and Perdue, 1988; Duncan and Zhang, 1991; Blake et al., 1986, Plesset and Chapman, 1970). This method has some inherent limitations. For example, compressibility in the gas cannot be included and in the study of bubble collapse near a surface, BEM can be used only up to the point of jet formation. To model the flow beyond the point of bubble collapse, BEM has been modified by introducing vortex elements (e.g., Zhang and Duncan, 1994; Zhang et al., 1993; Best, 1993). Furthermore, to set up this problem, recourse to experimental observation is required to obtain characteristic parameters. Such an approach is not general and cannot be used when the details of the explosion dynamics is unknown.

There are other assumptions used in past studies that are known to be in error. For example, significant compressible effects are known to occur in the collapse phase especially in deep sea strong explosions. Bubble shape is also known to quickly deviate from sphericity at bubble maximum, thereby, violating axisymmetric assumptions used in the past (e.g., Szymczak et al., 1993; Zhang and Duncan, 1994) and requiring full 3D treatment. Thus, simple 1-D or 2-D/axisymmetric analysis or incompressible methods cannot completely resolve the bubble and the flow dynamics. Furthermore, such simplified treatments also do not to provide details of the flow field inside and outside the bubble and cannot account for the interaction between the vapor and the liquid phases. Conventional numerical treatments (even using full 3D) such as Lagrangian or Eulerian techniques are also not practical, since the
expansion and collapse of bubbles create severe fluid motion so that a Lagrangian approach (in which the grid points move with the fluid resulting in severe grid distortion) becomes inappropriate, while in an Eulerian approach, adequate resolution in the regions of interest is very difficult to achieve since the bubble's shape changes very rapidly.

A numerical method that includes both compressibility and an ability to capture the entire bubble collapse in complex configuration is used in this study. This numerical code (ALE3D) combines Lagrangian eulerian features and is based on the Arbitary Lagrangian-Eulerian (ALE) scheme developed at the Lawrence Livermore Lab. Past applications include the 2D (e.g., Tipton et al., 1992) and the full 3D (Milligan et al., 1995) studies of bubble collapse. This paper reports some recent results using ALE3D, of both single and double bubble explosions in free field and in the vicinity of a rigid wall.

2. THE NUMERICAL METHOD

ALE3D (Anderson et al., 1994) is an explicit, 3D finite element code that simulates the fluid motion and elastic-plastic response on an unstructured grid. The grid may consist of arbitrarily connected hexahedral shell and beam elements. The ALE algorithm is implemented by carrying out a complete lagrangian calculation followed by an advection step. After each lagrangian step, a new mesh is created using a finite element based equipotential method to relax the distorted grid. In the eulerian advection step, the fluid variables such as mass, density, energy, momentum and pressure are reevaluated on the new mesh by allowing fluid motion. The details of the constitutive models are described elsewhere (e.g., Steinberg, 1991) and, therefore, are not described here for brevity.

The advection step uses a second order, monotonic advection algorithm. This can create mixed material elements (i.e., liquid and vapor). Material interfaces are not explicitly tracked but for the purpose of carrying out mixed element advection, they are inferred from volume fractions. Separate state variables are kept for each component of a mixed element.

3. RESULTS AND DISCUSSION

In this section, the results obtained for the various test cases are summarized and discussed. These studies serve to identify the capabilities and limitations of the ALE3D code and to identify areas for further study.

3.1 Free Field Single Bubble Oscillation

These simulations employed test conditions similar to the experimental set-up of Menon and Lal (1996). A freely oscillating bubble is modeled in the center of a 1.5 m x 1.5 m x 1.5 m tank filled with water. The initial bubble diameter is 0.34 cm and the initial explosion pressure is 9.34 atmospheres. The water pressure is 1 atmosphere. The ALE mesh treatment is applied to all the elements in the bubble and in the vicinity of the bubble. But away from the bubble where the bubble explosion does not cause much grid distortion, lagrangian mesh treatment is used. The number of elements used to resolve the bubble and the surrounding water was varied to confirm that the results are grid independent. For a typical 3D simulation, 4512 elements were used to discretize the domain, but as many as 150000 elements were used for carrying out the grid independence tests for this test case. Although various cases have been simulated, only characteristic results are discussed below.

The bubble grows after the explosion due to the high vapor pressure inside the bubble. Because of inertia, this results in an over expansion and the pressure inside the bubble falls below the ambient(water) pressure. As a result, the bubble collapses and reaches a bubble minimum at which time the internal pressure again exceeds the external pressure. Thus, an oscillation process is set up and continues as long as there is sufficient energy available. However, energy is continuously lost during the oscillation due to irreversible mechanical work done on water and vapor and due to the onset of various instabilities. Analysis of the losses and the instability mechanisms (Menon and Lal, 1996) suggest that during the collapse process the Rayleigh-Taylor (R-T) instability occurs at the interface. This results in a distortion of the vapor-water interface. This phenomenon has been captured in the numerical study. For example, Figs. 1a and 1b show snapshots of bubble at the first maximum and the first minimum. As can be seen, near the bubble minimum, wave-like distortion appears along the bubble interface. Figures 2a-b show the corresponding velocity vector field inside and outside the bubble. Figure 2a shows the outward motion of the bubble just before the bubble maximum and Fig. 2b shows the outward motion of the bubble just after the first bubble minimum. The magnitude of the velocity vectors also indicate that the acceleration of the fluid is minimum at the beginning and end of compression or expansion phases.

The deviation from sphericity and the formation of waves on the bubble interface are characteristics of Rayleigh-Taylor instability. To ensure that this interface distortion is not due to acoustic reflections from the wall, calculations were carried out by moving the wall further and by replacing the rectangular domain by a spherical domain. Results showed that, although there are changes in the bubble oscillation period, the interface distortion appears near the bubble minimum in all cases. The R-T instability can also be inferred by analyzing the variation of the radius with time. For example, Fig. 3 shows the region (near bubble minimum) where \( \frac{d^2R}{dt^2} > 0 \) (which is a necessary condition for R-T instability).

Figure 4 compares the pressure history in the bubble with experimental data (Menon and Lal, 1996). It can be seen that the computed first period of oscillation (around 15 ms), the peak pressure and the maximum radius agrees well with data. It can also be seen that the acoustics do not play an important role in dictating bubble dynamics as the time period and peak pressure hardly changed even when the walls are moved away by approximately eight times. During the contraction phase there are some differences between the calculations and experiments. However, the experimental set-up employed a glass globe (which contained the stoichiometric fuel-air mixture) with a metal insert that contained the pressure transducer and the spark generator while these features were ignored in the numerical model. In addition, the effect of glass fragments have not been included in the numerical model.

Figure 5a shows the time trace of pressure in the tank away from the bubble and close to a wall. It is very similar to the high frequency pressure oscillations as recorded by the tank transducer in the experiments (Menon and Lal, 1996) and is shown in Fig. 5b. The slight differences in the two plots may be attributed to the idealization of the tank as a cube with walls.
all around (whereas, for the experiment, the top surface was a free surface; see below).

As mentioned earlier, simulations were carried out to ensure grid independence, and to confirm that the walls do not affect the overall dynamics. It has been determined that the presence of walls does affect (decrease) the oscillation period (Fig. 4); however, the bubble dynamics are captured relatively accurately. To simulate true free field explosion will require using outflow boundary conditions. However, at present, the ALE3D code requires that the far field boundary be modeled as a solid reflecting wall. This limitation of the code can be removed only by modifying the source code. This is will be investigated in the future.

To extend the applicability of ALE3D to real underwater explosions is quite trivial. To demonstrate this capability a deep sea underwater explosion was simulated using pentolite as the explosive. The time period (not shown) scales as approximately two times the non-dimensional time based on the maximum radius of the bubble, the ambient (water) pressure and the water density \( \frac{R_{\text{max}}}{\sqrt{\frac{P_w}{P_\infty}}} \), as found in the above simulations and in earlier studies (e.g., Chahine and Perdue, 1988).

### 3.2 Bubble-Bubble Interactions

To investigate bubble-bubble interactions, a series of studies were carried out using bubbles of various sizes. A limitation of the current ALE3D code is that it does not allow phase difference between the two explosions to be incorporated into the model. However, by using different bubble sizes (or using different explosive strength) the net effective energy release from each bubble can be varied. The effect of inter-bubble distance on the interaction process was also studied. Due to space limitation only characteristic results are discussed below.

When two identical bubbles (of initial radii 3.17 cm and placed 8 cm apart) are exploded the bubbles expand and then collapse onto each other and a reentrant water jet with a high speed (30 m/s) is formed in both vertical and horizontal directions. Figures 6a-c show snapshots of the bubble-bubble interaction and the corresponding velocity fields are shown in Figs. 7a-c, respectively. Fig. 8a-c show photographs from the experiments (Lal and Menon, 1996a) for the present case with two bubbles of same size exploding in phase with each other. Although not clearly seen in the experimental Fig. 8c, studies have show the presence of vortex ring bubble. The jet directed towards the adjacent bubble impinges on its counterpart as in a stagnation point flow. As the bubble-bubble process continues, two counter vortex rings are formed with the velocity between the bubble increasing to as high as 50 m/s. There is reasonable agreement between the experimental observations and the present computations.

When same size bubbles were exploded at the same distance and time, but with one bubble containing four time more energy than the other, a similar result was obtained except that in this case, the weaker bubble is sucked into the other bubble with a velocity reaching a maximum of around 85 m/s (not shown). The reentrant water jet is first formed in the weaker bubble during the first oscillation and the vortex ring thus formed merges into the (still coherent) stronger bubble. The jet formation in the stronger bubble is delayed until the second oscillation, at which time the second bubble also collapses.

When two bubbles of different size (e.g., of radii 3.17 cm and 2.17 cm and thus, with different total explosion energy) are exploded, the results are quite similar to the case discussed above. During the expansion phase, the greater inertia and explosion strength of the bigger bubble inhibits the smaller bubble. During the collapse, the pressure drop in-between the bubbles is more than on the other sides and this pressure differential causes the smaller bubble to be engulfed into the larger bubble. The center of motion of the water jet directed towards the bubbles does not immediately adjust to the motion of the bubbles and, thus, the water motion is directed off-center of the bubble. This creates a high pressure on the side of the smaller bubble away from the larger bubble. This high pressure and the low pressure in-between the bubbles creates enough momentum to form a water jet through the bubbles which penetrates to the other side of the bubble. Final stage of the jet formation is shown in Figs. 9a and 9c.

The velocity vector field shows the formation of the water jet in agreement with experimental study (Lal and Menon, 1996a). A water jet was also observed in the experiments when two identical bubbles were exploded out-of-phase, as shown in Fig. 9b. Out-of-phase explosion essentially changes the relative strength of the bubble explosion during interaction and is, therefore, similar to the present case with two unequal bubbles exploding simultaneously and the similarities can be seen in Figs. 9a-c. At present, the ALE3D code cannot simulate phase difference between the adjacent explosions. This feature will be included in the code at a later date.

Finally, Fig. 10 compares the pressure between the two bubbles for the various test cases. All cases have the same period of oscillation. However, the case with increased energy content shows the strongest water jet formation (around 85 m/s) and the largest impact pressure at the first bubble minimum.

### 3.3 Bubble-Wall Interactions

Bubble collapse near a rigid wall is of significant interest due to a variety of reasons related to its ability to cause serious damage to the structure. This is because when the bubble collapses near a rigid surface, a strong reentrant water jet is formed that is directed towards the wall. The peak impact pressure on the wall due to this water jet can be substantially higher than the explosion pressure especially when the initial explosion energy is very large. Various simulations were performed by varying the explosion strength and distance of the bubble from the rigid plate. However, only characteristic results are discussed here to highlight the pertinent observations.

Two cases are discussed here with bubble placed 8.34 cm above (buoyancy inhibiting jet formation) and 8.34 cm below (buoyancy aiding the jet formation) the wall. Figures 11a-d show the velocity field at various stages of the collapse for the first case. Initially, the bubble is almost spherical but begins to distort as it collapses. The physics of the jet formation is quite similar to the bubble-bubble case. Since there is less volume of water between the wall and the bubble during the collapse, the pressure drop is quite large relative to the pressure on other sides of the bubble. This pressure differential further forces the bubble towards the wall. Since steam is lighter, the bubble tends to move further away from the wall (due to buoyancy) for the case where the gravitational force is inhibiting the jet formation, while for the second case, the bubble is further
accelerated towards the wall (Fig. 12). The water surrounding the bubble is directed off-center relative to the bubble geometric center, thereby, creating a higher pressure on the side of the bubble away from the wall. The combination of these effects causes the water to penetrate the bubble from the high pressure side and to form a high-speed water jet that impacts the rigid surface. As this jet impacts on the rigid plate, a ring bubble vortex is formed as shown. The maximum jet velocity obtained is around 70m/s. It scales as approximately 7 times the non-dimensional velocity scale based on the ambient(water) pressure and water density \( \sqrt{\frac{P_w}{\rho_w}} \) and this scaling is in good agreement with earlier results (e.g., Chahine and Perdue, 1988).

The effect of buoyancy in the formation of jet is very evident in Fig. 13 where the impact pressure on the wall is plotted versus time. The buoyancy aided case increases the impact pressure than for the buoyancy inhibited case and is as much as two times that of the peak explosion pressure.

The present study captures the vortex ring bubble after the jet impact, as shown above. This ring bubbles have been also observed both in experiments (e.g., Tomita and Shima, 1986; Vogel, et al., 1989) and in numerical studies (Best, 1993; Szmyczak, et al., 1992; Zhang and Duncan 1994).

4. CONCLUSIONS
These studies show that the ALE3D code can be used for bubble explosions. The basic code has been validated using shallow water explosion data. In addition to isolated bubbles, bubble-bubble and bubble-wall interaction studies were also performed. It has been shown that all the features observed in experiments have been captured in these studies. The formation of reentrant waterjet when the bubble collapses near a rigid surface and the formation of a ring vortex bubble have been captured and are in good agreement with experimental data.

Some limitations of the current ALE3D code have also been identified. For example, the current code is unable to simulate bubble-bubble interaction with a phase difference between the explosions. However, such features can be incorporated by proper modifications to the code.

ACKNOWLEDGMENTS
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Figure 1a: Freely oscillating bubble at 8ms (Near Maximum)

Figure 1b: Freely oscillating bubble at 15ms (Minimum)

Figure 2a: Vector field corresponding to Fig. 1a.

Figure 2b: Vector field corresponding to Fig. 1b.

Figure 3: $d^2r/dt^2$ versus time plot to identify the regions of R-T stable and unstable regions.

Figure 4: Time trace of pressure in the freely oscillating bubble (for near and far walls) compared with the experiment.
Figure 5: The acoustic pressure signature in the tank away from the bubble and near the walls. a) Numerical Simulation and b) Experiments (Menon and Lal, 1996).

Figure 6: Time sequence of two bubbles of same size interacting:
(a) At bubble maximum,
(b) Just before the jet formation
(c) Formation of the toroidal double ring bubble.

Figure 7: Velocity vectors for the cases discussed in fig. 6.
Figure 8: Snapshots of the bubbles in the expt. (Lal and Menon, 1996) for in-phase explosions of same size. a) Corresponds to bubble maximum, b) During Collapse & c) During rebound.

Figure 9: ALE3D Bubble shape(a). Corresponding to expt. (Lal and Menon) snapshot for out of phase explosion(b) and corresponding velocity field(c) at the time of jet formation for the bubbles of different sizes interacting.

Figure 10: Time trace of the pressure in between the bubbles for different cases of double bubble interactions.
Figure 11: Velocity field around a bubble collapsing near a wall (Buoyancy Inhibiting). a) At bubble maximum, b) & c) Just before the jet formation and d) After the toroidal bubble is formed during rebound.

Figure 12: Velocity field around the bubble collapsing near a rigid wall with buoyancy aiding the collapse.

Figure 13: Impact pressure on the wall for both the buoyancy aiding and inhibiting cases.
NUMERICAL STUDY OF BUBBLE COLLAPSE AND REBOUND NEAR A WALL

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ABSTRACT
The collapse and rebound of an explosion bubble near a wall are numerically investigated using an Arbitrary Lagrangian Eulerian three-dimensional code and the results are compared with the experimental data. The final stages of the collapse including the formation of a high velocity re-entrant jet are successfully captured in the simulations. The jet velocity and the impact pressure on the wall are functions of the explosion pressure and the distance of the bubble from the wall and to some extent on gravity. The results indicate that, for a given explosion pressure and initial conditions (gravity aided or inhibited), there is an optimal distance of the bubble from the wall, which gives the maximum impact pressure. This trend is in good agreement with the experimental results. The evolution of the vortex ring bubble, reported in earlier experimental and numerical studies is accurately predicted. The applicability of the available scaling laws for the time period and peak velocities is also reviewed.

INTRODUCTION:
The physics of cavitation erosion of marine propellers and turbine blades and the destruction caused by underwater explosions is of great practical interest but is not very well understood. Bubble (cavitation or explosion) collapse near a rigid wall is of significant interest due to a variety of reasons related to its ability to cause serious damage to the structure. This is because when the bubble collapses near a rigid surface, a strong reentrant jet is formed that is directed towards the wall. The peak impact pressure on the wall due to this water jet can be substantially higher than the initial explosion pressure (especially when it is very large). Detailed reviews (e.g., Blake and Gibson, 1987; Prosperetti, 1982) have summarized past experimental and numerical results. Experimental studies are too many to list completely; however, in general, only limited amount of information can be obtained due to various difficulties encountered in acquiring data. The most recent and notable experiments conducted to study bubble-wall interactions are those reported by Vogel et. al. (1989) and Tomita and Shima (1986). Recently, experiments were also carried out to investigate large-scale bubble explosions (Menon and Lal, 1996; Lal and Menon, 1996a, b). These experiments were conducted in shallow water due to interest in understanding the dynamics of bubble-wall interaction in such flows and to investigate the feasibility of targeting buried mines for destruction in beaches. The data obtained from these bubble-wall experiments is used in this paper to validate the numerical results.

Numerical studies in the past range from simple 1-D analytic solutions to more complex 2D/3D studies. Most of the earlier work has been done using the Boundary Element Methods (BEM) (e.g., Chahine and Perdue, 1988; Duncan and Zhang, 1991; Blake et. al., 1986; Wilkerson, 1989). The original BEM methodology cannot handle the phenomena of jet formation as the geometry changes from simple domain to a doubly connected domain. To overcome this short coming, many variants of the BEM methodology are used to capture the collapse and rebound of (e.g., Zhang and Duncan, 1994; Zhang et. al., 1993; Best and Kucera, 1992; Best, 1993) the bubble. Such procedures are often ad hoc and recourse to the experiments is required to model the problem correctly. Further they cannot be easily extended to other complex problems like the collapse of bubbles near deformable surface or collapse of non-spherical bubbles. Incompressible flow assumption made in these studies is not completely valid for the collapse of bubble near a wall as the pressures and velocities encountered are very high. The axisymmetric assumption also can not be generally used without apriori knowledge of the problem at hand.

Conventional Lagrangian or Eulerian techniques (even using full 3D) are also not practical, since the expansion and collapse of bubbles, the jet formation and the rebound of the ring vortex bubble create severe fluid motion. In a Lagrangian approach grid points move with the fluid, resulting in severe grid distortion (skewness, overlap of grid etc...), becomes inappropriate. In an Eulerian approach, adequate resolution in
the regions of interest (e.g., the bubble surface) is very difficult to achieve since the bubble's shape changes very rapidly.

A numerical method that includes both compressibility and an ability to capture the entire bubble collapse and rebound in complex configuration is used in this study. This numerical code (ALE3D), developed at the Lawrence Livermore National Laboratory, combines Lagrangian and eulerian features and is based on the Arbitrary Lagrangian-Eulerian (ALE) scheme. Past applications of this method include the 2D (e.g., Tipton et al., 1992) and the full 3D (Milligan et al., 1995 and Couch et al., 1996) studies of bubble collapse. This paper reports results of the interactions of the explosion bubbles with a rigid wall. The effects of various parameters are also analyzed.

**NUMERICAL METHODOLOGY**

ALE3D (Anderson et al., 1994) is an explicit, 3D finite element code that simulates the fluid motion and elastic-plastic response on an unstructured grid. The grid may consist of arbitrarily connected hexahedral shell and beam elements. The ALE algorithm is implemented by carrying out a complete Lagrangian calculation followed by an advection step. After each lagrangian step, a new mesh is created using a finite element based equitpotential method to relax the distorted grid. In the eulerian advection step, the fluid variables such as mass, density, energy, momentum and pressure are reevaluated on the new mesh by allowing fluid motion. The details of the constitutive models are described elsewhere (e.g., Steinberg, 1991) and, therefore, are not described here for brevity.

The advection step uses a second order, monotonic advection algorithm. This can create mixed material elements (i.e., liquid and vapor). Material interfaces are not explicitly tracked but for the purpose of carrying out mixed element advection, they are inferred from volume fractions. Separate state variables are kept for each component of a mixed element.

**RESULTS AND DISCUSSION**

In this section, the various test cases studied are summarized and specific trends are discussed.

The simulations employed here are similar to the experimental set-up of Lal and Menon (1996b). The experimental case is modeled as a bubble placed at a distance (d) above the wall in the vertical direction in a 1.5 m x 1.5 m x .75m tank filled with water. The initial diameter of the bubble is 6.34 cm and the initial explosion pressure is 13 atmospheres. The water pressure is 1 atmosphere. The experimental set-up employed a glass globe with a metal insert that contained pressure transducer and the spark generator, while these features were ignored in the numerical model. There are inherent limitations in the models employed for the equations of state for water and vapor. Further it is assumed that viscosity and surface tension are negligible. The ALE mesh treatment is applied to all the elements in the bubble, in the vicinity of the bubble and in between the bubble and the nearby wall. The other parts of the domain, where the bubble explosion and collapse does not cause much grid distortion, the standard lagrangian mesh treatment is used. The resolution of the grid is varied to ensure grid independence. The full 3D simulation of the bubble collapse against a rigid wall is carried out and the results compare well with the simulation using only one-quarter of the domain and using two symmetry planes. This indicates that the present problem is symmetric about the axis and the results reported here are from the reduced problem.

Three cases are analyzed here to highlight the various physical phenomena associated with the bubble collapse near a wall and its dependence on the proximity to the wall. These cases correspond to an initial bubble location of 5 cm, 6.34 cm and 4 cm above the wall (gravity inhibiting case as in the experiments), respectively. Figures 1a-c show the velocity field at various stages of the collapse for the first case. At the bubble maximum, the bubble is almost spherical but begins to distort as it collapses. Since there is less volume of water between the wall and the bubble during the collapse, the pressure drop is quite large relative to the drop on other sides of the bubble. This pressure differential further forces the bubble towards the wall. This migration of bubble causes the water surrounding the bubble to be directed off-center relative to the bubble geometric center, thereby, creating a higher pressure on the side of the bubble away from the wall. This results in the well known Bjerknes force. The iterative combination of these effects causes the water to penetrate the bubble from the high pressure side and to form a high-speed water jet that impacts the rigid surface. As this jet impacts the rigid plate, a toroidal vortex ring bubble is formed, as shown in the Fig. 3. This ring bubble qualitatively compares well with those observed in both experiments (e.g., Tomita and Shima, 1986; Vogel et al., 1989) and in numerical studies (Best, 1993; Szymczak et al., 1993; Zhang and Duncan, 1994). In Fig. 4, the two different views of the bubble are shown after the jet has penetrated it and compares well with the recent data reported in Jin et al. (1996).

In Fig. 5, the velocity fields for the cases 2 and 3 are shown at a time before the jet formation. Comparison of Fig. 1b and Fig. 5, indicates that for case 2, where the bubble is the farthest has the provision to collapse to a smaller volume and thus the higher velocities. In case 3, the bubble cannot collapse as much as the other two cases and thus, lower velocities and in turn the lower pressures are observed.

Figure 6 compares the time traces of peak impact pressure on the rigid wall for the three cases. The time periods scaled with non-dimensional time based on the maximum radius of the bubble, the ambient water pressure and the water density

\[ \frac{R_{\text{max}} \sqrt{\rho_w}}{\rho_w} \]

are 2.021, 2.017 and 2.048 respectively for the three cases. The peak velocities are observed slightly before the jet impacts the wall. These velocities scaled with non-dimensional velocity scale based on the ambient water pressure and water density (\(\sqrt{\rho_w} \)) are 6.9, 7 and 6.4 respectively.

These scaling laws compare well with earlier results (e.g., Chahine and Perdue, 1988; Blake et al., 1986). It is observed that even though maximum velocities are seen in case 2, it does not correspond to the maximum peak impact pressure as the jet formed loses some of its energy as it impinges on the wall through the water layer between the toroidal bubble and the wall. In case 1, the lower surface and the upper surfaces are attached to the wall before the impingement of the jet and thus, the jet does not transmit energy to the water layer as in case 2. This explains the higher impact pressures observed. The other fact to be noted in this figure is that the impact pressure on the wall due to the first pressure pulse is higher for case 3, and is inversely proportional to proximity of the bubble from the wall. The strength of the pressure wave generated by the bubble decreases radially outwards and thus, a greater impact pressure is felt on the wall in case 3. It can be seen that the time periods of
jet formation is not a strong function of the bubble distance from the wall, since the pressure peak is very steep, appropriately small time steps have to be taken to ensure numerical accuracy.

Figure 7 compares the variation of peak impact pressure (scaled with initial explosion pressure) versus separation distance (scaled with the initial radius of the bubble) of the present simulations to that of the experiments (Lal and Menon, 1996b). It can be seen clearly that the present numerical study captures the trends and the magnitudes of the peak impact pressures. The differences between the gravity inhibiting case and the experiment can be due to many reasons associated with the idealization of the experimental setup in addition to numerical errors. The gravity aided case (bubble placed below the wall) shows similar behavior to the gravity inhibited case and the impact pressure peaks at a distance slightly greater than for the other case. This follows the physical intuition that the buoyancy aids the migration of the bubble towards the wall and thus the peak shifts to the right.

Figure 8 shows the variation of impact pressure along the wall for case 1, just after the jet formation. This indicates that the jet is very narrow and even millimeters away from the jet center the pressure falls of very rapidly. This kind of behavior is very typical of stagnation point flows and makes it very difficult to simulate the process accurately, unless the jet is properly resolved.

CONCLUSIONS

The above studies demonstrate the potential of ALE3D code to capture the physical phenomena associated with the collapse and rebound of bubble near a rigid wall. The procedure adopted is general and the code is applicable to more complex cases than those discussed here. The results obtained are in good quantitative and qualitative agreement with the experiments and other numerical work. In addition the formation of re-entrant jet and a ring vortex bubble has been captured without any ad hoc procedure.

ACKNOWLEDGMENTS

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Figure 1: Velocity vector field around the bubble collapsing near wall (5 cm). a) Initial phase of collapse, b) & c) Just before the jet formation.
Figure 2: Pressure contours showing the pressure build on the side of the bubble away from the wall (case 1).

Figure 3: Velocity vector field in and around the vortex ring bubble formed during rebound.

Figure 4: Different views of the bubble just after the jet formation.

Figure 5: The velocity field in and around the bubble just before the jet formation.
   a) Case 2 and b) case 3
Figure 6: Comparison of the time traces of the impact pressure on the wall for different distances of the bubble center from the wall.

Figure 7: Variation of peak impact pressures on the wall with distance of the bubble from wall for simulations (gravity aiding and inhibiting) and experiments.

Figure 8: Variation of the impact pressure along the wall from the stagnation point of the reentrant jet showing the spread of impact.
INTERACTION OF TWO UNDERWATER EXPLOSION BUBBLES

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ABSTRACT

Underwater explosion bubbles are created by detonating a mixture of oxygen and Carbon Monoxide or Hydrogen in glass globes submerged in a water tank. A cinematographic technique is employed to capture entire interaction process in both horizontal and vertical configurations. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or they may violently repel each other. A violent interaction leads to an increased instability of the bubbles. When a coalesced bubble is formed by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble. Water free surface repels the bubble and the bubble migration speed and direction change smoothly as the explosion depth is continuously decreased.

INTRODUCTION

Much of the research activities in the area of underwater bubble dynamics has been focused on the behavior of cavitation bubbles. Cavitation bubble dynamics play a very important role in underwater acoustics and in predicting and preventing propeller and turbine blade damage. These bubbles, however, seldom occur singly. Actual cavitation fields contain several thousands of oscillating and translating microbubbles. Study of the behavior of a cloud of bubbles thus becomes inevitable and experimental (Chahine and Sirian, 1985), numerical (Chahine and Liu, 1985; Chahine, 1991; Chahine and Duraiswami, 1992) and analytical (Van Wijngaarden, 1972) techniques have all been developed. The simplest model that has been studied by the researchers is the interaction between two bubbles. Theoretical and numerical studies of the interaction of two spherical or nonspherical bubbles of same or different sizes (Fujikawa et al., 1985; Fujikawa and Takahira, 1986; Fujikawa and Takahira, 1988; Morioka, 1974; Shima, 1971) have been carried out. Interesting experimental observations of the interaction of a gas bubble with a pressure wave (Shima and Tomita, 1988) or with a vapor bubble (Smith and Mesler, 1972) have also been made.

Large bubbles, such as those formed during underwater explosion, owing to their tremendous inherent destructive capabilities upon collapse near a rigid boundary, find practical applications in underwater weaponry. These bubbles were recently simulated experimentally on a laboratory scale in a free field configuration (Menon and Lal, 1996).

The interaction of two underwater explosion bubbles is a very interesting and complex phenomenon, because of the fact that one bubble is influenced by time-delayed pressure or shock wave radiated from the adjacent bubble. Radial motion of the bubble may be greatly excited or subdued due to the interaction depending on their spatial and temporal separations. Though some experimental work have been done on the interaction of gas bubbles with two adjacent underwater explosion bubbles, and it has been shown that strong and complicated interactions ensue, it appears that no detailed results on the interaction of two underwater explosion bubbles have been published in the public domain literature.

This paper reports the results of the experiments carried out in a laboratory water tank to study the interaction between two adjacent bubbles created by underwater explosion of flammable gas contained in glass globes. The globes were placed side-by-side either in a horizontal or a vertical plane. The distance between the two globes and their sizes were both varied. This paper will also deal with the interaction of single and double explosion bubble(s) with the water free surface.

EXPERIMENTAL PROCEDURES

Underwater explosion experiments were conducted in a wooden tank of dimension 2 m x 1.5 m x 1.5 m, coated with fiberglass resin from inside. The tank, as shown in Fig. 1, has three windows on three sides. The underwater explosion bubble is generated by centrally igniting a mixture of an explosive gas (either Hydrogen or Carbon Monoxide) and oxygen contained in a hand-blown glass globe. Three different sizes of glass globes have been used for present experiments with average radii of 2.9 cm, 3.2 cm, and 3.8
cm. The glass globe, as shown in Fig. 2, has an electric spark ignition system connected to a 3000V DC power supply. The explosion takes place at a constant volume until the globe bursts. It has been shown (Menon and Lal, 1996) with the help of geometric and dynamic scaling rules that the explosion bubble thus formed is a reasonable subscale approximation of the deep sea underwater explosion bubble.

![Top View](image1)

**Fig. 1. Experimental setup**

The pressure responses in the water around the bubbles are recorded during the experiments by means of 4 KISTLER dynamic piezoelectric pressure transducers fitted at the ends of 4 stainless steel (1.27 cm diameter) tubes bent at right angle, as shown in Fig. 1. A hydrophone is also mounted in the tank and is used for measuring acoustic pressure. Pressure inside the bubble during its oscillation is measured by another KISTLER transducer which is mounted inside the plug, as shown in Fig. 2. Signals from these six pressure transducers and the hydrophone are digitized using National Instrument’s AT-MIO-16X analog-to-digital converter board, and are recorded into a microcomputer. Eight channel data recording are performed with a sustained sampling rate of 10,000 samples per second per channel.

The tank is illuminated by either direct overhead flood lights or an argon-ion laser sheet which lies in a vertical plane perpendicular to the camera axis. The optical recording of the bubble motion is performed by a CCD enhanced digital video camera with a maximum speed of 6000 frames per second. Since the viewable picture size is inversely proportional to the recording speed, the maximum speed was limited to 2000 frames per second as the image size reduces to half at this speed. Most of the experiments, however, were performed at the speed of 1000 frames per second (full screen image).

![Top view of the test plug](image2)

**Fig. 2. Glass bulb and test plug instrumentation**

The two glass globes are supported inside the tank by means of a modified sting which makes the pressure transducers mounted inside the two globes to face each other. This facilitates the direct measurement of the fluctuation in the pressure inside one bubble due to its interaction with the other. In order to provide a means for altering the distance between the two bubbles, six holes were drilled in the supporting copper pipe (1.6 cm diameter) of Fig. 1 at equal spatial separation from either ends. Experiments were conducted in primarily two configurations: a horizontal configuration, when the supporting pipe was horizontal, and a vertical configuration, when it was vertical. The former configuration prohibited the use of laser light sheet and only the flood lights were used for imaging, while the latter allowed the use of laser light sheet.

Experiments were also conducted to study the interaction of water free surface with the explosion bubble(s). The motivation for this kind of experiments has obvious reasons as detailed in the next section. The free surface provides a constant pressure boundary in close proximity of the oscillating bubble. It is known that the bubble moves away from the free surface and a reentrant jet is formed which pierces the bubble in the direction of its migration (Birkhoff, 1957; Blake and Gibson, 1961; Chahine, 1962; Wilkerson, 1969). Wilkerson (1969) developed a boundary integral technique for the analysis of expansion and collapse of an explosion bubble in free field, near a rigid surface, or near a free surface. To verify the accuracy of his method for predicting reentrant jet tip velocities for a bubble near a free surface, he compared his results with a PISCES code calculation and expressed his inability to predict an accurate estimation of the error involved in his method because of the unavailability of any such experimental data. This observed lack of data motivated the current experiments to study the interaction of a free surface with an explosion bubble.
RESULTS AND DISCUSSION

The interaction process is highly dependent on the time delays between the two explosions. These time delays are related to a variety of hardware and bubble response characteristics. The two globes have independent power supplies and there is always a time delay between the occurrence of the spark and when the bubble starts expanding. The bubble expansion occurs immediately after the glass globe bursts. The time delay depends primarily on the gas volume (globe size) and the nature of gas mixture inside the globe. A bigger globe size will create a larger time delay. Similarly, fuel-rich or fuel-lean mixtures will also create larger delays as compared to the one associated with a stoichiometric mixture. In addition to this delay, there is another time delay which is associated with the spark system itself. This delay is between the instant when the power is turned on and when the spark actually fires. This delay primarily depends on the actual gap between the two spark wires since a bigger gap creates a larger delay. The actual delay (temporal separation) between two explosions is therefore measured from the recorded video images as the time elapsed between the instant when the individual globe bursts. It was therefore deemed necessary to conduct several experiments to collect statistical information about the range of bubble behavior with respect to the delay between two explosions.

The entire spectrum of delay can be classified into two broad regions and they are called in-phase oscillation and out-of-phase oscillation (Morikoka, 1974; Shima, 1971; Smith and Mesler, 1972). In most of the past analytical, numerical or even experimental work on the interaction of two cavitation bubbles, interest has been focused primarily on the contraction phase of the bubble oscillation. This yields an in-phase oscillation of the bubbles as they both start collapsing at the same time following a sudden change in the ambient pressure. In-phase oscillation is obtained when there is strictly no delay between the explosions and both the bubbles start their oscillation cycle at the same time. Of course, it is assumed that the two bubbles have identical time period of oscillation and they are in identical phase at any instant throughout their oscillation cycle. This is the simplest scenario and has been analyzed comprehensively.

Another interesting scenario is 180° out-of-phase oscillation and it can be best understood in the interaction of two identical explosion bubbles as a case where one bubble starts its oscillation cycle when the other has already reached its maximum radius. In fact, these are the two scenarios predicted by the analytical theories (Morikoka, 1974). Theoretical analysis of natural frequencies of two pulsating bubbles predicts the existence of two natural frequencies corresponding to in-phase and 180° out-of-phase oscillations, respectively. Of course, in an experimental setup one can have any amount of delay between zero to 180°, or beyond.

The behavior of explosion bubbles under these two scenarios have long been predicted (Birkhoff, 1957; Bjerknes, 1966; Cole, 1948; Young, 1989) and it is called the laws of Bjerknes. Bjerknes bowed in 1868 that two spheres pulsating in-phase attract each other, and those pulsating 180° out-of-phase repel each other. Two spheres collapsing in-phase are equivalent to a single sphere collapsing near a rigid surface at a distance which is exactly equal to be half of the distance between two spheres. Similarly, two spheres oscillating 180° out-of-phase are equivalent to a single sphere pulsating near a free surface at a distance equal to half of the distance between two spheres. It has been shown (Birkhoff, 1957) that the migration speed of the bubble towards a rigid surface or away from a free surface is inversely proportional to \( r \), where \( r \) is the instantaneous bubble radius. Therefore, most of the migration would take place when the bubble radius is small (i.e., when the bubble is collapsing and approaching its minimum radius). Also, Bjerknes (Bjerknes, 1906; Young, 1989) showed as an analogy with gravitational and electromagnetic forces that the attractive force, \( F \), between two bubbles of volumes \( V_1 \) and \( V_2 \) a distance \( d \) apart in a pressure field is given by \( F = \frac{V_1 V_2 d^2}{2} \). Since the bubble migration velocity is directly proportional to the attractive force, its magnitude is expected to increase for bigger bubbles pulsating out-of-phase at a shorter inter-bubble distance, and decrease for smaller bubbles at a larger distance.

Fig. 3 shows a perfect example for two underwater explosion bubbles oscillating in-phase. The numbers in the parentheses denote the frame numbers, with frame number 1 corresponding to the instant when the sparks are visible for the first time. The initial volume of the right glass globe is 94 ml and that of the left glass globe is 90.5 ml. They are in a horizontal configuration and are initially separated by a distance \( d \), where \( d/R_o = 2.32 \). Here \( R_o \) is the initial bubble radius, which is taken to be the radius of the glass globe. Both are filled with stoichiometric mixture of Carbon Monoxide and Oxygen. It can be seen in Fig. 3 that there is virtually no delay between two explosions. Since the initial spatial separation between the globes was intentionally kept to a very small value so that a violent interaction can ensue, the bubbles soon come in contact with each other. They deviate more and more from sphericity as they expand with time. Fig. 4(a) shows how different radii of the left bubble change with time. A similar behavior is exhibited by the right bubble. The deviation from sphericity is more clearly demonstrated in Fig. 4(b), where the time history of the ratio of major and minor radius is plotted. As is evident from this figure, the bubbles quickly deviate from sphericity and the maximum deviation is attained at around frame no. 11. This deviation slowly diminishes and the bubbles become nearly spheric in shape at around frame no. 21. When the bubbles merge together to form a single coalesced bubble, it becomes difficult to isolate them individually.

Fig. 3. In-phase oscillation

The surface where the two bubbles come in contact in Fig. 3 grows in size as the bubbles grow and is initially curved. The bubbles reach their maximum radii at around frame no. 18 and start collapsing thereafter. The surface of contact slowly becomes a plane surface around frame no. 21 and remains so thereafter until the bubbles collapse near frame no. 28. This plane surface of contact may be considered as a rigid surface in an equivalent single bubble.
The collapse is very violent (as recorded by two plug transducers) and the coalesced bubble quickly disintegrates into a cloud of bubbles. The bubble contour is traced and 360 bubble radii are obtained at equal azimuth locations. A mean radius is obtained from this data, which is used to normalize the radii data. These data are then Fourier analyzed and the results are shown in Fig. 5, which shows the power spectral density of the coalesced bubble's interface at three instants: just after it is formed and 1 and 5 ms after it collapses. Here \( c \) is the bubble circumference and \( \lambda \) is the wavelength. The coalesced bubble is therefore very unstable.

The pressure traces measured around the bubbles show that the bubble behavior is symmetrical. A pressure fluctuation of about 700 kPa exhibited by the plug transducers near bubble minimum demonstrates the severity of the collapse of the jets formed in the two bubbles onto each other. The pressure drops exponentially as one moves away from the bubble (Cole, 1948). A pressure drop of 70\% (from 1000 kPa to 300 kPa) over a distance of 14 cm, and that of 80\% (from 1000 kPa to 200 kPa) over a distance of 34 cm from the bubble center have been recorded by tank transducers.

Fig. 5. Power spectral density of the bubble.
why the left bubble does not have sufficient energy to expand to its maximum radius. But, it certainly aids its predecessor to form a coalesced bubble with a greater energy to collapse violently and this fact is captured by all the transducers and hydrophone in the form of elevated collapse peaks. This time, the right plug transducer lies inside the coalesced bubble as it collapses (see Fig. 6, frame no. 27). The collapse pressure recorded by the transducer is very high (2500 kPa). The pressure traces recorded by the right and left transducers are once again almost identical, indicating a symmetrical bubble behavior.

Fig. 6. Bubble formation by merging energies.

Similar dynamic behavior is exhibited by the interaction of bubbles formed by glass globes of initially different sizes. It is not possible, however, to obtain an in-phase oscillation because of a simple reason that the two bubbles have different time periods of oscillation. On the other hand, the bubble dynamics observed where the coalesced bubble is formed by merging the energies of two bubbles have been exhibited in the bubbles of different sizes where the small bubble has been gobbled up by its predecessor large bubble. This kind of bubble dynamics is not feasible for large inter-bubble distance.

When the inter-globe distance is sufficiently large, the bubbles start repelling each other for a nonzero time delay. The repulsion force increases with the delay between the explosions, up to the point where they are oscillating 180° out-of-phase. In such cases, the formation of a strong reentrant water jet in the successor bubble is observed with increasing magnitude. Figure 7 shows how this reentrant jet travels with time for a case of 180° out-of-phase oscillation. Here x denotes the location of the jet tip relative to the inertial frame (the tank) and x = 0 corresponds to the instant when the jet tip becomes visible for the first time. As the phase delay between two explosions increase beyond 180°, the repulsion force as well as the water jet velocity decrease in magnitude. If the phase delay between two explosions increase beyond 360° (i.e., if one bubble has already completed one oscillation cycle when the other bubble forms), the resulting interaction is very weak. In this case, though the predecessor bubble manages to create a depression in successor bubble at its maximum radius, neither a jet nor a repulsion force is formed.

The horizontal configuration is very important from a practical standpoint, as it can set a catastrophic bending vibration to a nearby rigid body if tuned properly. The research effort is being pursued in this direction. The vertical configuration is also equally important, as it can dramatically enhance the impact pressure of a single bubble when collapsed near a rigid body. If two bubbles are placed close to each other along an axis perpendicular to a nearby rigid body, and if these two bubbles are tuned to oscillate 180° out-of-phase with each other, a water jet will be formed and directed towards the rigid surface with a velocity which will be higher than that formed by the collapse of a single bubble under similar circumstances. This is also being studied.

The bubble dynamics in vertical configuration in the present study (in the absence of any nearby rigid body), however, does not show a remarkable difference from that of horizontal counterpart. This is because of the fact that only the orientation of the gravitational force changes between two configurations. In practical cases, the force field generated by the bubbles is much more stronger to be affected negligibly by the gravitational force.

![Fig. 7. Water jet tip location and velocity](image)

**Bubble(s)-Free Surface Interaction**

Since it was shown earlier that two bubbles oscillating 180° out-of-phase are equivalent to a single bubble oscillating near a free surface, experiments were carried out to study the interaction of a free surface with explosion bubble(s). Figure 8 shows the transition of bubble migration velocity with water depth.

![Fig. 8. Bubble migration velocity with water depth](image)
A simple sting mount was used to support a glass globe and the depth of water in the tank was decreased in a step of 2.54 cm. It was found that the bubble migration velocity smoothly changes its direction as well as its magnitude. The transition point determines the maximum inter-bubble distance \( 2d \) for which two identical bubbles will start interacting while oscillating 180° out-of-phase with each other. It is found that \( d = 3.4R_b \).

Both out-of-phase and in-phase oscillation of two explosion bubbles in a horizontal configuration near a free surface were also studied. The bubbles oscillating out-of-phase with each other repel each other and the effect of the free surface becomes apparent only after they have been repelled by each other. This once again indicates that the force field generated by the interaction between two bubbles is much stronger than that of a free surface and there have been instances where one of the bubbles actually migrates upward.

CONCLUSIONS

This paper discusses results obtained in an experimental investigation of the dynamical interaction of two underwater explosion bubbles in both horizontal and vertical configurations. The former configuration can excite a nearby submerged structure in a bending vibration mode, while the latter can easily be tailored for the directionality and enhancement of the impact pressure resulting from the collapse of an underwater explosion bubble near a solid boundary. Depending on the delay between two explosions and inter-bubble distance, the bubbles may either attract each other to form a single coalesced bubble, or they may violently repel each other. A violent interaction between the bubbles leads to an increased instability of the bubbles. If a coalesced bubble is formed by merging the energies of two bubbles, the resulting bubble has more residual energy and is more stable for successive oscillations. An out-of-phase oscillation generates a reentrant water jet which pierces the bubble. Water free surface repels the bubble and the transition point of bubble migration velocity determines the maximum inter-bubble distance required for the initiation of interaction between two out-of-phase pulsating bubbles.

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